Lepton Mixing and Neutrino Mass

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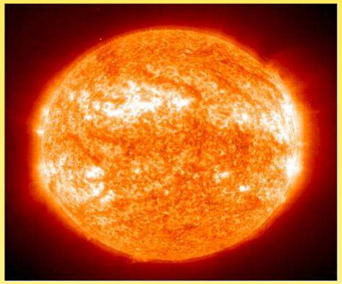
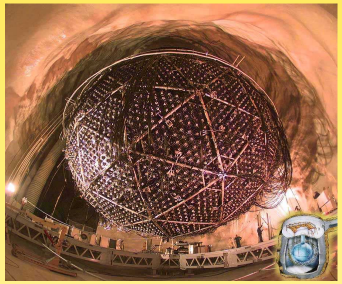
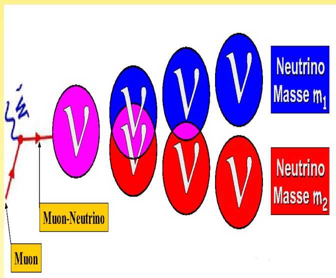
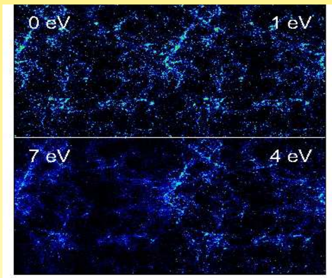
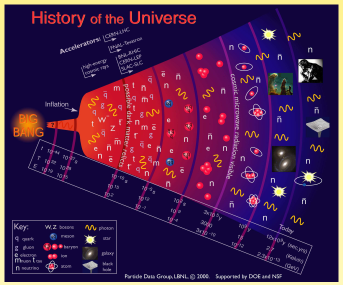
MANITOP
Massive Neutrinos: Investigating their Theoretical Origin and Phenomenology

$m_\nu = m_L - m_D^T M_R^{-1} m_D$





Neutrinos!

Literature

- ArXiv:
  - Bilenky, Giunti, Grimus: *Phenomenology of Neutrino Oscillations*, `hep-ph/9812360`
  - Akhmedov: *Neutrino Physics*, `hep-ph/0001264`
  - Grimus: *Neutrino Physics – Theory*, `hep-ph/0307149`
- Textbooks:
  - Fukugita, Yanagida: *Physics of Neutrinos and Applications to Astrophysics*
  - Kayser: *The Physics of Massive Neutrinos*
  - Giunti, Kim: *Fundamentals of Neutrino Physics and Astrophysics*
  - Schmitz: *Neutrinophysik*

Contents

I Basics

Contents

II Neutrino Oscillations

Contents

III Neutrino Mass

Contents

I Basics

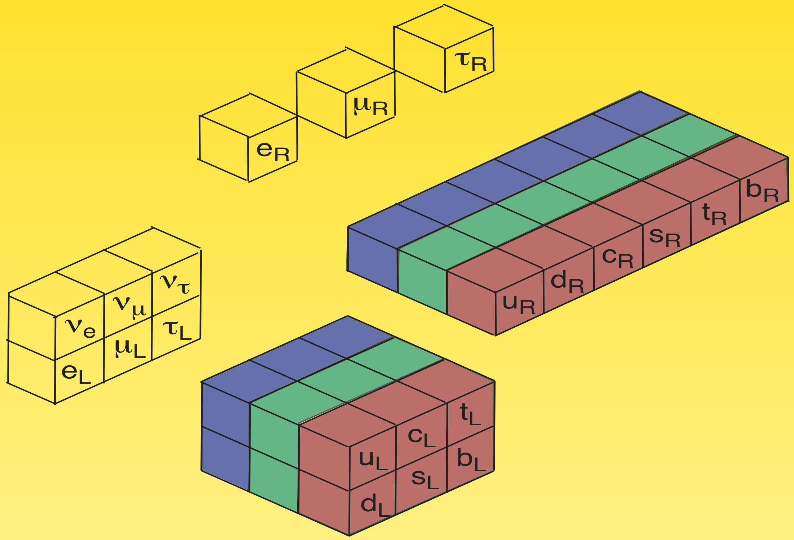
I1) **Introduction**

I2) **History of the neutrino**

I3) **Fermion mixing, neutrinos and the Standard Model**

I1) Introduction

Standard Model of Elementary Particle Physics: $SU(3)_C \times SU(2)_L \times U(1)_Y$

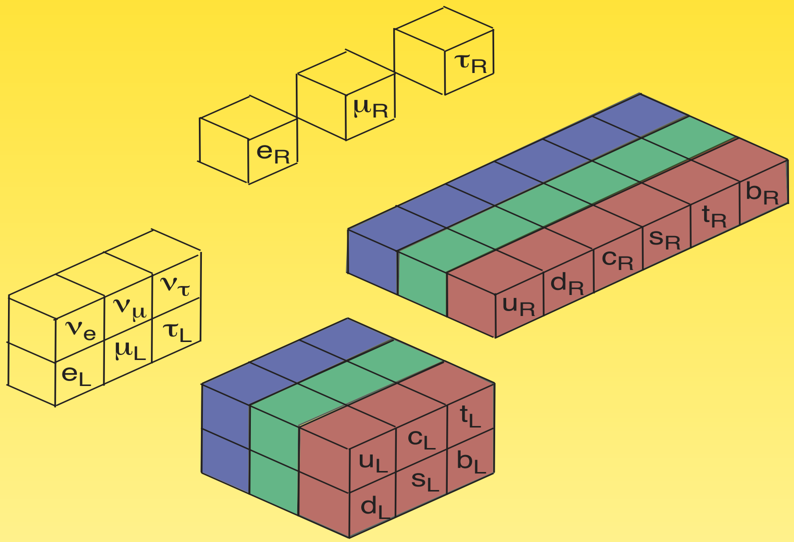


Species	#	$\sum$
Quarks	10	10
Leptons	3	13
Charge	3	16
Higgs	2	18

18 free parameters...

+ Gravitation
+ Dark Energy
+ Dark Matter
+ Baryon Asymmetry

Standard Model of Elementary Particle Physics: $SU(3)_C \times SU(2)_L \times U(1)_Y$



Species	#	$\sum$
Quarks	10	10
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+ Neutrino Mass m_ν

Standard Model* of Particle Physics

add neutrino mass matrix m_ν (and a new energy scale?)

Species	#	$\sum$
Quarks	10	10
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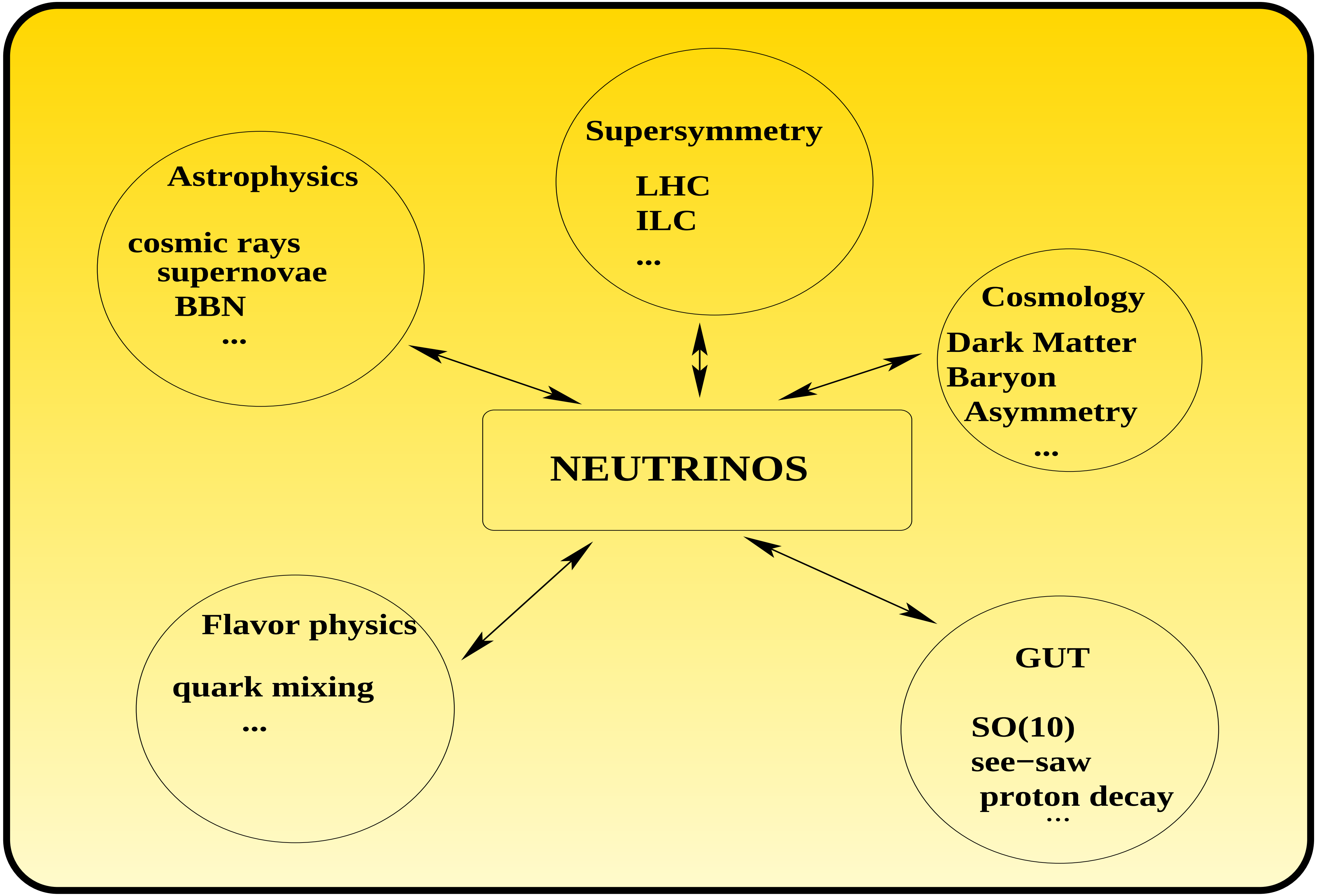
Standard Model* of Particle Physics

add neutrino mass matrix m_ν (and a new energy scale?)

Species	#	$\sum$
Quarks	10	10
Leptons	3	13
Charge	3	16
Higgs	2	18

$\longrightarrow$

Species	#	$\sum$
Quarks	10	10
Leptons	12 (10)	22 (20)
Charge	3	25 (23)
Higgs	2	27 (25)


Supersymmetry
LHC
ILC
...
Astrophysics
cosmic rays
supernovae
BBN
...
Cosmology
Dark Matter
Baryon
Asymmetry
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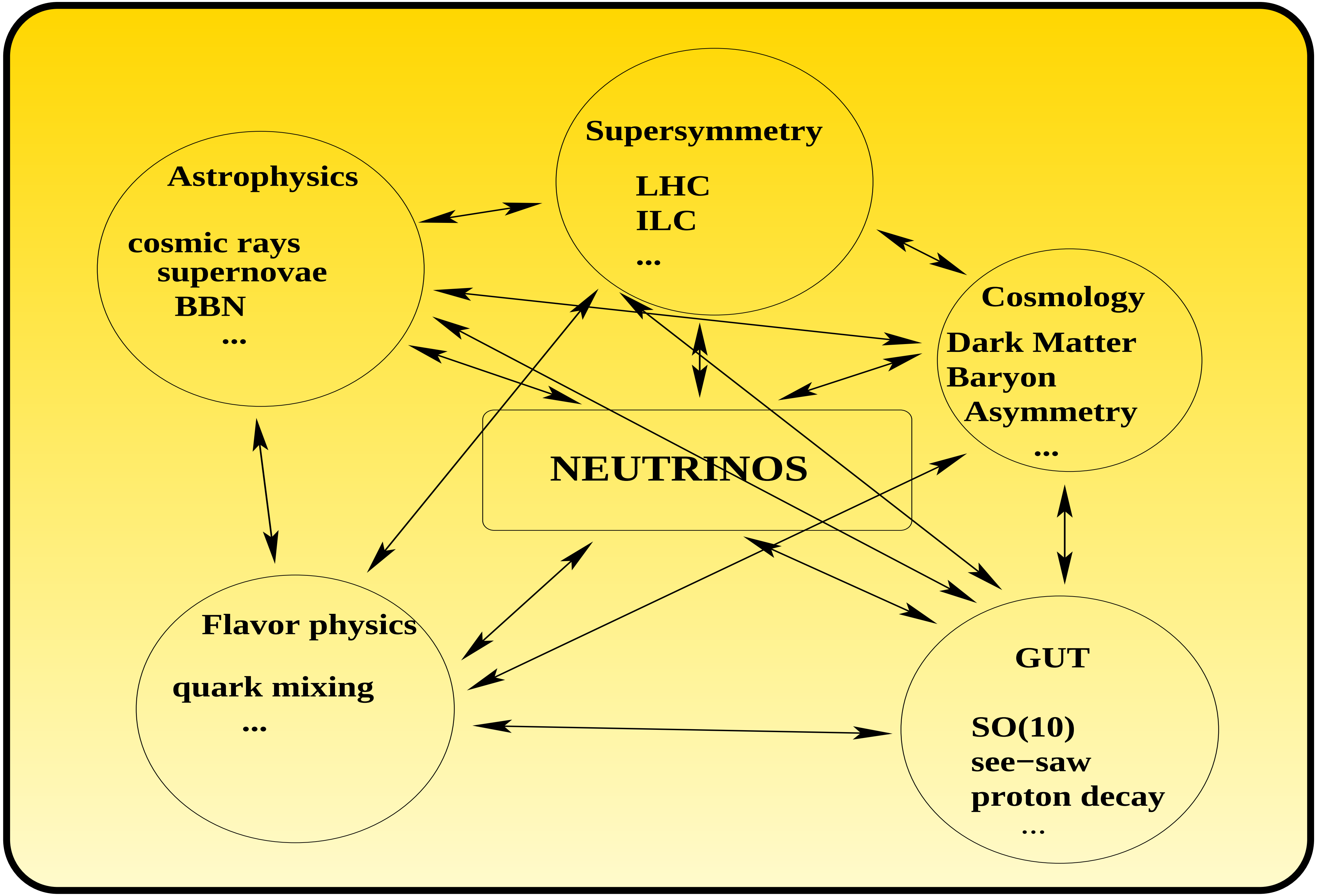
NEUTRINOS
Flavor physics
quark mixing
...
GUT
SO(10)
see−saw
proton decay
...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General Remarks

- Neutrinos interact weakly: can probe things not testable by other means
  - solar interior
  - geo-neutrinos
  - cosmic rays
- Neutrinos have no mass in SM
  - probe scales $m_\nu \propto 1/\Lambda$
  - happens in GUTs
  - connected to new concepts, e.g. new representations of SM gauge group, Lepton Number Violation

$\Rightarrow$ particle and source physics

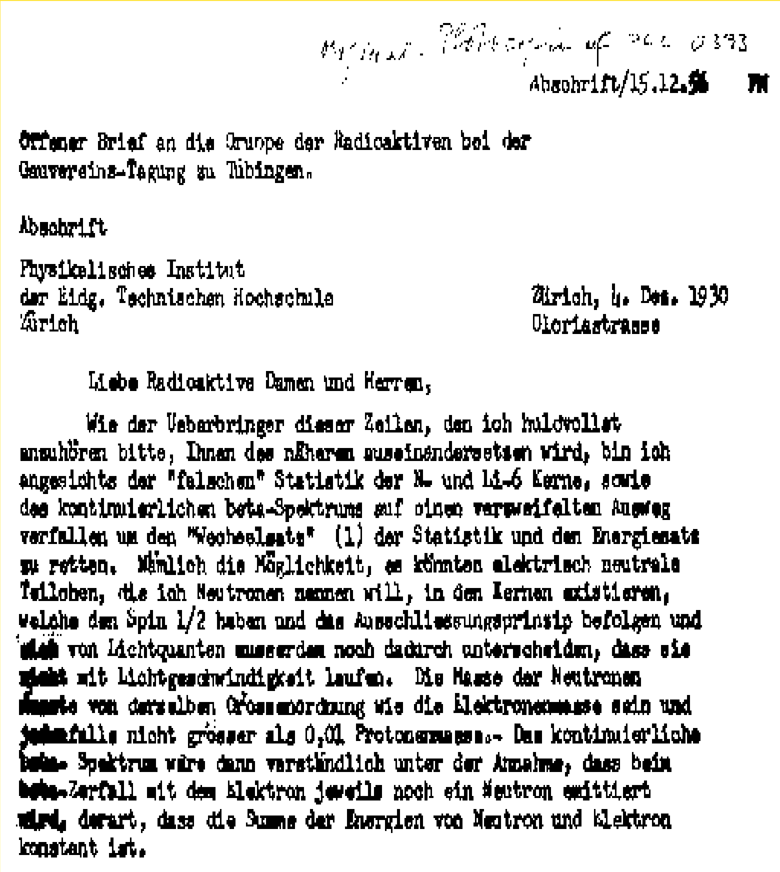
Contents

I Basics

I2) History

1926 problem in spectrum of β-decay

1930 Pauli postulates “neutron”



Abschrift/15.12.56 PM

Offener Brief an die Gruppe der Radioaktiven bei der Gauvereins-Tagung zu Tübingen.

Abschrift

Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dez. 1930
Gloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grössenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist.

1932 Fermi theory of β-decay

1956 discovery of $\bar{\nu}_e$ by Cowan and Reines (NP 1985)

1957 Pontecorvo suggests neutrino oscillations

1958 helicity $h(\nu_e) = -1$ by Goldhaber $\Rightarrow V - A$

1962 discovery of ν_μ by Lederman, Steinberger, Schwartz (NP 1988)

1970 first discovery of solar neutrinos by Ray Davis (NP 2002); solar neutrino problem

1987 discovery of neutrinos from SN 1987A (Koshiba, NP 2002)

1991 $N_\nu = 3$ from invisible Z width

1998 SuperKamiokande shows that atmospheric neutrinos oscillate

2000 discovery of ν_τ

2002 SNO solves solar neutrino problem

2010 the third mixing angle

Future History

20ab CP shown to be violated

20cd mass ordering determined

20ef Dirac/Majorana nature shown

20gh value of neutrino mass measured

20ij new physics discovered

20kl ???

Contents

I Basics

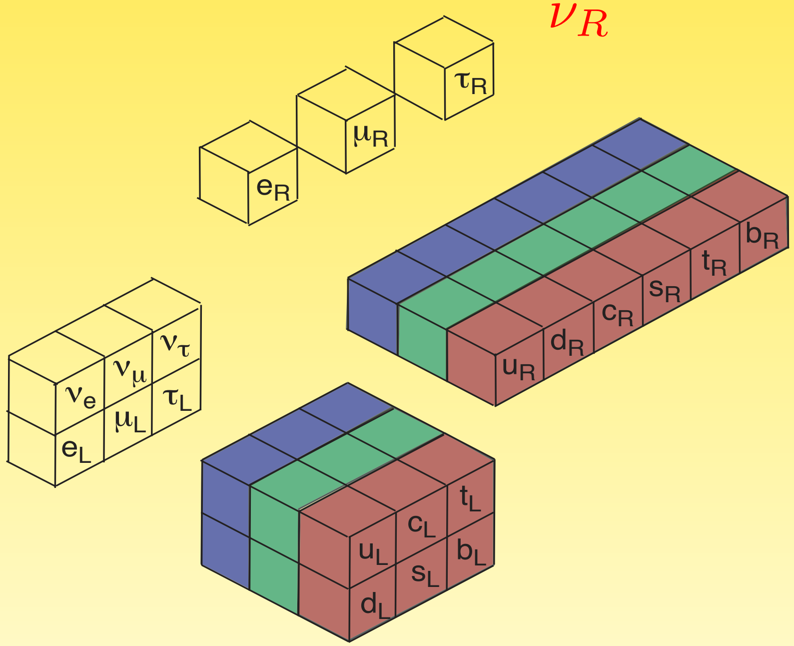
I3) Neutrinos and the Standard Model

$SU(3)_c \times SU(2)_L \times U(1)_Y \to SU(3)_c \times U(1)_{em}$ with $Q = I_3 + \frac{1}{2}\, Y$

$$L_e = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad \sim (2,-1)$$

$$e_R \qquad \sim (1,-2)$$

$$\nu_R \qquad \sim (1,0)$$ total SINGLET!!



Masses in the SM:

$$-\mathcal{L}_Y = g_e\,\overline{L}\,\Phi\,e_R + g_\nu\,\overline{L}\,\tilde{\Phi}\,\nu_R + h.c.$$

with

$$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \text{and} \quad \tilde{\Phi} = i\tau_2\,\Phi^* = i\tau_2 \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}^* = \begin{pmatrix} \phi^0 \\ -\phi^+ \end{pmatrix}^*$$

after EWSB: $\langle\Phi\rangle \to (0, v/\sqrt{2})^T$ and $\langle\tilde{\Phi}\rangle \to (v/\sqrt{2}, 0)^T$

$$-\mathcal{L}_Y = g_e\,\frac{v}{\sqrt{2}}\,\overline{e_L}\,e_R + g_\nu\,\frac{v}{\sqrt{2}}\,\overline{\nu_L}\,\nu_R + h.c. \equiv m_e\,\overline{e_L}\,e_R + m_\nu\,\overline{\nu_L}\,\nu_R + h.c.$$

$\Leftrightarrow$ in a renormalizable, lepton number conserving model with Higgs doublets the absence of ν_R means absence of m_ν

Quark Mixing

$$-\mathcal{L}_{\rm CC} = \frac{g}{\sqrt{2}} W_\mu^+ \, \overline{u_L} \, \gamma^\mu \, V \, d_L$$

with $u = (u, c, t)^T$, $d = (d, s, b)^T$

Quarks have masses $\leftrightarrow$ they mix

$\leftrightarrow$ mass matrices need to be diagonalized

$\leftrightarrow$ flavor states not equal to mass states

results in non-trivial CKM (Cabibbo-Kobayashi-Maskawa) matrix:

$$|V| = \left(\begin{array}{ccc} 0.97427 \pm 0.00015 & 0.22534 \pm 0.00065 & 0.00351^{+0.00015}_{-0.00014} \\ 0.22520 \pm 0.00065 & 0.97344^{+0.00016}_{-0.00016} & 0.0412^{+0.0011}_{-0.0005} \\ 0.00867^{+0.00029}_{-0.00031} & 0.0404^{+0.0011}_{-0.0005} & 0.999146^{+0.000021}_{-0.000046} \end{array} \right)$$

small mixing related to hierarchy of masses?

Contents

II Neutrino Oscillations

II1) The PMNS matrix

Analogon of CKM matrix:

$$-\mathcal{L}_{\rm CC} = \frac{g}{\sqrt{2}} \, \overline{\ell_L} \, \gamma^\mu \, U \, \nu_L \, W_\mu^-$$

Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix

$$\nu_\alpha = U_{\alpha i}^* \, \nu_i$$

connects flavor states ν_α $(\alpha = e, \mu, \tau)$ to mass states ν_i $(i = 1, 2, 3)$

mass terms go like $\overline{\ell_L} \, \ell_R$ and $\overline{\nu_L} \, \nu_R$ (?)

(flavor = mass if $m_\nu = 0 \Rightarrow U = \mathbb{1}$)

Number of parameters in U for N families:

complex $N \times N$	$2\,N^2$	$2\,N^2$
unitarity	$-N^2$	N^2
rephase ν_i, ℓ_i	$-(2\,N-1)$	$(N-1)^2$

a real matrix would have $\frac{1}{2}\,N\,(N-1)$ rotations around ij-axes

in total:

families	angles	phases
2	1	0
3	3	1
4	6	3
N	$\frac{1}{2}\,N\,(N-1)$	$\frac{1}{2}\,(N-2)\,(N-1)$

this assumes $\bar{\nu}\nu$ mass term, what if **Majorana mass term** $\nu^T\nu$?

Number of parameters in U for N families:

complex $N \times N$	$2\,N^2$	$2\,N^2$
unitarity	$-N^2$	N^2
rephase ℓ_α	$-N$	$N(N-1)$

a real matrix would have $\frac{1}{2}\,N\,(N-1)$ rotations around ij-axes

in total:

families	angles	phases	extra phases
2	1	1	1
3	3	3	2
4	6	6	3
N	$\frac{1}{2}\,N\,(N-1)$	$\frac{1}{2}\,N\,(N-1)$	$N-1$

Extra $N-1$ "Majorana phases" because of mass term $\nu^T\nu$

(absent for Dirac neutrinos)

Majorana Phases

- connected to Majorana nature, hence to Lepton Number Violation
- I can always write: $U = \tilde{U}\,P$, where all Majorana phases are in $P = \mathrm{diag}(1, e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}, \ldots)$:
- 2 families:

$$U = \left(\begin{array}{cc} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{array} \right) \left(\begin{array}{cc} 1 & 0 \\ 0 & e^{i\alpha} \end{array} \right)$$

- 3 families: $U = R_{23}\,\tilde{R}_{13}\,R_{12}\,P$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\,e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\,e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} P$$

$$= \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{i\delta} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{-i\delta} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{-i\delta} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{-i\delta} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{-i\delta} & c_{23}\,c_{13} \end{pmatrix} P$$

with $P = \mathrm{diag}(1, e^{i\alpha}, e^{i\beta})$

Contents

II Neutrino Oscillations

II2) Neutrino Oscillations (in Vacuum)

Neutrino produced with charged lepton α is **flavor state**

$$|\nu(0)\rangle = |\nu_\alpha\rangle = U^*_{\alpha j}\,|\nu_j\rangle$$

evolves with time as

$$|\nu(t)\rangle = U^*_{\alpha j}\,e^{-i\,E_j\,t}\,|\nu_j\rangle$$

amplitude to find state $|\nu_\beta\rangle = U^*_{\beta i}\,|\nu_i\rangle$:

$$\mathcal{A}(\nu_\alpha \to \nu_\beta, t) = \langle\nu_\beta|\nu(t)\rangle = U_{\beta i}\,U^*_{\alpha j}\,e^{-i\,E_j\,t}\quad \underbrace{\langle\nu_i|\nu_j\rangle}_{\delta_{ij}}$$

$$= U^*_{\alpha i}\,U_{\beta i}\,e^{-i\,E_i\,t}$$

Probability:

$$P(\nu_\alpha \to \nu_\beta, t) \equiv P_{\alpha\beta} = |\mathcal{A}(\nu_\alpha \to \nu_\beta, t)|^2$$

$$= \sum_{ij} \underbrace{U^*_{\alpha i}\, U_{\beta i}\, U^*_{\beta j}\, U_{\alpha j}}_{\mathcal{J}^{\alpha\beta}_{ij}} \quad \underbrace{e^{-i\,(E_i - E_j)\,t}}_{e^{-i\Delta_{ij}}}$$

$= \ldots =$ **EXERCISE!**

$$\boxed{P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{j>i} \mathrm{Re}\{\mathcal{J}^{\alpha\beta}_{ij}\}\, \sin^2 \frac{\Delta_{ij}}{2} + 2 \sum_{j>i} \mathrm{Im}\{\mathcal{J}^{\alpha\beta}_{ij}\} \sin \Delta_{ij}}$$

with phase

$$\tfrac{1}{2}\Delta_{ij} = \tfrac{1}{2}\,(E_i - E_j)\,t \simeq \tfrac{1}{2}\left(\sqrt{p_i^2 + m_i^2} - \sqrt{p_j^2 + m_j^2}\right) L$$

$$\simeq \tfrac{1}{2}\left(p_i\,(1 + \tfrac{m_i^2}{2p_i^2}) - p_j\,(1 + \tfrac{m_j^2}{2p_j^2})\right) L \simeq \tfrac{m_i^2 - m_j^2}{4E} L$$

$$\boxed{\frac{1}{2}\Delta_{ij} = \frac{m_i^2 - m_j^2}{4\,E}\, L \simeq 1.27 \left(\frac{\Delta m^2_{ij}}{\mathrm{eV}^2}\right) \left(\frac{L}{\mathrm{km}}\right) \left(\frac{\mathrm{GeV}}{E}\right)}$$

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{j>i} \mathrm{Re}\{\mathcal{J}_{ij}^{\alpha\beta}\}\, \sin^2\frac{\Delta_{ij}}{2} + 2\sum_{j>i} \mathrm{Im}\{\mathcal{J}_{ij}^{\alpha\beta}\} \sin\Delta_{ij}$$

- $\alpha = \beta$: survival probability
- $\alpha \neq \beta$: transition probability
- requires $U \neq \mathbb{1}$ and $\Delta m_{ij}^2 \neq 0$
- $\sum_\alpha P_{\alpha\beta} = 1 \leftrightarrow$ conservation of probability
- $\mathcal{J}_{ij}^{\alpha\beta}$ invariant under $U_{\alpha j} \to e^{i\phi_\alpha}\, U_{\alpha j}\, e^{i\phi_j}$

  $\Rightarrow$ Majorana phases drop out!

CP Violation

In oscillation probabilities: $U \to U^*$ for anti-neutrinos

Define asymmetries:

$$\Delta_{\alpha\beta} = P(\nu_\alpha \to \nu_\beta) - P(\overline{\nu}_\alpha \to \overline{\nu}_\beta) = P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha)$$
$$= 4 \sum_{j>i} \mathrm{Im}\{\mathcal{J}_{ij}^{\alpha\beta}\} \sin \Delta_{ij}$$

- 2 families: U is real and $\mathrm{Im}\{\mathcal{J}_{ij}^{\alpha\beta}\} = 0 \; \forall \alpha, \beta, i, j$
- 3 families:

$$\Delta_{e\mu} = -\Delta_{e\tau} = \Delta_{\mu\tau} = \left(\sin \frac{\Delta m_{21}^2}{2\,E} L + \sin \frac{\Delta m_{32}^2}{2\,E} L + \sin \frac{\Delta m_{13}^2}{2\,E} L \right) J_{\mathrm{CP}}$$

where $J_{\mathrm{CP}} = \mathrm{Im}\left\{ U_{e1}\, U_{\mu 2}\, U_{e2}^*\, U_{\mu 1}^* \right\}$

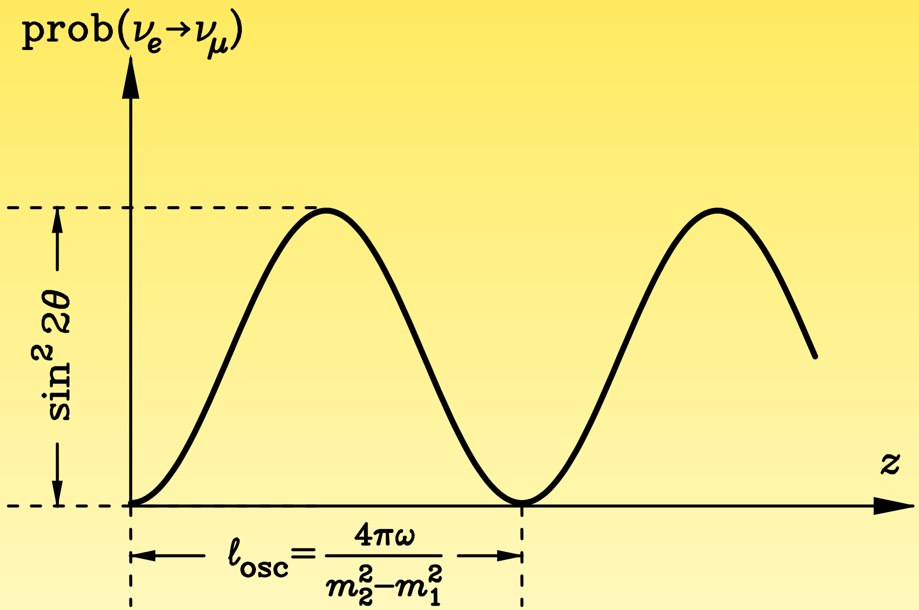
$$= \tfrac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta$$

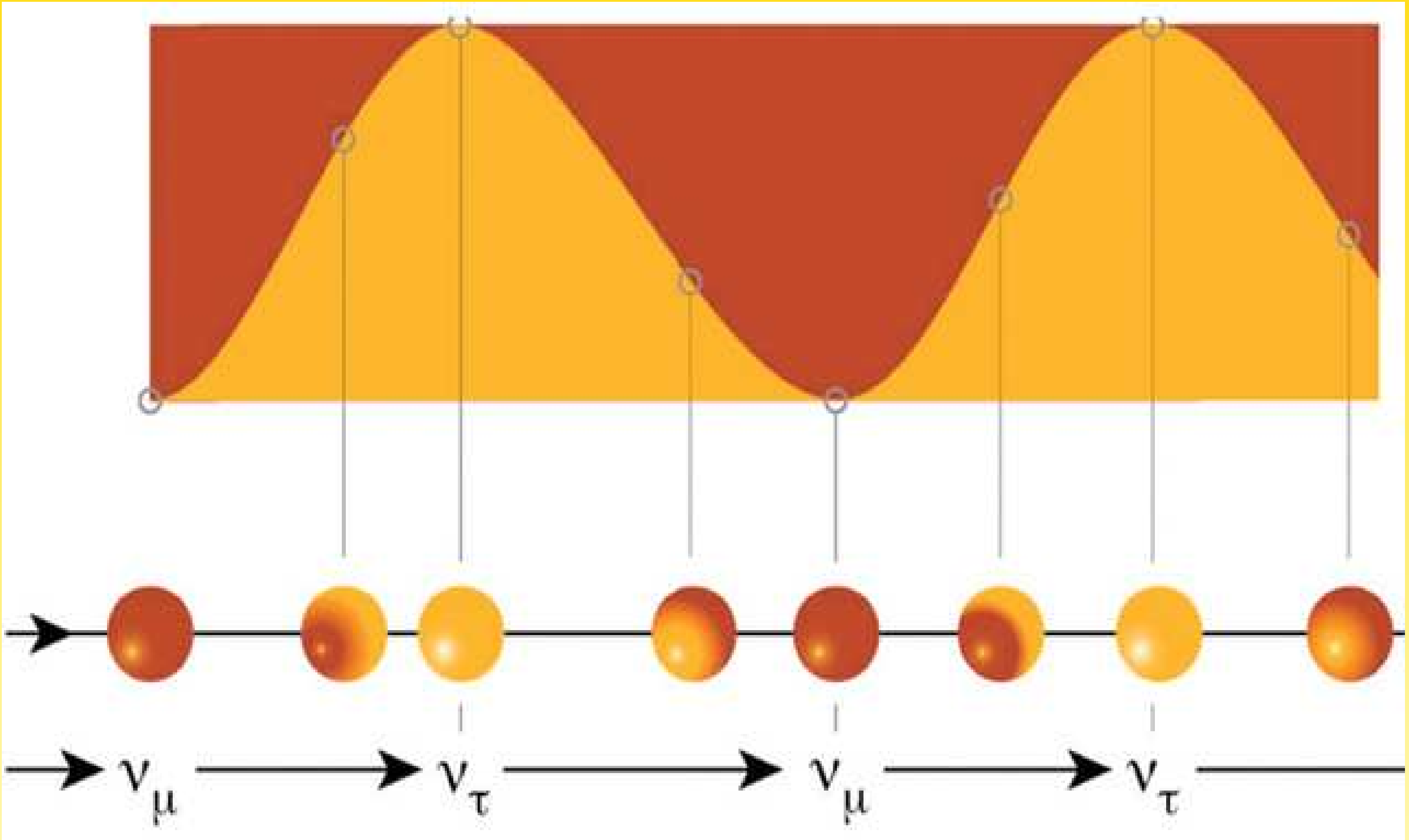
vanishes for one $\Delta m_{ij}^2 = 0$ or one $\theta_{ij} = 0$ or $\delta = 0, \pi$

Two Flavor Case

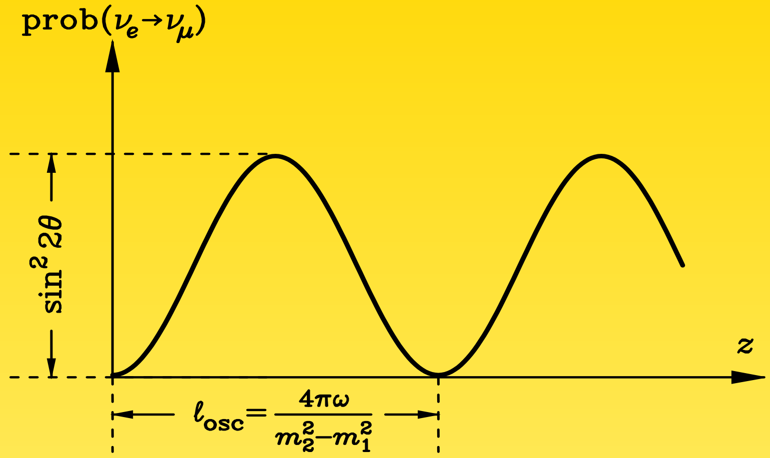
$$U = \begin{pmatrix} U_{\alpha 1} & U_{\alpha 2} \\ U_{\beta 1} & U_{\beta 2} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \Rightarrow \mathcal{J}_{12}^{\alpha\alpha} = |U_{\alpha 1}|^2 |U_{\alpha 2}|^2 = \frac{1}{4}\sin^2 2\theta$$

and transition probability is $$P_{\alpha\beta} = \sin^2 2\theta \, \sin^2 \frac{\Delta m_{21}^2}{4\,E} L$$





ν_μ
ν_τ
ν_μ
ν_τ

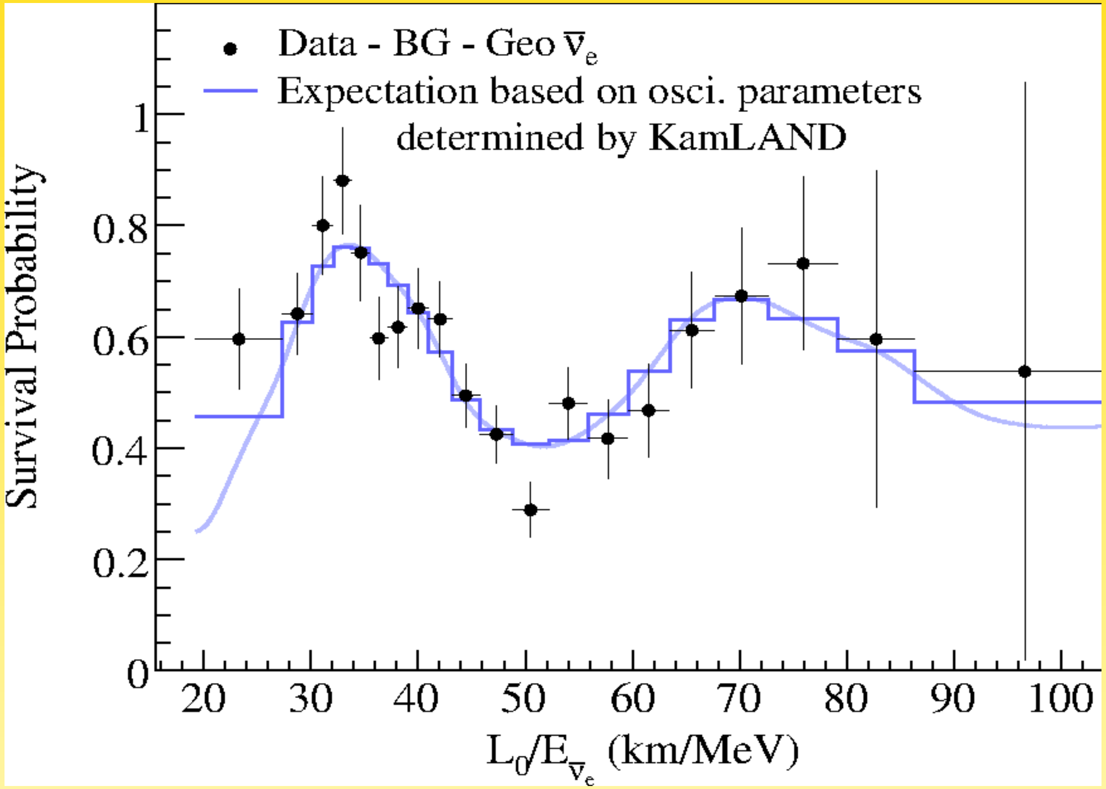


- amplitude $\sin^2 2\theta$
- maximal mixing for $\theta = \pi/4 \Rightarrow \nu_\alpha = \sqrt{\frac{1}{2}}\,(\nu_1 + \nu_2)$
- oscillation length $L_{\rm osc} = 4\pi\, E/\Delta m_{21}^2 = 2.48\, \frac{E}{\rm GeV}\, \frac{\rm eV^2}{\Delta m_{21}^2}$ km

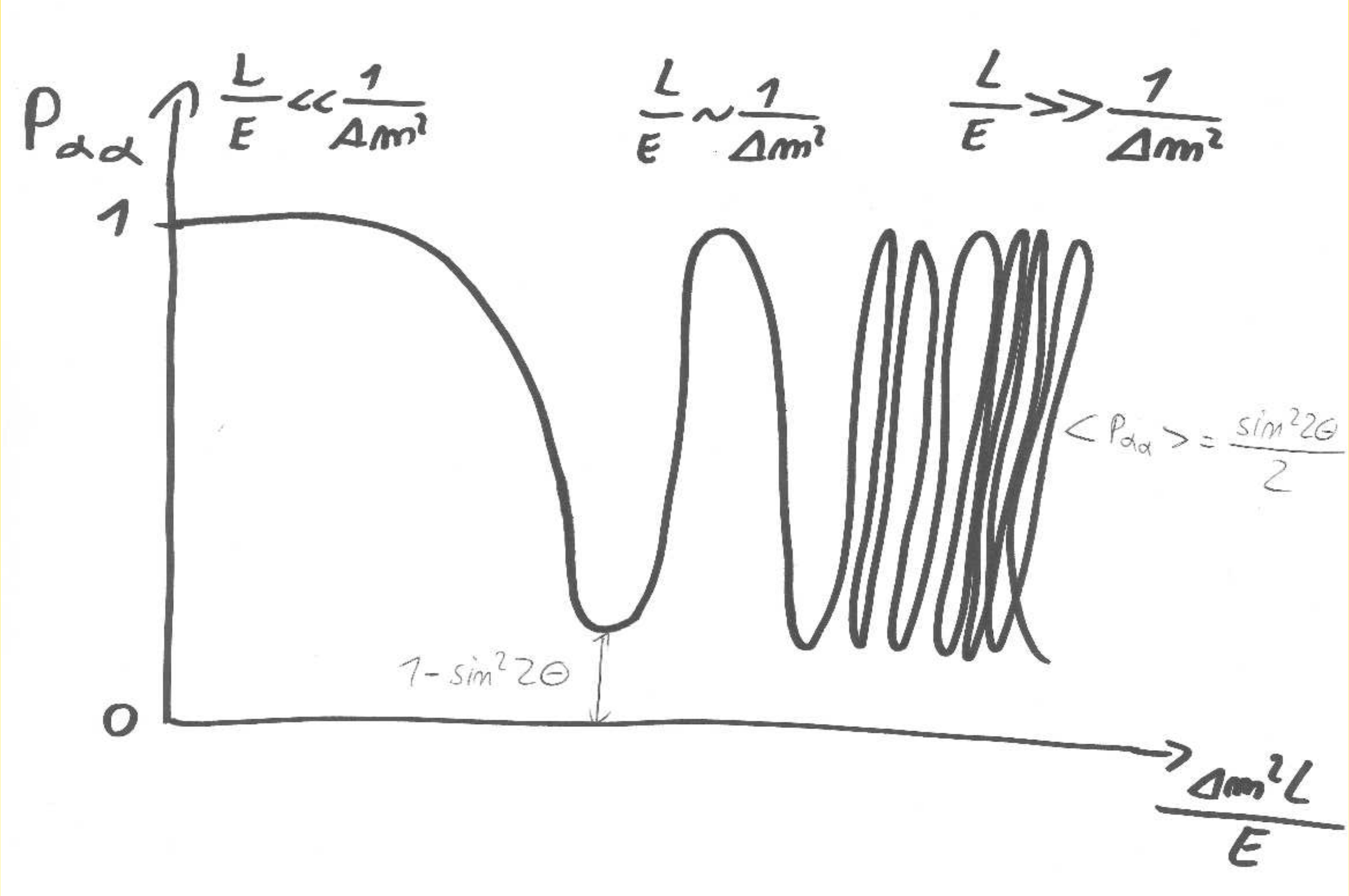
  $\Rightarrow P_{\alpha\beta} = \sin^2 2\theta\, \sin^2 \pi \frac{L}{L_{\rm osc}}$

  is distance between two maxima (minima)

  e.g.: $E =$ GeV and $\Delta m^2 = 10^{-3}$ eV^2: $L_{\rm osc} \simeq 10^3$ km



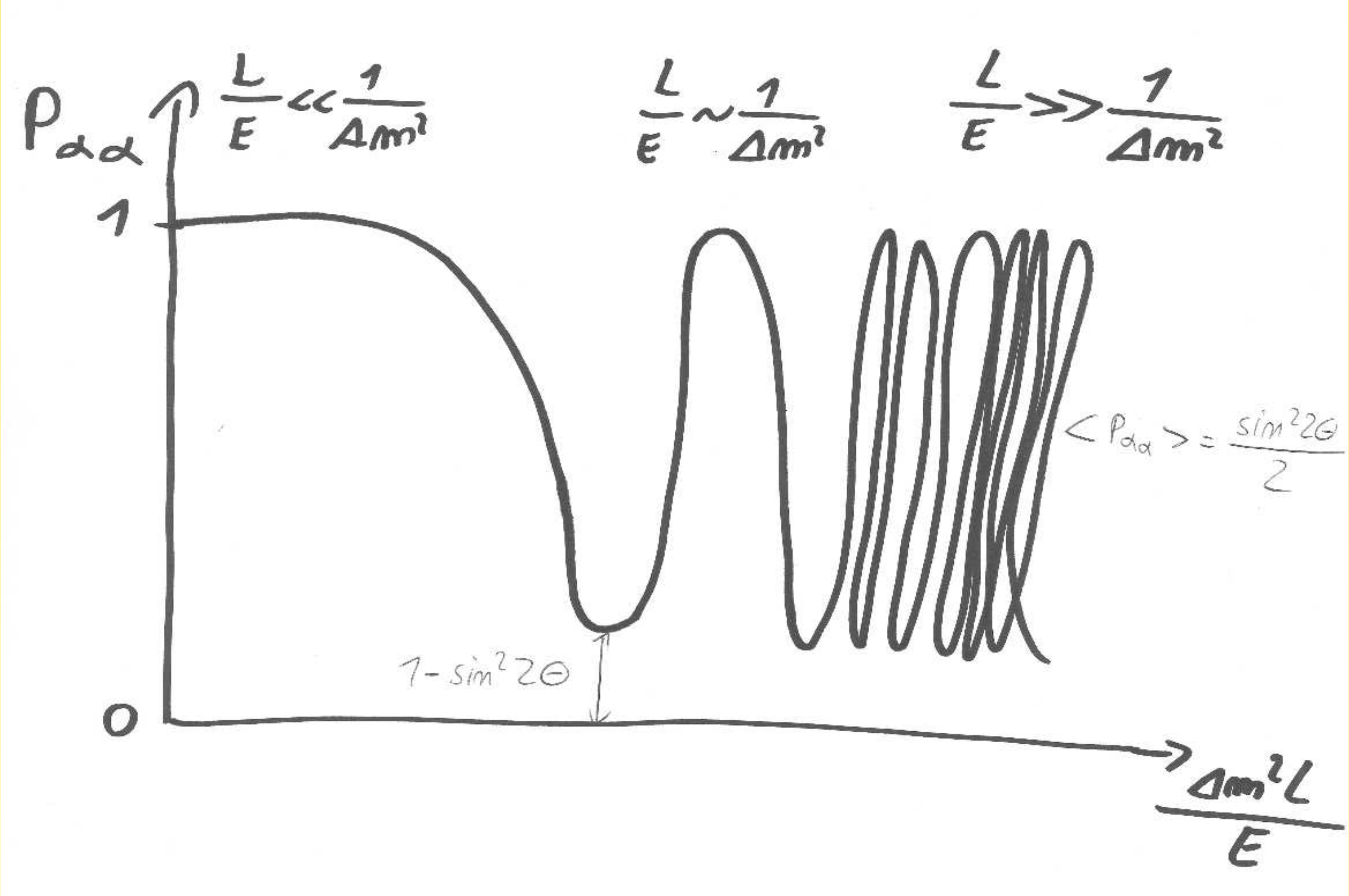
Data - BG - Geo $\bar{\nu}_e$
Expectation based on osci. parameters determined by KamLAND
Survival Probability
0
0.2
0.4
0.6
0.8
1
20
30
40
50
60
70
80
90
100
$L_0/E_{\bar{\nu}_e}$ (km/MeV)



$L \gg L_{\rm osc}$: fast oscillations $\langle \sin^2 \pi L/L_{\rm osc} \rangle = \frac{1}{2}$

and $P_{\alpha\alpha} = 1 - 2\,|U_{\alpha 1}|^2\,|U_{\alpha 2}|^2 = |U_{\alpha 1}|^4 + |U_{\alpha 2}|^4$

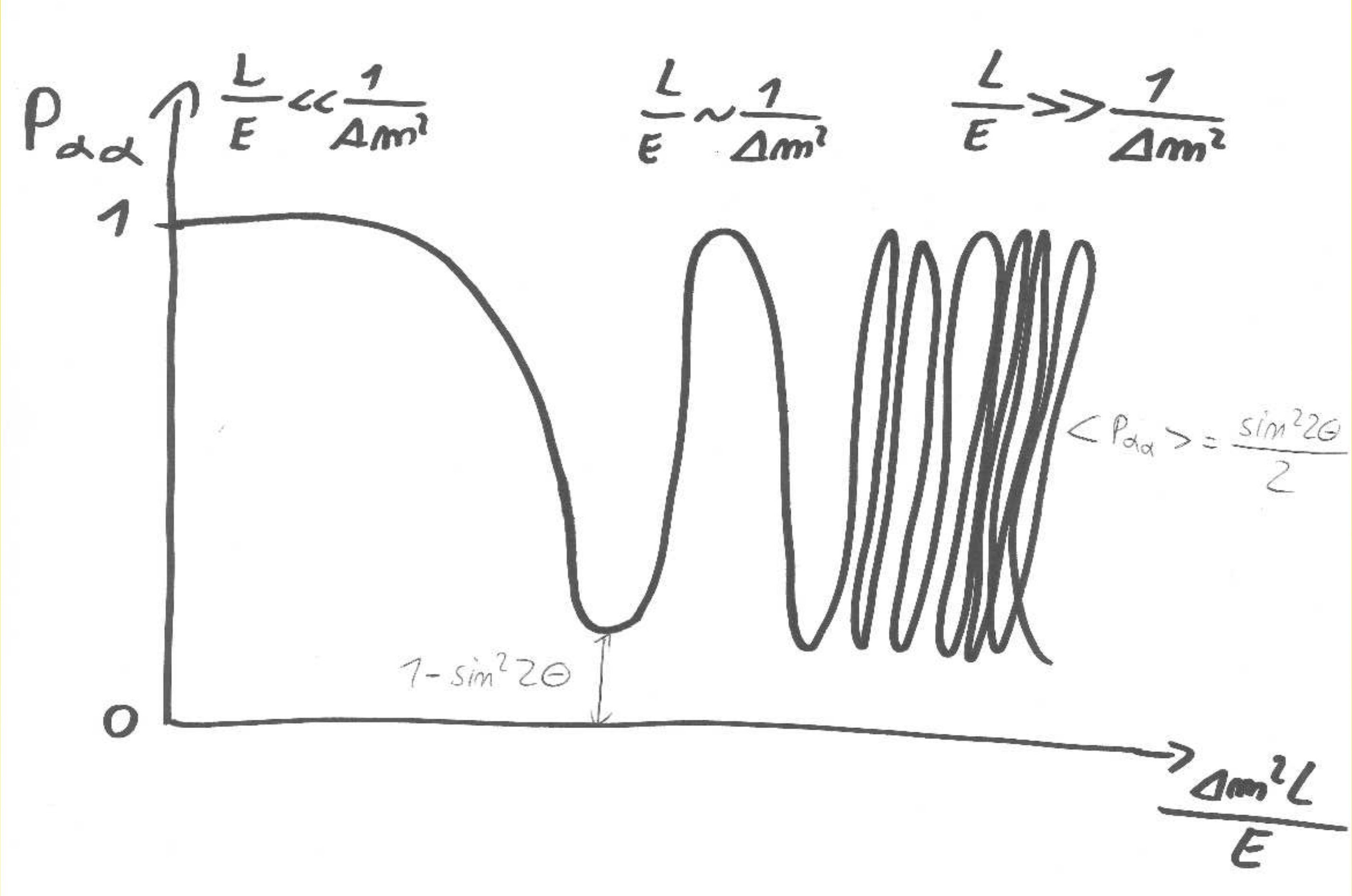
sensitivity to mixing



$L \gg L_{\rm osc}$: fast oscillations $\langle \sin^2 \pi L / L_{\rm osc} \rangle = \frac{1}{2}$

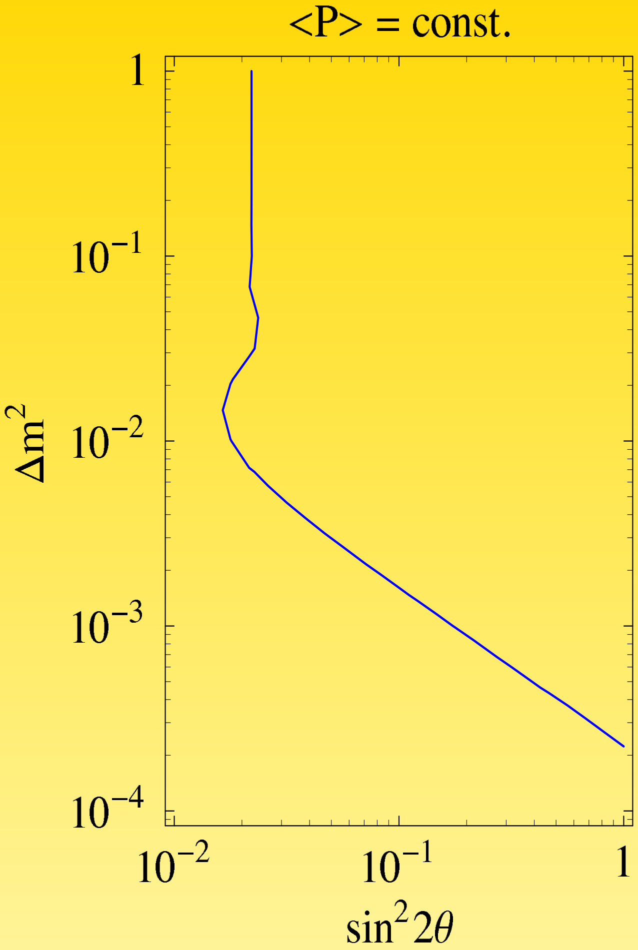
and $P_{\alpha\beta} = 2\,|U_{\alpha 1}|^2\,|U_{\alpha 2}|^2 = |U_{\alpha 1}|^2\,|U_{\beta 1}|^2 + |U_{\alpha 2}|^2\,|U_{\beta 2}|^2 = \frac{1}{2}\sin^2 2\theta$

sensitivity to mixing



$L \ll L_{\rm osc}$: hardly oscillations and $P_{\alpha\beta} = \sin^2 2\theta \, (\Delta m^2 L/(4E))^2$

sensitivity to product $\sin^2 2\theta \, \Delta m^2$



large Δm^2: sensitivity to mixing

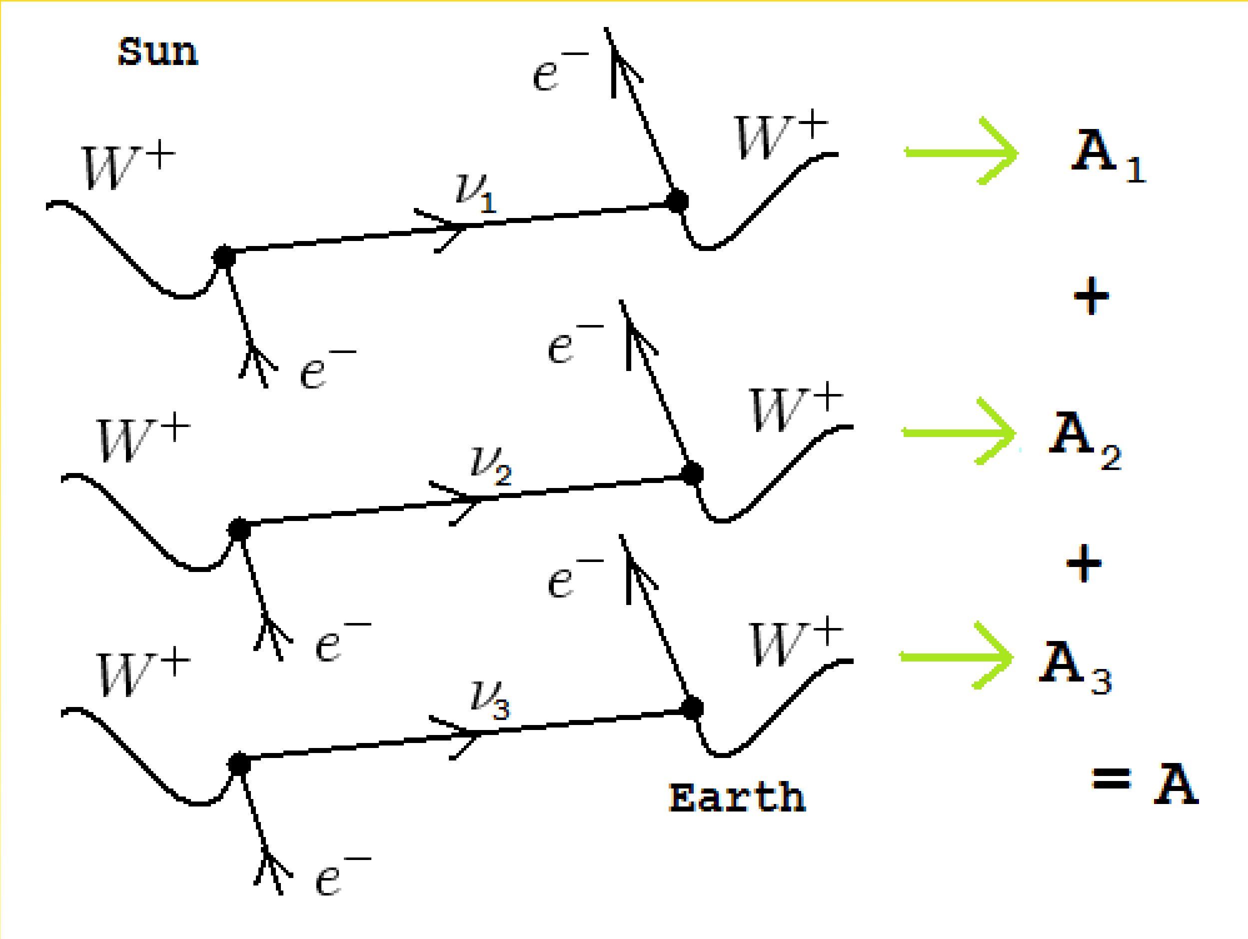
small Δm^2: sensitivity to $\sin^2 2\theta\, \Delta m^2$

maximal sensitivity when $\Delta m^2 L/E \simeq 2\pi$

Characteristics of typical oscillation experiments

Source	Flavor	E [GeV]	L [km]	$(\Delta m^2)_{\rm min}$ [eV^2]
Atmosphere	$\overset{(-)}{\nu}_e, \overset{(-)}{\nu}_\mu$	$10^{-1} \ldots 10^2$	$10 \ldots 10^4$	10^{-6}
Sun	ν_e	$10^{-3} \ldots 10^{-2}$	10^8	10^{-11}
Reactor SBL	$\bar{\nu}_e$	$10^{-4} \ldots 10^{-2}$	10^{-1}	10^{-3}
Reactor LBL	$\bar{\nu}_e$	$10^{-4} \ldots 10^{-2}$	10^2	10^{-5}
Accelerator LBL	$\overset{(-)}{\nu}_e, \overset{(-)}{\nu}_\mu$	$10^{-1} \ldots 1$	10^2	10^{-1}
Accelerator SBL	$\overset{(-)}{\nu}_e, \overset{(-)}{\nu}_\mu$	$10^{-1} \ldots 1$	1	1

Quantum Mechanics



Can't distinguish the individual m_i: coherent sum of amplitudes and *interference*

Quantum Mechanics

Textbook calculation is completely wrong!!

- $E_i - E_j$ is not Lorentz invariant
- massive particles with different p_i and same E violates energy and/or momentum conservation
- definite p: in space this is e^{ipx}, thus no localization

Quantum Mechanics

consider E_j and $p_j = \sqrt{E_j^2 - m_j^2}$:

$$p_j \simeq E + m_j^2 \left.\frac{\partial p_j}{\partial m_j^2}\right|_{m_j=0} \equiv E - \xi \frac{m_j^2}{2E} , \quad \text{with } \xi = -2E \left.\frac{\partial p_j}{\partial m_j^2}\right|_{m_j=0}$$

$$E_j \simeq p_j + m_j^2 \left.\frac{\partial E_j}{\partial m_j^2}\right|_{m_j=0} = p_j + \frac{m_j^2}{2p_j} = E + \frac{m_j^2}{2E}(1-\xi)$$

in pion decay $\pi \to \mu\nu$:

$$E_j = \frac{m_\pi^2}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_j^2}{2m_\pi^2}$$

thus,

$$\xi = \frac{1}{2}\left(1 + \frac{m_\mu^2}{m_\pi^2}\right) \simeq 0.8 \text{ in } E_i - E_j \simeq (1-\xi)\frac{\Delta m_{ij}^2}{2E}$$

wave packet with size $\sigma_x (\gtrsim 1/\sigma_p)$ and group velocity $v_i = \partial E_i / \partial p_i = p_i / E_i$:

$$\psi_i \propto \exp\left\{-i(E_i\, t - p_i\, x) - \frac{(x - v_i\, t)^2}{4\sigma_x^2}\right\}$$

1) wave packet separation should be smaller than σ_x!

$$L\, \Delta v < \sigma_x \Rightarrow \frac{L}{L_{\rm osc}} < \frac{p}{\sigma_p}$$

(loss of *coherence*: interference impossible)

2) m_ν^2 should NOT be known too precisely!

if known too well: $\Delta m^2 \gg \delta m_\nu^2 = \frac{\partial m_\nu^2}{\partial p_\nu}\, \delta p_\nu \Rightarrow \delta x_\nu \gg \frac{2\, p_\nu}{\Delta m^2} = \frac{L_{\rm osc}}{2\pi}$

(I know which state ν_i is exchanged, *localization*)

In both cases: $P_{\alpha\alpha} = |U_{\alpha 1}|^4 + |U_{\alpha 2}|^4$ (same as for $L \gg L_{\rm osc}$)

Quantum Mechanics

total amplitude for $\alpha \to \beta$ should be given by

$$A \propto \sum_j \int \frac{d^3p}{2E_j} \mathcal{A}^*_{\beta j} \, \mathcal{A}_{\alpha j} \exp\left\{-i(E_j t - px)\right\}$$

with production and detection amplitudes

$$\mathcal{A}_{\alpha j} \, \mathcal{A}^*_{\beta j} \propto \exp\left\{-\frac{(p-\tilde{p}_j)^2}{4\sigma_p^2}\right\}$$

we expand around $\tilde{p}_j$:

$$E_j(p) \simeq E_j(\tilde{p}_j) + \left.\frac{\partial E_j(p)}{\partial p}\right|_{p=\tilde{p}_j} (p-\tilde{p}_j) = \tilde{E}_j + v_j \, (p-\tilde{p}_j)$$

and perform the integral over p:

$$A \propto \sum_j \exp\left\{-i(\tilde{E}_j t - \tilde{p}_j x) - \frac{(x-v_j t)^2}{4\sigma_x^2}\right\}$$

the probability is the integral of $|A|^2$ over t:

$$P = \int dt\, |A|^2 \propto \exp\left\{-i\left[(\tilde{E}_j - \tilde{E}_k)\frac{v_j + v_k}{v_j^2 + v_k^2} - (\tilde{p}_j - \tilde{p}_k)\right] x\right\}$$

$$\times \exp\left\{-\frac{(v_j - v_k)^2 x^2}{4\sigma_x^2 (v_j^2 + v_k^2)} - \frac{(\tilde{E}_j - \tilde{E}_k)^2}{4\sigma_p^2 (v_j^2 + v_k^2)}\right\}$$

now express average momenta, energy and velocity as

$$\tilde{p}_j \simeq E - \xi \frac{m_j^2}{2E}$$

$$\tilde{E}_j \simeq E + (1 - \xi)\frac{m_j^2}{2E}\ , \qquad v_j = \frac{\tilde{p}_j}{\tilde{E}_j} \simeq 1 - \frac{m_j^2}{2E^2}$$

this we insert in first exponential of P:

$$\left[(\tilde{E}_j - \tilde{E}_k)\frac{v_j + v_k}{v_j^2 + v_k^2} - (\tilde{p}_j - \tilde{p}_k)\right] = \frac{\Delta m_{jk}^2 L}{2E}$$

the second exponential (damping term) can also be rewritten and the final probability is

$$P \propto \exp\left\{-i\frac{\Delta m_{ij}^2}{2E}L - \left(\frac{L}{L_{jk}^{\mathrm{coh}}}\right)^2 - 2\pi^2(1-\xi)^2\left(\frac{\sigma_x}{L_{jk}^{\mathrm{osc}}}\right)^2\right\}$$

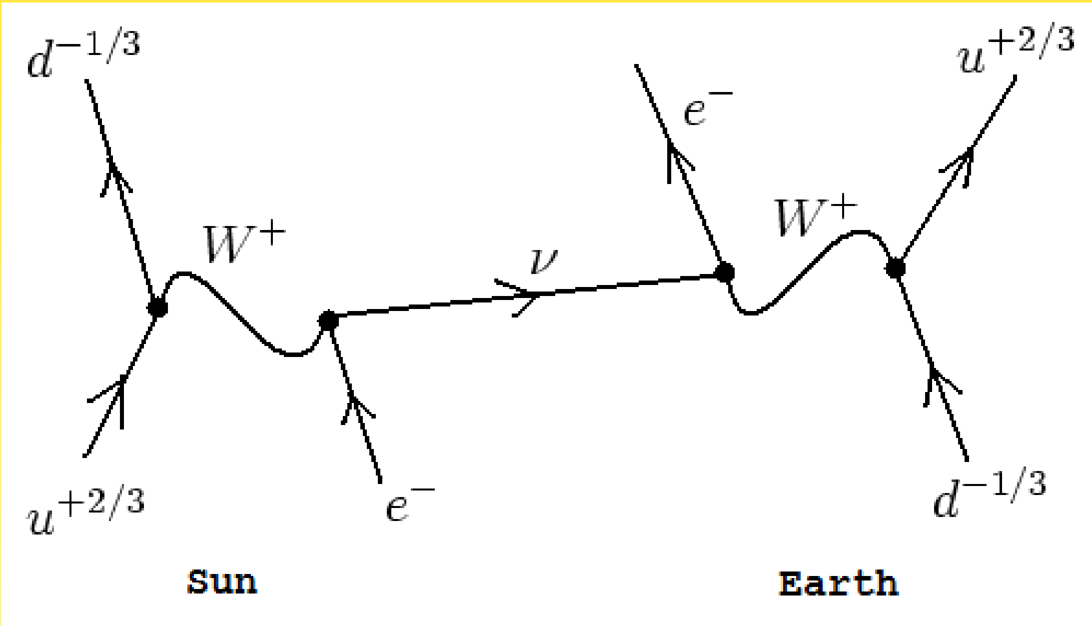
with

$$L_{jk}^{\mathrm{coh}} = \frac{4\sqrt{2}E^2}{|\Delta m_{jk}^2|}\sigma_x \text{ and } L_{jk}^{\mathrm{osc}} = \frac{4\pi E}{|\Delta m_{jk}^2|}$$

expressing the two conditions (coherence and localization) for oscillation discussed before

Quantum Mechanics

derivation of formula also works in QFT, when everything is a big Feynman diagram:



(Lorentz invariance, energy and momentum conservation at every vertex, etc.)

Contents

II Neutrino Oscillations

III3) Results and their interpretation – what have we learned?

- Early main results as by-products:
  - check solar fusion in Sun $\rightarrow$ solar neutrino problem
  - look for nucleon decay $\rightarrow$ atmospheric neutrino oscillations
- data for long time describable by 2-flavor formalism
- genuine 3-flavor effects since 2011
  - third mixing angle!
  - mass ordering?
  - CP violation?
- have entered precision era

3 families: $U = R_{23}\,\tilde{R}_{13}\,R_{12}\,P$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\,e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\,e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} P$$

$$= \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{i\delta} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{-i\delta} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{-i\delta} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{-i\delta} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{-i\delta} & c_{23}\,c_{13} \end{pmatrix} P$$

with $P = \mathrm{diag}(1, e^{i\alpha}, e^{i\beta})$

Characteristics of typical oscillation experiments

Source	Flavor	E [GeV]	L [km]	$(\Delta m^2)_{\min}$ [eV^2]
Atmosphere	$\overset{(-)}{\nu}_e,\ \overset{(-)}{\nu}_\mu$	$10^{-1}\ldots 10^2$	$10\ldots 10^4$	10^{-6}
Sun	ν_e	$10^{-3}\ldots 10^{-2}$	10^8	10^{-11}
Reactor SBL	$\bar{\nu}_e$	$10^{-4}\ldots 10^{-2}$	10^{-1}	10^{-3}
Reactor LBL	$\bar{\nu}_e$	$10^{-4}\ldots 10^{-2}$	10^2	10^{-5}
Accelerator LBL	$\overset{(-)}{\nu}_e,\ \overset{(-)}{\nu}_\mu$	$10^{-1}\ldots 1$	10^2	10^{-1}
Accelerator SBL	$\overset{(-)}{\nu}_e,\ \overset{(-)}{\nu}_\mu$	$10^{-1}\ldots 1$	1	1

Interpretation in 3 Neutrino Framework

assume $\Delta m^2_{21} \ll |\Delta m^2_{31}| \simeq |\Delta m^2_{32}|$ and small θ_{13}:

- atmospheric and accelerator neutrinos: $\Delta m^2_{21} L/E \ll 1$

$$P(\nu_\mu \to \nu_\tau) \simeq \sin^2 2\theta_{23} \, \sin^2 \frac{\Delta m^2_{31}}{4\,E}\, L$$

- solar and KamLAND neutrinos: $|\Delta m^2_{31}| L/E \gg 1$

$$P(\nu_e \to \nu_e) \simeq 1 - \sin^2 2\theta_{12} \, \sin^2 \frac{\Delta m^2_{21}}{4\,E}\, L$$

- short baseline reactor neutrinos: $\Delta m^2_{21} L/E \ll 1$

$$P(\nu_e \to \nu_e) \simeq 1 - \sin^2 2\theta_{13} \, \sin^2 \frac{\Delta m^2_{31}}{4\,E}\, L$$

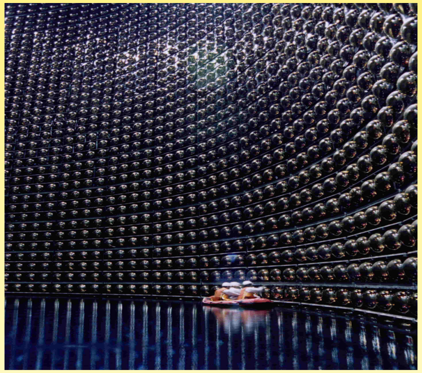
EXERCISE!

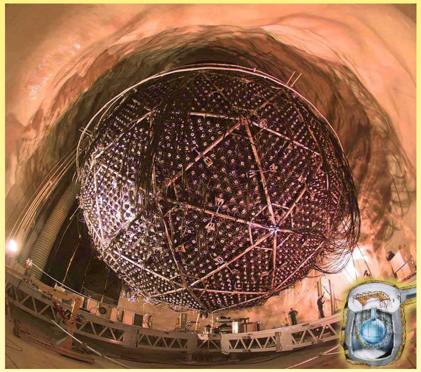
Summing up

$$U = \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{i\delta} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{i\delta} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{i\delta} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{i\delta} & c_{23}\,c_{13} \end{pmatrix} =$$

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{atmospheric and LBL accelerator}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}\,e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\,e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{SBL reactor}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{solar and LBL reactor}}$$

atmospheric and LBL accelerator — SBL reactor — solar and LBL reactor





$$U = \begin{pmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{-i\delta} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{i\delta} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{i\delta} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{i\delta} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, e^{i\delta} & c_{23}\, c_{13} \end{pmatrix} =$$

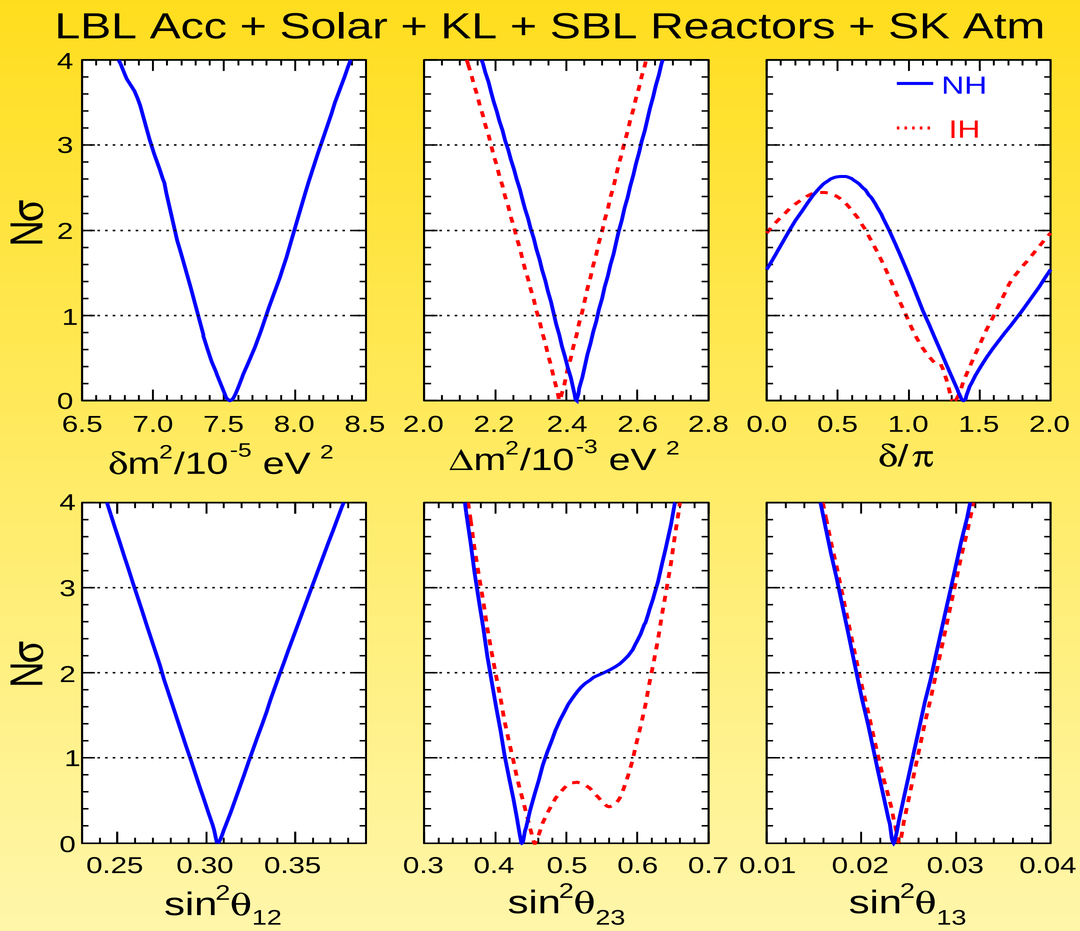
$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{atmospheric and LBL accelerator}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}\, e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\, e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{SBL reactor}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{solar and LBL reactor}}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}}+\epsilon \\ 0 & \sqrt{\frac{1}{2}}+\epsilon & \sqrt{\frac{1}{2}} \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & \epsilon \\ 0 & 1 & 0 \\ \epsilon & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}}+\epsilon & 0 \\ -\sqrt{\frac{1}{3}}+\epsilon & \sqrt{\frac{2}{3}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(\sin^2\theta_{23} = \tfrac{1}{2} - \epsilon) \quad (\sin^2\theta_{13} = \epsilon^2) \quad (\sin^2\theta_{12} = \tfrac{1}{3} - \epsilon^2)$$

$$\Delta m^2_{\mathrm{A}} \quad \Delta m^2_{\mathrm{A}} \quad \Delta m^2_{\odot}$$

Status of global Fits



Fogli, Lisi *et al.*, March 2014

PMNS matrix is given by

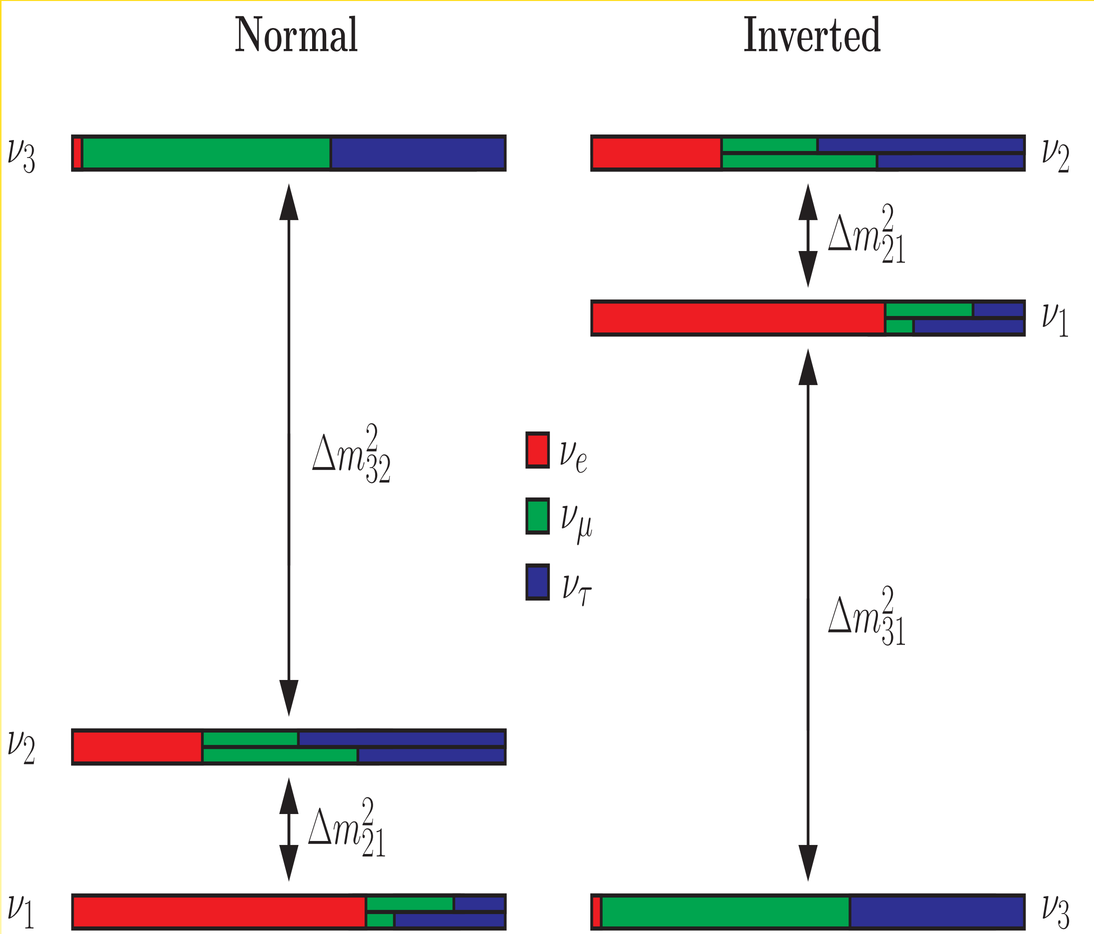
$$|U| = \begin{pmatrix} 0.779\ldots0.848 & 0.510\ldots0.604 & 0.122\ldots0.190 \\ 0.183\ldots0.568 & 0.385\ldots0.728 & 0.613\ldots0.794 \\ 0.200\ldots0.576 & 0.408\ldots0.742 & 0.589\ldots0.775 \end{pmatrix}$$

compare with CKM:

$$|V| = \begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.00065 & 0.00351^{+0.00015}_{-0.00014} \\ 0.22520 \pm 0.00065 & 0.97344^{+0.00016}_{-0.00016} & 0.0412^{+0.0011}_{-0.0005} \\ 0.00867^{+0.00029}_{-0.00031} & 0.0404^{+0.0011}_{-0.0005} & 0.999146^{+0.000021}_{-0.000046} \end{pmatrix}$$

$\leftrightarrow$ WTF? large mixing??

Two Mass Orderings



unknown sign of larger Δm^2

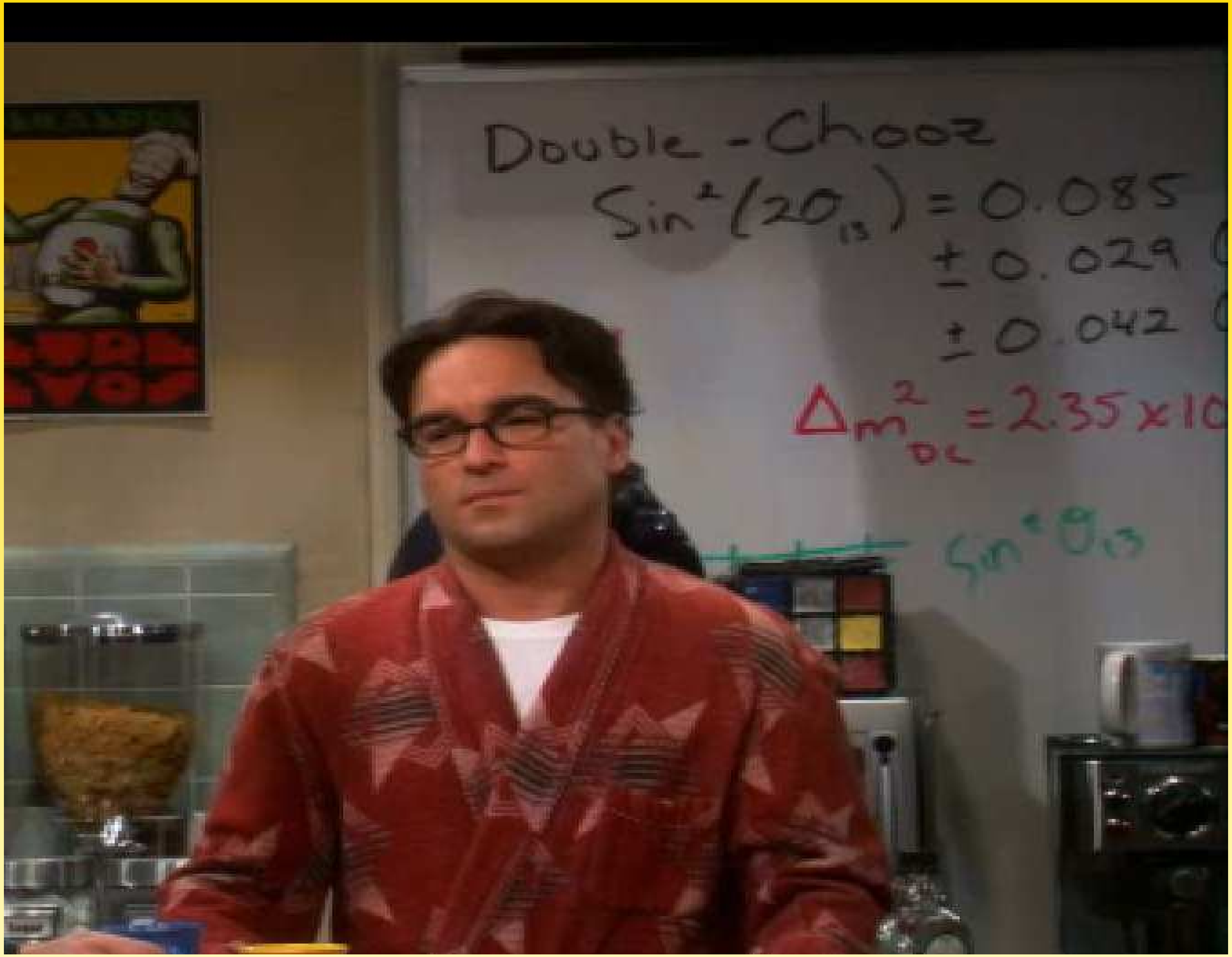
Present knowledge

$$\mathcal{L} = \frac{1}{2}\,\nu^T\, m_\nu\, \nu \;\; \text{with} \;\; m_\nu = U\,\mathrm{diag}(m_1, m_2, m_3)\, U^T$$

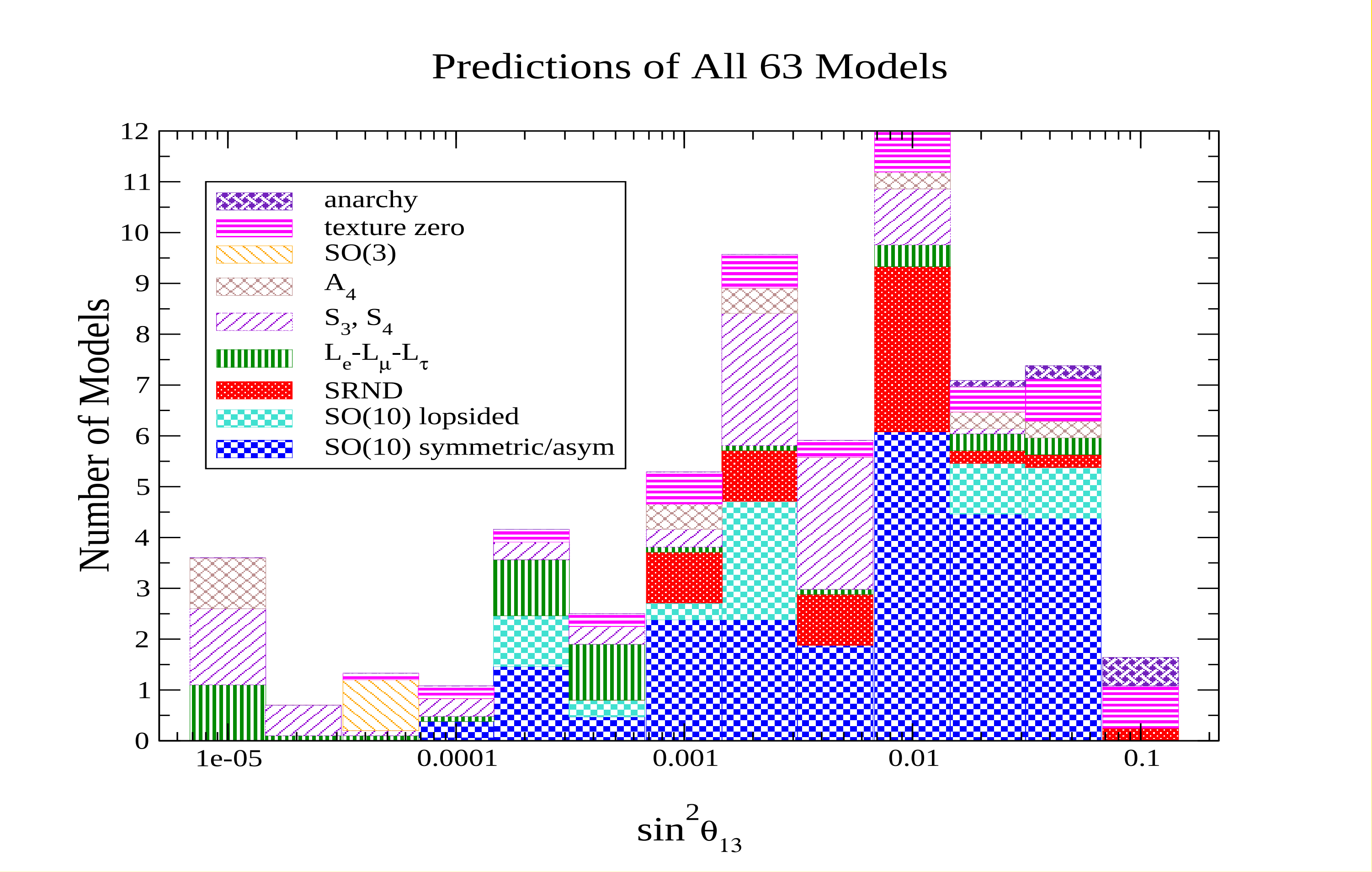
9 physical parameters in m_ν

- θ_{12} and $m_2^2 - m_1^2$
- θ_{23} and $|m_3^2 - m_2^2|$
- θ_{13} (or $|U_{e3}|$)
- m_1, m_2, m_3
- sgn$(m_3^2 - m_2^2)$
- Dirac phase δ
- Majorana phases α and β

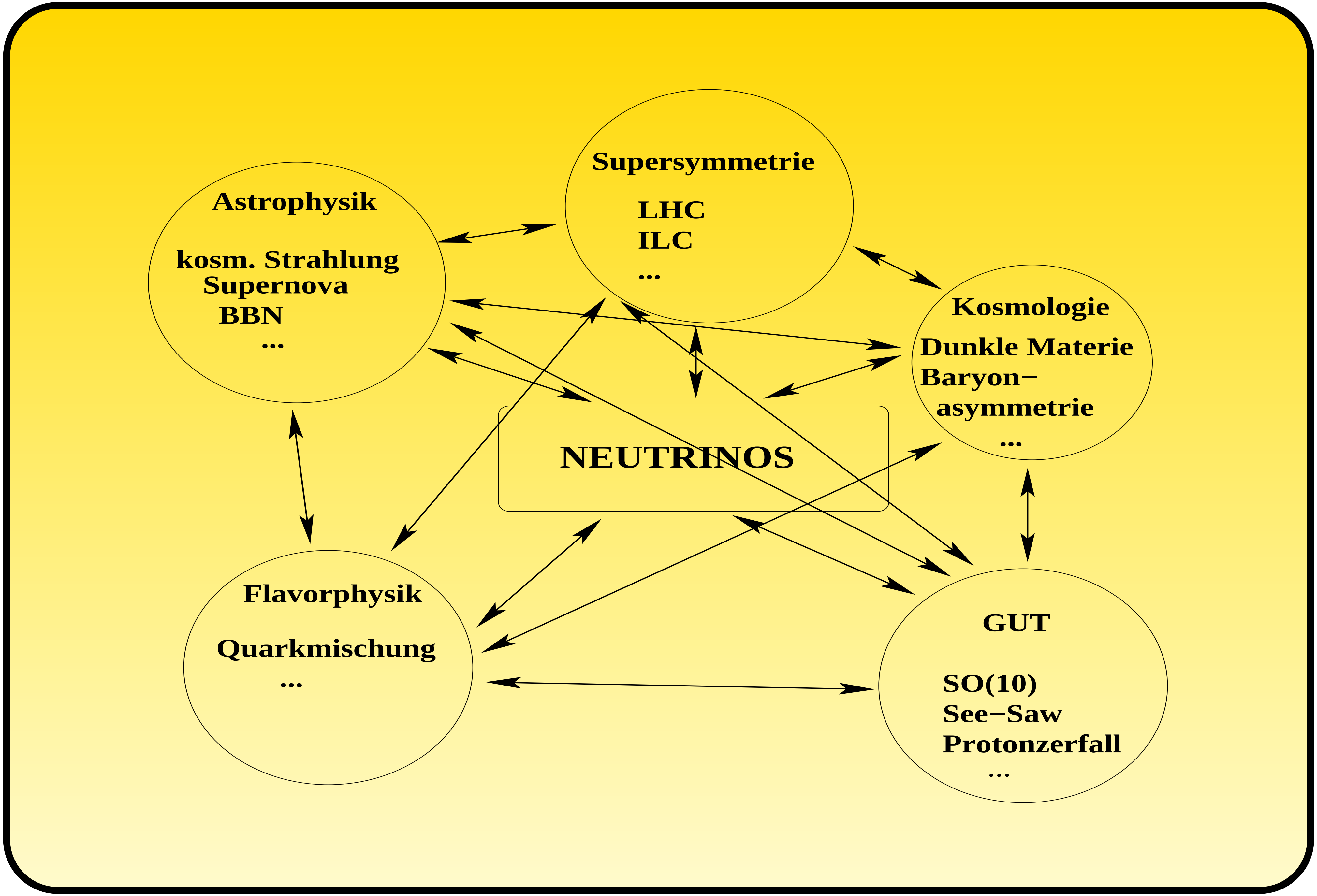

45°
40°
35°
30°
25°
20°
15°
10°
23
12
?


Double - Chooz
Sin²(2θ13) = 0.085
± 0.029
± 0.042
Δm²DC = 2.35 x 10
sin²θ13

What's that good for?



Albright, Chen


Supersymmetrie
LHC
ILC
...
Astrophysik
kosm. Strahlung
Supernova
BBN
...
Kosmologie
Dunkle Materie
Baryon−
asymmetrie
...
NEUTRINOS
Flavorphysik
Quarkmischung
...
GUT
SO(10)
See−Saw
Protonzerfall
...

EXERCISE: Lepton Flavor Violation

Suppose $\mathrm{BR}(\mu \to e\gamma) \propto |(m_\nu m_\nu^\dagger)_{e\mu}|^2$.

In general, can it be zero? What does the experimental value $|U_{e3}| \simeq 0.15$ imply for the BR?

EXERCISE: Neutrino Telescopes

$$\begin{pmatrix} \Phi_e \\ \Phi_\mu \\ \Phi_\tau \end{pmatrix} = \begin{pmatrix} P_{ee} & P_{e\mu} & P_{e\tau} \\ P_{e\mu} & P_{\mu\mu} & P_{\mu\tau} \\ P_{e\tau} & P_{\mu\tau} & P_{\tau\tau} \end{pmatrix} \begin{pmatrix} \Phi_e^0 \\ \Phi_\mu^0 \\ \Phi_\tau^0 \end{pmatrix}$$

Transition amplitude after loss of coherence:

$$P(\nu_\alpha \to \nu_\beta) = \sum |U_{\alpha i}|^2 \, |U_{\beta i}|^2$$

if $\theta_{23} = \pi/4$ and $\theta_{13} = 0$: what happens to a "pion source"
$(\Phi_e^0 : \Phi_\mu^0 : \Phi_\tau^0) = (1 : 2 : 0)$?

Obtain the first order corrections ($\theta_{23} = \pi/4 - \epsilon_1$ and $\theta_{13} = \epsilon_2$). How large can they become?

Contents

II Neutrino Oscillations

Degeneracies

Expand *3 flavor* oscillation probabilities in terms of $R = \Delta m^2_\odot / \Delta m^2_{\rm A}$ and $|U_{e3}|$:

$$P(\nu_e \to \nu_\mu) \simeq \sin^2 2\theta_{13}\, \sin^2\theta_{23}\, \frac{\sin^2(1-\hat{A})\Delta}{(1-\hat{A})^2} + R^2\, \sin^2 2\theta_{12}\, \cos^2\theta_{23} \frac{\sin^2 \hat{A}\Delta}{\hat{A}^2}$$

$$+\sin\delta \sin 2\theta_{13}\, R\, \sin 2\theta_{12}\, \cos\theta_{13}\, \sin 2\theta_{23} \sin\Delta\, \frac{\sin\hat{A}\Delta\, \sin(1-\hat{A})\Delta}{\hat{A}(1-\hat{A})}$$

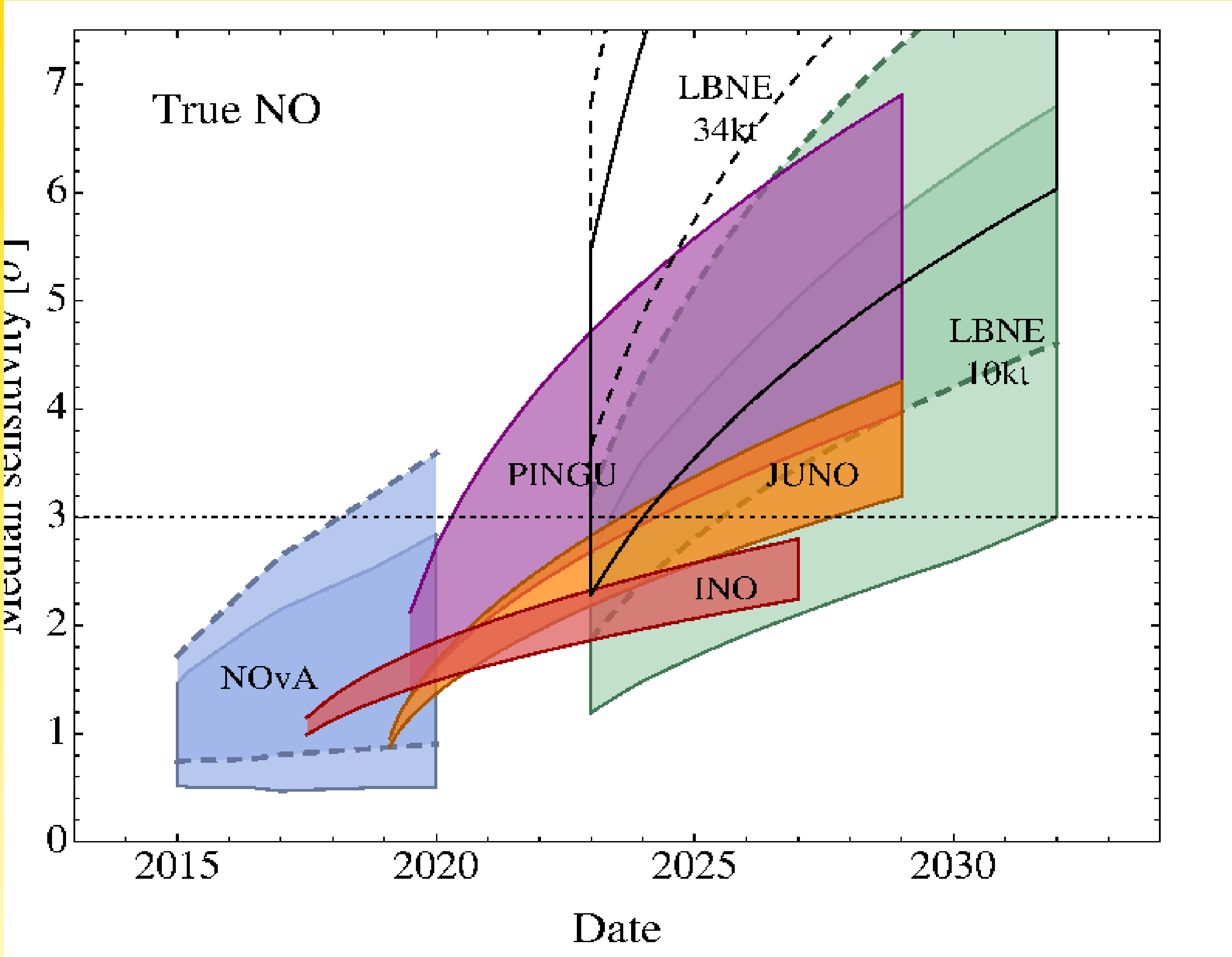
$$+\cos\delta \sin 2\theta_{13}\, R\, \sin 2\theta_{12}\, \cos\theta_{13}\, \sin 2\theta_{23}\, \cos\Delta \frac{\sin\hat{A}\Delta\, \sin(1-\hat{A})\Delta}{\hat{A}(1-\hat{A})}$$

with $\hat{A} = 2\sqrt{2}\, G_F\, n_e\, E / \Delta m^2_{\rm A}$ and $\Delta = \frac{\Delta m^2_{\rm A}}{4\,E}\, L$

- $\theta_{23} \leftrightarrow \pi/2 - \theta_{23}$ degeneracy
- θ_{13}-δ degeneracy
- δ-sgn$(\Delta m^2_{\rm A})$ degeneracy

Solutions: more channels, different L/E, high precision,...

Typical time scale



3 Tasks

Now that neutrino mass is introduced:

- Determine parameters
- Explain values of parameters
- Check if minimal description is correct

Contents

III Neutrino Mass

III1) Dirac vs. Majorana masses

Observation shows that neutrinos possess non-vanishing rest mass

Upper limits on masses imply $m_\nu \lesssim$ eV

How can we introduce neutrino mass terms?

a) Dirac masses

add $\nu_R \sim (1,0)$:

$$\mathcal{L}_D = g_\nu \, \overline{L} \, \tilde{\Phi} \, \nu_R \stackrel{\text{EWSB}}{\longrightarrow} \frac{v}{\sqrt{2}} \, g_\nu \, \overline{\nu_L} \, \nu_R = m_\nu \, \overline{\nu_L} \, \nu_R$$

But $m_\nu \lesssim$ eV implies $g_\nu \lesssim 10^{-12} \lll g_e$

highly unsatisfactory fine-tuning...

actually, $m_e = 10^{-6}\, m_t$, so WTF?

point is that

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \text{with} \quad m_u \simeq m_d$$

has to be contrasted with

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \text{with} \quad m_\nu \simeq 10^{-6}\, m_e$$

b) Majorana masses (long version)

need charge conjugation:

electron e^-: $[\gamma_\mu (i\partial^\mu + e\, A^\mu) - m]\, \psi = 0 \quad (1)$

positron e^+: $[\gamma_\mu (i\partial^\mu - e\, A^\mu) - m]\, \psi^c = 0 \quad (2)$

Try $\psi^c = S\, \psi^*$, evaluate $(S^*)^{-1}\, (2)^*$ and compare with (1):

$$S = i\gamma_2$$

and thus

$$\boxed{\psi^c = i\gamma_2\, \psi^* = i\gamma_2\, \gamma_0 \overline{\psi}^T \equiv C\, \overline{\psi}^T}$$

flips all charge-like quantum numbers

Properties of C:

$$C^\dagger = C^T = C^{-1} = -C$$

$$C\,\gamma_\mu\,C^{-1} = -\gamma_\mu^T$$

$$C\,\gamma_5\,C^{-1} = \gamma_5^T$$

$$C\,\gamma_\mu\gamma_5\,C^{-1} = (\gamma_\mu\gamma_5)^T$$

properties of charged conjugate spinors:

$$(\psi^c)^c = \psi$$

$$\overline{\psi^c} = \psi^T\,C$$

$$\overline{\psi_1}\,\psi_2^c = \overline{\psi_2^c}\,\psi_1$$

$$\boxed{(\psi_L)^c = (\psi^c)_R}$$

$$\boxed{(\psi_R)^c = (\psi^c)_L}$$

C flips chirality: LH becomes RH

b) Majorana masses (short version)

charge conjugation: $\psi^c = i\gamma_2\,\psi^* = i\gamma_2\,\gamma_0 \overline{\psi}^T \equiv C\,\overline{\psi}^T$

flips all charge-like quantum numbers

connection to chirality:

$$(\psi_L)^c = (\psi^c)_R$$

$$(\psi_R)^c = (\psi^c)_L$$

C flips chirality: LH becomes RH

Mass terms

$$\mathcal{L} = m_\nu \, \overline{\nu_L} \, \nu_R + h.c. = m_\nu \, (\overline{\nu_L} \, \nu_R + \overline{\nu_R} \, \nu_L)$$

both chiralities a must for mass term!

Possibilities for $\overline{L}R$:

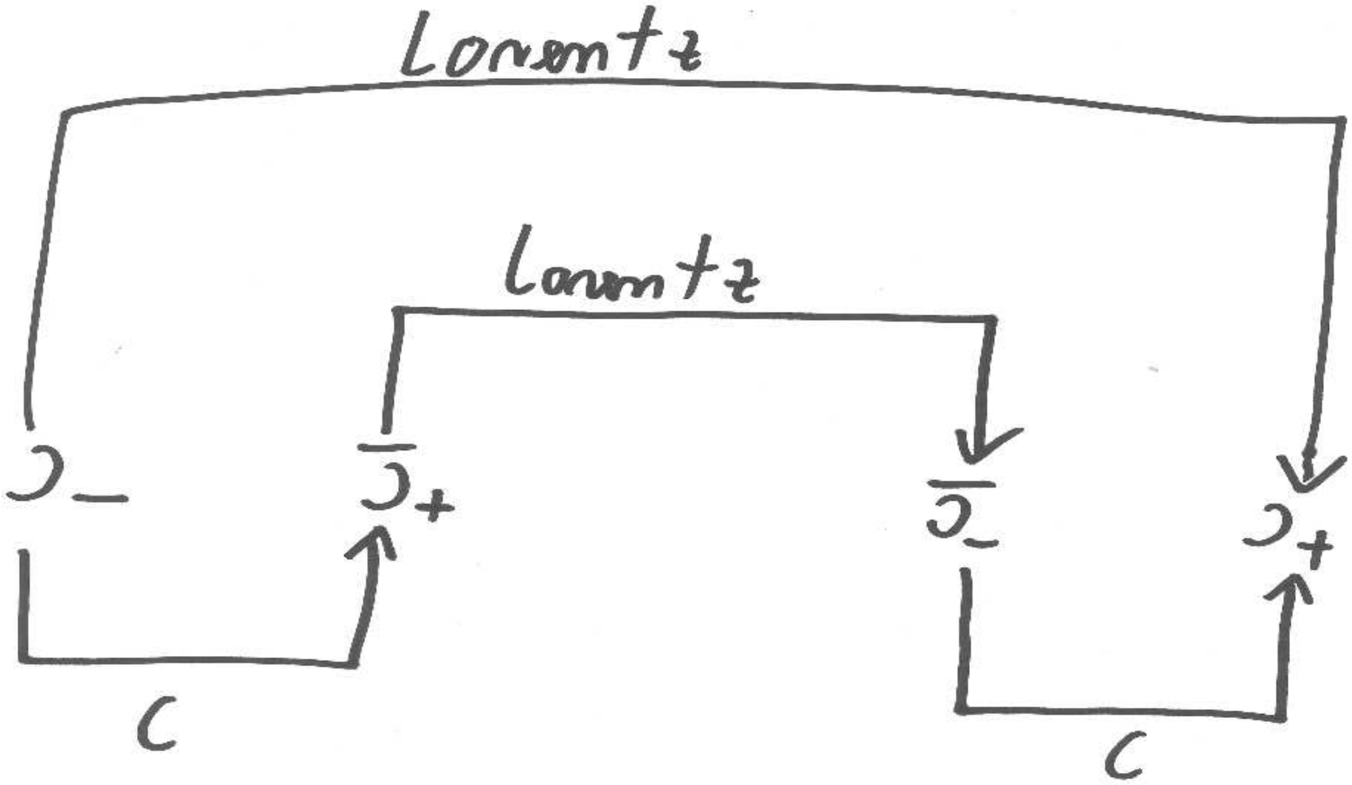
(i) ν_R independent of ν_L: **Dirac particle**

(ii) $\nu_R = (\nu_L)^c$: **Majorana particle**

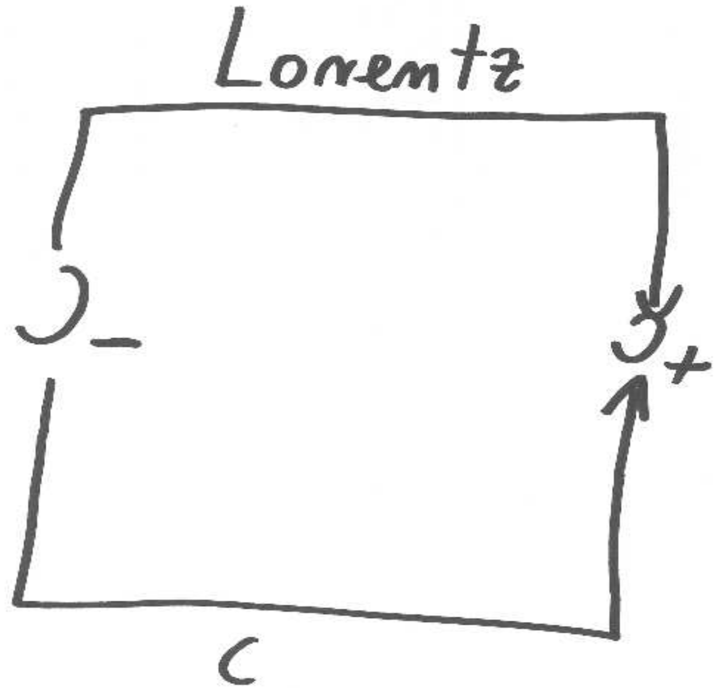
$$\Rightarrow \nu^c = (\nu_L + \nu_R)^c = (\nu_L)^c + (\nu_R)^c = \nu_R + \nu_L = \nu : \boxed{\nu^c = \nu}$$

$\Rightarrow$ Majorana fermion is identical to its antiparticle, a truly neutral particle

$\Rightarrow$ all additive quantum numbers (Q, L, B,...) are zero in terms of helicity states $\pm$: 4 d.o.f. for **Dirac particles:**



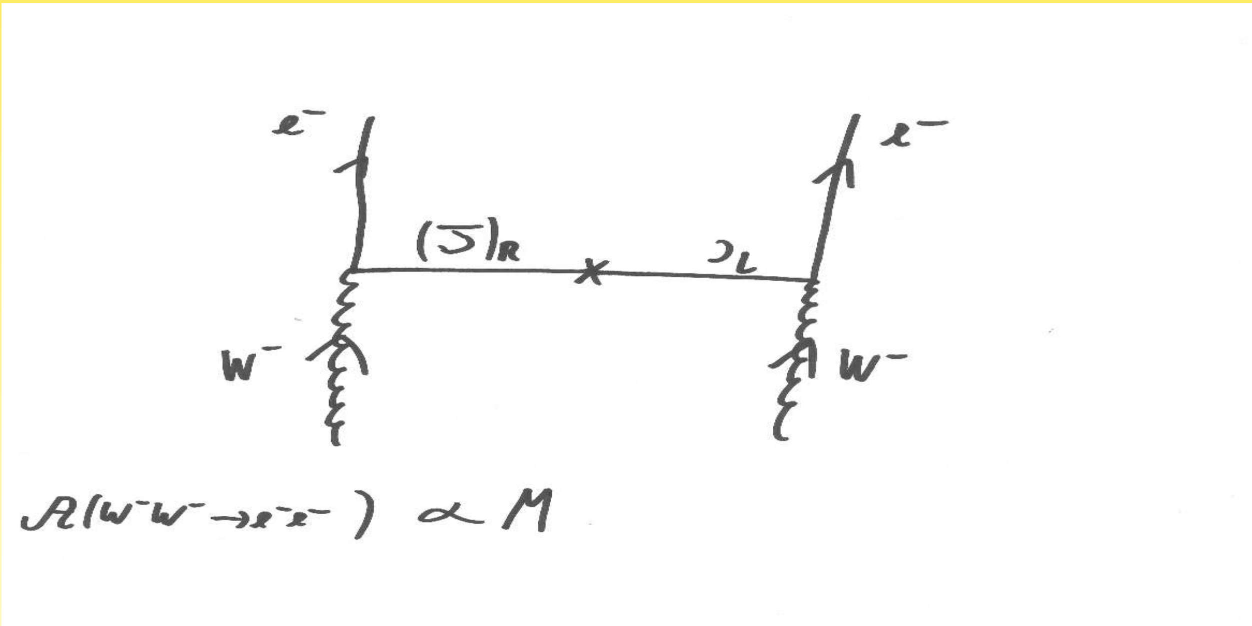
in terms of helicity states $\pm$: 2 d.o.f. for **Majorana particles:**



Mass term for Majorana particles:

$$\mathcal{L}_M = \frac{1}{2}\,\overline{\nu}\, m_\nu\, \nu = \frac{1}{2}\,\overline{\nu_L + (\nu_L)^c}\, m_\nu\, (\nu_L + (\nu_L)^c) = \frac{1}{2}\,\overline{\nu_L}\, m_\nu\, \nu_L^c + h.c.$$

- Majorana mass term
- $\mathcal{L}_M \propto \nu^\dagger\, \nu^* \Rightarrow$ NOT invariant under $\nu \to e^{i\alpha}\, \nu$
  $\Rightarrow$ breaks Lepton Number by 2 units



Dirac vs. Majorana

in $V - A$ theories: observable **always** suppressed by $(m/E)^2$

- suppose beam from π^+ decays: $\pi^+ \to \mu^+ \nu_\mu$
- can we observe $\overline{\nu}_\mu + p \to n + \mu^+$?
- chirality is not a good quantum number: "spin flip"
- emitted ν_μ (negative helicity) is not purely left-handed:

$$u_\downarrow(p) = u_L^{(m=0)}(p) + \frac{m}{2E}\, u_R^{(m=0)}(-p)$$

- $P_R\, u_\downarrow \neq 0$ and μ^+ can be produced IF $u \propto v$ ($\leftrightarrow$ Majorana!)

$\Rightarrow$ amplitude $\propto (m/E)$ $\Rightarrow$ probability $\propto (m/E)^2$

$\Rightarrow$ only N_A can save the day!

Dirac vs. Majorana

- Z-decay:

$$\frac{\Gamma(Z \to \nu_{\rm D}\nu_{\rm D})}{\Gamma(Z \to \nu_{\rm M}\nu_{\rm M})} \simeq 1 - 3\frac{m_\nu^2}{m_Z^2}$$

- Meson decays

$$\mathrm{BR}(K^+ \to \pi^- \, e^+ \, \mu^+) \propto |m_{e\mu}|^2 = \left|\sum U_{ei}\, U_{\mu i}\, m_i\right|^2 \sim 10^{-30} \, \left(\frac{|m_{e\mu}|}{\mathrm{eV}}\right)^2$$

- neutrino-antineutrino oscillations

$$P(\nu_\alpha \to \bar{\nu}_\beta) = \frac{1}{E^2}\left|\sum_{i,j} U_{\alpha j} U_{\beta j} U_{\alpha i}^* U_{\beta i}^* \, m_i m_j \, \mathrm{e}^{-i(E_j - E_i)t}\right|$$

Contents

III Neutrino Mass

III2) Realization of Majorana masses beyond the SM

a) Higher dimensional operators

Renormalizability: only dimension 4 terms in $\mathcal{L}$

SM has several problems $\rightarrow$ there is a theory beyond SM, whose low energy limit is the SM $\rightarrow$ higher dimensional operators:

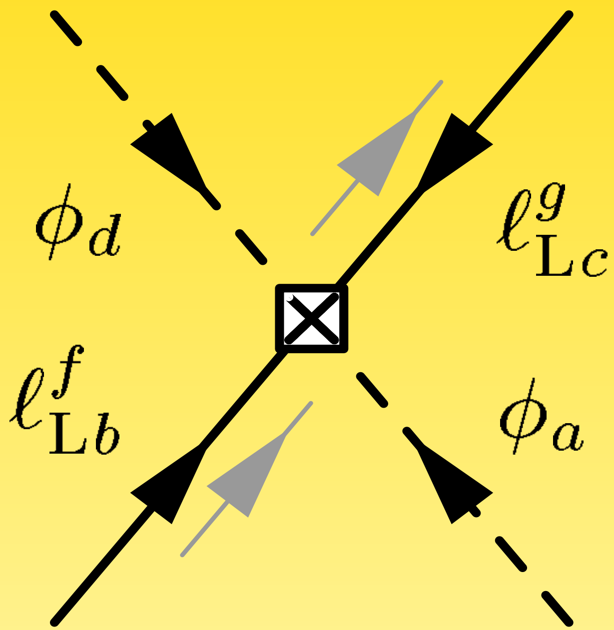
$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_5 + \mathcal{L}_6 + \ldots = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda}\,\mathcal{O}_5 + \frac{1}{\Lambda^2}\,\mathcal{O}_6 + \ldots$$

gauge and Lorentz invariant, only SM fields:

$$\frac{1}{\Lambda}\,\mathcal{O}_5 = \frac{c}{\Lambda}\,\overline{L^c}\,\tilde{\Phi}^*\,\tilde{\Phi}^\dagger\,L \stackrel{\rm EWSB}{\longrightarrow} \frac{c\,v^2}{2\,\Lambda}\,\overline{(\nu_L)^c}\,\nu_L \equiv m_\nu\,\overline{(\nu_L)^c}\,\nu_L$$

it follows $\boxed{\Lambda \gtrsim c\left(\frac{0.1~\rm eV}{m_\nu}\right)10^{14}~\text{GeV}}$ Weinberg 1979

Weinberg operator is $LL\Phi\Phi$



Seesaw Mechanisms are realizations of this effective operator by integrating out heavy physics

Master Formula: $2 + 2 = 3 + 1$

General remarks: $SU(2)_L \times U(1)_Y$ with $2 \otimes 2 = 3 \oplus 1$:

$$\overline{L}\,\tilde{\Phi} \sim (2, +1) \otimes (2, -1) = (3, 0) \oplus (1, 0)$$

To make a singlet, couple $(1, 0)$ or $(3, 0)$, because $3 \otimes 3 = 5 \oplus 3 \oplus 1$

Alternatively:

$$\overline{L}\,L^c \sim (2, +1) \otimes (2, +1) = (3, +2) \oplus (1, +2)$$

To make a singlet, couple to $(1, -2)$ or $(3, -2)$. However, singlet combination is $\overline{\nu}\,\ell^c - \overline{\ell}\,\nu^c$, which cannot generate neutrino mass term

$\Longrightarrow$	$(1, 0)$	or	$(3, -2)$	or	$(3, 0)$
	type I		type II		type III

b) Fermion singlets (type I)

introduce $N_R \sim (1,0)$ and couple to $g_\nu \overline{L}\,\tilde{\Phi} \sim (1,0)$

Hence, $g_\nu \overline{L}\,\tilde{\Phi}\,N_R$ is also singlet and becomes $g_\nu\, v/\sqrt{2}\,\overline{\nu_L}\,N_R \equiv m_D\,\overline{\nu_L}\,N_R$

in addition: Majorana mass term for N_R

$$\begin{aligned}\mathcal{L} &= \overline{\nu_L}\,m_D\,N_R + \tfrac{1}{2}\,\overline{N_R^c}\,M_R\,N_R + h.c. \\ &= \tfrac{1}{2}\,(\overline{\nu_L}, \overline{N_R^c}) \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix} + h.c. \\ &\equiv \tfrac{1}{2}\,\overline{\Psi}\,\mathcal{M}_\nu\,\Psi^c + h.c.\end{aligned}$$

b) Fermion singlets (type I)

$$\mathcal{L} = \frac{1}{2}\,(\overline{\nu_L}, \overline{N_R^c})\begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix}\begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix} + h.c.$$
$$\equiv \frac{1}{2}\,\overline{\Psi}\,\mathcal{M}_\nu\,\Psi^c + h.c.$$

Diagonalization with $\mathcal{U}^\dagger\,\mathcal{M}_\nu\,\mathcal{U}^* = D = \mathrm{diag}(m_\nu, M)$ and

$$\mathcal{U} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\mathcal{L} = \tfrac{1}{2}\,(\overline{\nu_L}, \overline{N_R^c}) \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}$$

$$= \tfrac{1}{2} \quad \underbrace{(\overline{\nu_L}, \overline{N_R^c})\,\mathcal{U}}_{(\overline{\nu}, \overline{N^c})} \;\; \underbrace{\mathcal{U}^\dagger \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \mathcal{U}^*}_{\mathrm{diag}(m_\nu, M)} \;\; \underbrace{\mathcal{U}^T \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}}_{(\nu^c, N)^T}$$

with

general formula:	if $m_D \ll M_R$:
$\tan 2\theta = \frac{2\, m_D}{M_R - 0}$	$\ll 1$
$m_\nu = \frac{1}{2}\left[(0 + M_R) - \sqrt{(0 - M_R)^2 + 4\, m_D^2}\right]$	$\simeq -m_D^2/M_R$
$M = \frac{1}{2}\left[(0 + M_R) + \sqrt{(0 - M_R)^2 + 4\, m_D^2}\right]$	$\simeq M_R$

Note: m_D associated with EWSB, part of SM, bounded by $v/\sqrt{2} = 174$ GeV

M_R is SM singlet, does whatever it wants: $\Rightarrow M_R \gg m_D$

Hence, $\theta \simeq m_D/M_R \ll 1$

$$\nu = \nu_L \cos\theta - N_R^c \sin\theta \simeq \nu_L \qquad \text{with mass} \quad m_\nu \simeq -m_D^2/M_R$$

$$N = N_R \cos\theta + \nu_L^c \sin\theta \simeq N_R \qquad \text{with mass} \quad M \simeq M_R$$

in effective mass terms

$$\mathcal{L} \simeq \frac{1}{2}\, m_\nu\, \overline{\nu_L}\, \nu_L^c + \frac{1}{2}\, M_R\, \overline{N_R^c}\, N_R$$

compare with Weinberg operator:

$$\boxed{\Lambda = -\frac{c\, v^2}{m_D^2}\, M_R}$$

also: integrate N_R away with Euler-Lagrange equation matrix case: block diagonalization

$$\mathcal{L} = \frac{1}{2}\,(\overline{\nu_L}, \overline{N_R^c}) \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}$$

$$= \frac{1}{2} \quad \underbrace{(\overline{\nu_L}, \overline{N_R^c})\,\mathcal{U}}_{(\overline{\nu}, \overline{N^c})} \;\; \underbrace{\mathcal{U}^\dagger \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \mathcal{U}^*}_{\mathrm{diag}(m_\nu, M)} \;\; \underbrace{\mathcal{U}^T \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}}_{(\nu^c, N)^T}$$

with 6×6 diagonal matrix

$$\mathcal{U} = \begin{pmatrix} 1 & -\rho \\ \rho^\dagger & 1 \end{pmatrix}, \;\; \mathcal{U}^\dagger = \begin{pmatrix} 1 & \rho \\ -\rho^\dagger & 1 \end{pmatrix}, \;\; \mathcal{U}^* = \begin{pmatrix} 1 & -\rho^* \\ \rho^T & 1 \end{pmatrix}$$

write down individual components:

write down individual components:

$$\begin{aligned} m_\nu &= \rho\, m_D^T + m_D\, \rho + \rho\, M_R\, \rho^T \\ 0 &= m_D - \rho\, m_D^T\, \rho^* + \rho\, M_R \\ M &= -\rho^\dagger\, m_D - m_D^T\, \rho^* + M_R \end{aligned}$$

now, ρ (aka θ from before) will be of order m_D/M_R:

$$\begin{aligned} m_\nu &= \rho\, m_D^T + m_D\, \rho + \rho\, M_R\, \rho^T \\ 0 &\simeq m_D + \rho\, M_R \Rightarrow \rho = -m_D\, M_R^{-1} \\ M &\simeq M_R \end{aligned}$$

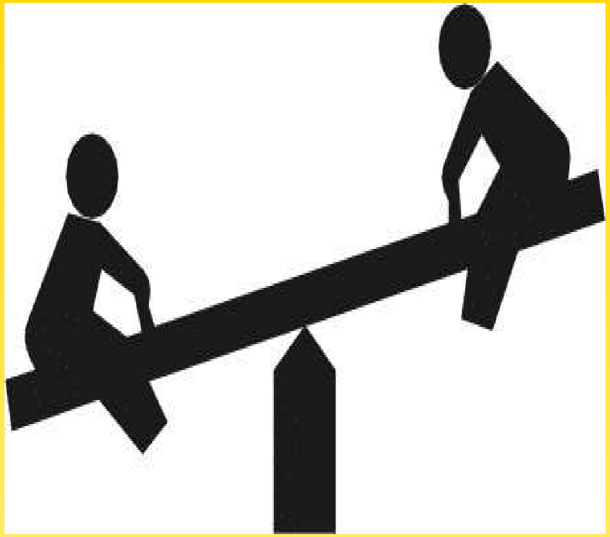
insert ρ in m_ν to find:

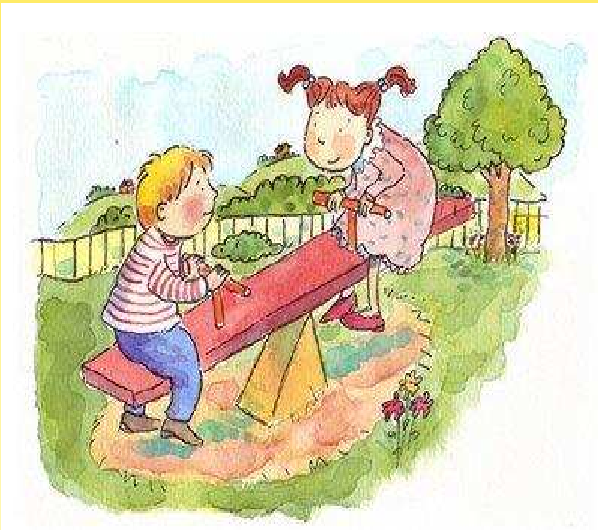
$$m_\nu = -m_D\, M_R^{-1}\, m_D^T$$

$$m_\nu = -\frac{m_D^2}{M_R}$$

(type I) See-Saw Mechanism

Minkowski; Yanagida; Glashow; Gell-Mann, Ramond, Slansky; Mohapatra, Senjanović (77-80)





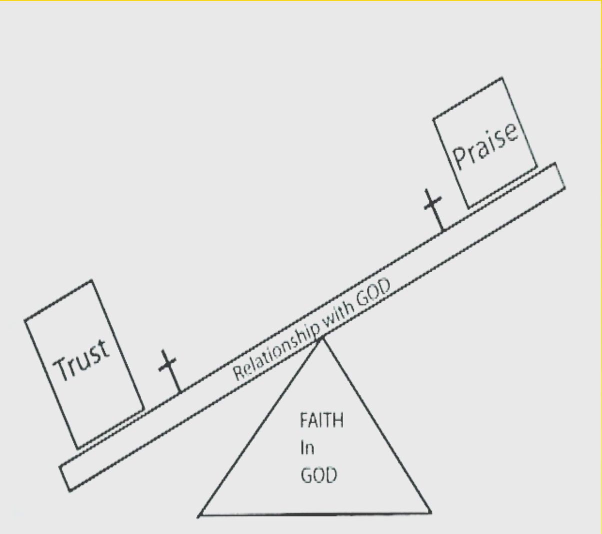


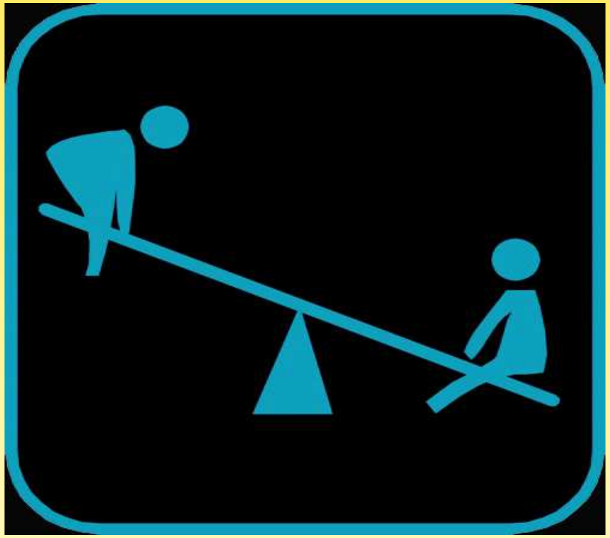
QUARKS
SEESAW
MECHANISM


SEE-SAW DON COVAY
DON COVAY SEE-SAW


THE SEESAW
Waiting


joe-ks.com


Praise
Trust
Relationship with GOD
FAITH
In
GOD



SEE SAW

See-Saw Scale

See-saw formula:

$$m_\nu = m_D^2/M_R \simeq v^2/M_R$$

with $m_\nu \simeq \sqrt{\Delta m_{\rm A}^2}$ it follows

$$\boxed{M_R \simeq 10^{15}\ \mathrm{GeV}}$$

Note: not necessarily correct. . .

Parameter Count

- m_ν: $9 = 6 + 3$
- m_D and M_R: $18 = 9 + 9$

Parametrization

$$m_\nu = -m_D\, M_R^{-1}\, m_D^T = U\, m_\nu^{\rm diag}\, U^T$$

is "solved" by

$$m_D = i\, U\, \sqrt{m_\nu^{\rm diag}}\, R\, \sqrt{M_R}$$

where complex orthogonal R contains the unknown parameters

$$R = \begin{pmatrix} \tilde c_{12}\tilde c_{13} & \tilde s_{12}\tilde c_{13} & \tilde s_{13} \\ -\tilde s_{12}\tilde c_{23} - \tilde c_{12}\tilde s_{23}\tilde s_{13} & \tilde c_{12}\tilde c_{23} - \tilde s_{12}\tilde s_{23}\tilde s_{13} & \tilde s_{23}\tilde c_{13} \\ \tilde s_{12}\tilde s_{23} - \tilde c_{12}\tilde c_{23}\tilde s_{13} & -\tilde c_{12}\tilde s_{23} - \tilde s_{12}\tilde c_{23}\tilde s_{13} & \tilde c_{23}\tilde c_{13} \end{pmatrix}$$

Note: these are now complex angles, $\tilde c_{12} = \cos\omega_{12} = \cos(\rho_{12} + i\sigma_{12})$

Higgs and Seesaw

1.) relies on Yukawa coupling with Higgs

2.) sterile neutrinos N_R couple to Higgs $\leftrightarrow$ vacuum stability bound

Paths to Neutrino Mass

approach	ingredient	$SU(2)_L \times U(1)_Y$ quantum number of messenger	$\mathcal{L}$	m_ν	scale
"SM" (Dirac mass)	RH ν	$N_R \sim (1,0)$	$h \overline{N_R} \Phi L$	hv	$h = \mathcal{O}(10^{-12})$
"effective" (dim 5 operator)	new scale + LNV	–	$h \, \overline{L^c} \, \Phi \, \Phi \, L$	$\frac{h\, v^2}{\Lambda}$	$\Lambda = 10^{14}$ GeV
"direct" (type II seesaw)	Higgs triplet + LNV	$\Delta \sim (3,-2)$	$h \overline{L^c} \Delta L + \mu \Phi \Phi \Delta$	$h v_T$	$\Lambda = \frac{1}{h\mu} M_\Delta^2$
"indirect 1" (type I seesaw)	RH ν + LNV	$N_R \sim (1,0)$	$h \overline{N_R} \Phi L + \overline{N_R} M_R N_R^c$	$\frac{(hv)^2}{M_R}$	$\Lambda = \frac{1}{h} M_R$
"indirect 2" (type III seesaw)	fermion triplets + LNV	$\Sigma \sim (3,0)$	$h \overline{\Sigma} \, L \Phi + \mathrm{Tr} \overline{\Sigma} M_\Sigma \Sigma$	$\frac{(hv)^2}{M_\Sigma}$	$\Lambda = \frac{1}{h} M_\Sigma$

plus seesaw variants (linear, double, inverse,...)

plus radiative mechanisms

plus extra dimensions

plusplusplus

c) Higgs triplet (type II)

$$\mathcal{L} \propto \overline{L}\, L^c \to \overline{\nu}\, \nu^*$$

has isospin $I_3 = +1$ and transforms as $\sim (3, +2)$

$\Rightarrow$ introduce Higgs triplet $\sim (3, -2)$ with $(I_3 = Q - Y/2)$:

$$\Delta = \begin{pmatrix} H^+ & \sqrt{2}\, H^{++} \\ \sqrt{2}\, H^0 & -H^+ \end{pmatrix} \to \begin{pmatrix} 0 & 0 \\ v_T & 0 \end{pmatrix}$$

with $SU(2)$ transformation property $\Delta \to U\, \Delta\, U^\dagger$:

$$\mathcal{L} = g_\nu\, \overline{L}\, i\tau_2\, \Delta\, L^c \xrightarrow{\text{vev}} g_\nu\, v_T\, \overline{\nu_L}\, \nu_L^c \equiv m_\nu\, \overline{\nu_L}\, \nu_L^c$$

Constraints on v_T

- $m_\nu = g_\nu\, v_T \lesssim$ eV $\Rightarrow v_T \lesssim \mathrm{eV}/g_\nu$
- ρ-parameter

$$\left(\frac{M_W}{M_Z\, \cos\theta_W}\right)^2 = \rho = \frac{\sum I_i\,(I_i + 1 - \frac{1}{4}\,Y_i^2)\,v_i^2}{\frac{1}{2}\,\sum v_i^2\,Y_i^2}$$
$$= \begin{cases} 1 & I = \frac{1}{2} \text{ and } Y = 1 \\ \dfrac{v^2 + 2\,v_T^2}{v^2 + 4\,v_T^2} & I = 1 \text{ and } Y = 2 \end{cases}$$
$$\Rightarrow v_T \lesssim 8\ \mathrm{GeV}$$

$v_T \ll v$ because

$$V = -M_\Delta^2 \, \mathrm{Tr}\left(\Delta\,\Delta^\dagger\right) + \mu\,\Phi^\dagger\, i\tau_2\,\Delta\,\Phi$$

with $\frac{\partial V}{\partial \Delta} = 0$ one has $\boxed{v_T = \frac{\mu\, v^2}{M_\Delta^2}}$

coupling of SM Higgs with triplet drives minimum v_T away from zero

v_T can be suppressed by M_Δ and/or μ

compare with Weinberg operator:

$$\boxed{\Lambda = \frac{c\, M_\Delta^2}{g_\nu\, \mu}}$$

Type II (or Triplet) See-Saw Mechanism

Magg, Wetterich; Mohapatra, Senjanovic; Lazarides, Shafi, Wetterich; Schechter, Valle (80-82)

d) Fermion triplets (type III)

The term

$$\mathcal{L} \propto \overline{L}\, \Sigma^c\, \tilde{\Phi}$$

is a singlet if $\Sigma \sim (3,0)$: "hyperchargeless triplet"

$$\Sigma = \begin{pmatrix} \Sigma^0/\sqrt{2} & \Sigma^+ \\ \Sigma^- & -\Sigma^0/\sqrt{2} \end{pmatrix}$$

additional terms in Lagrangian

$$\overline{L}\, \sqrt{2}\, Y_\Sigma\, \Sigma^c\, \tilde{\Phi} + \frac{1}{2} \mathrm{Tr}\left\{ \overline{\Sigma}\, M_\Sigma\, \Sigma^c \right\}$$

give a "Dirac mass term" $m_D^\Sigma = v\, Y_\Sigma$ and Majorana mass term M_Σ for neutral component of Σ

overall mass term for neutrinos

$$m_\nu = -\frac{(m_D^\Sigma)^2}{M_\Sigma}$$

same structure as type I see-saw

Type III See-Saw Mechanism

Foot, Lew, He, Joshi (1989)

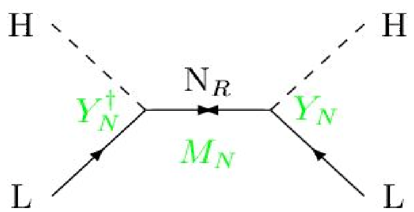
compare with Weinberg operator:

$$\Lambda = -\frac{c\,v^2}{(m_D^\Sigma)^2}\,M_\Sigma$$

The 3 basic seesaw models

→ i.e. tree level ways to generate the dim 5 operator

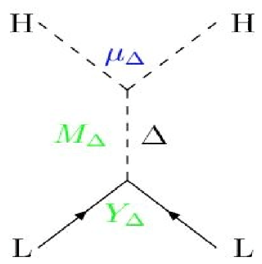
Right-handed singlet: (type-I seesaw)



$$m_\nu = Y_N^T \frac{1}{M_N} Y_N v^2$$

Minkowski; Gellman, Ramon, Slansky; Yanagida;Glashow; Mohapatra, Senjanovic

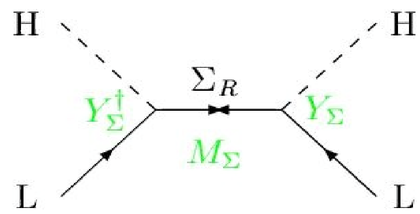
Scalar triplet: (type-II seesaw)



$$m_\nu = Y_\Delta \frac{\mu_\Delta}{M_\Delta^2} v^2$$

Magg, Wetterich; Lazarides, Shafi; Mohapatra, Senjanovic; Schechter, Valle

Fermion triplet: (type-III seesaw)



$$m_\nu = Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma v^2$$

Foot, Lew, He, Joshi; Ma; Ma, Roy;T.H., Lin, Notari, Papucci, Strumia; Bajc, Nemevsek, Senjanovic; Dorsner, Fileviez-Perez;....

slide by T. Hambye

Remarks

- Higgs and fermion triplets have SM charges $\Rightarrow$ coupling to gauge bosons in kinetic terms:
  - RG effects
  - production at colliders
  - FCNC
  - ...
- naturalness in GUTs: type I $\simeq$ type II $\gg$ type III
- note: one, two or three of the see-saw terms may be present in m_ν

Remarks

- Higgs and fermion triplets have SM charges $\Rightarrow$ coupling to gauge bosons in kinetic terms:
  - RG effects
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  - ...
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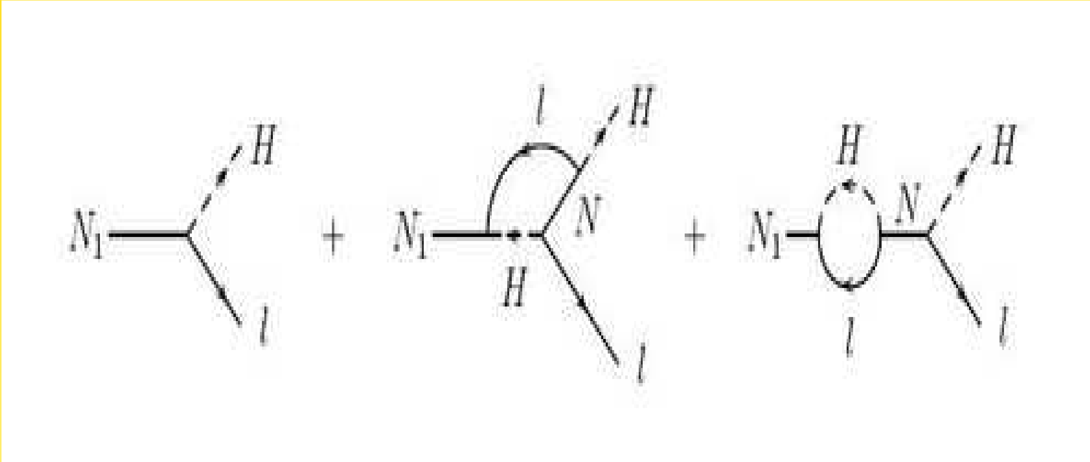
Seesaw Phenomenology: Leptogenesis

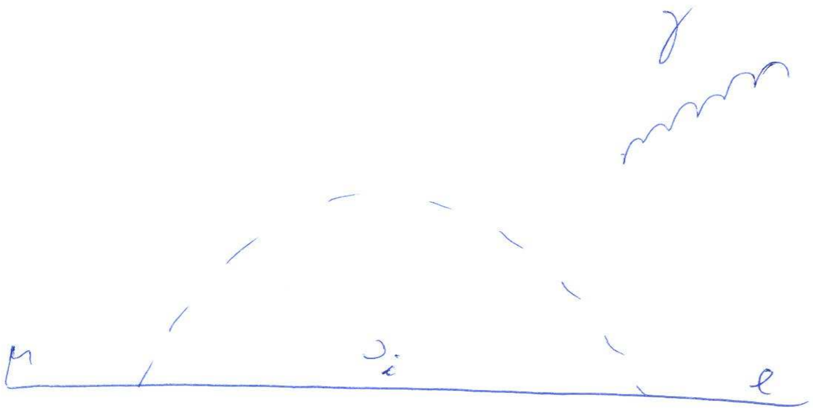


Take advantage of (L-Higgs-N) vertex in early Universe!

$$Y_B \propto \varepsilon_i^\alpha = \frac{\Gamma(N_i \to \Phi\,\bar{L}^\alpha) - \Gamma(N_i \to \Phi^\dagger\,L^\alpha)}{\Gamma(N_i \to \Phi\,\bar{L}) + \Gamma(N_i \to \Phi^\dagger\,L)}$$

Fukugita and Yanagida (1986)

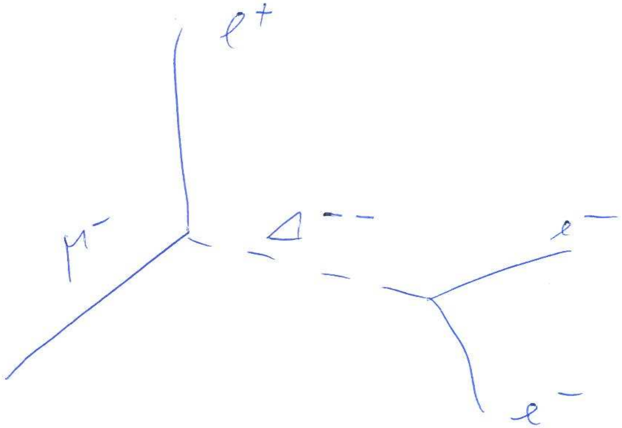
Lepton Flavor Violation in Type II seesaw



$\mu \to e\gamma$

$\sum Y_{\mu i} Y^*_{ei} \propto (m_\nu m_\nu^\dagger)_{e\mu}$

because $Y v_L = m_\nu$



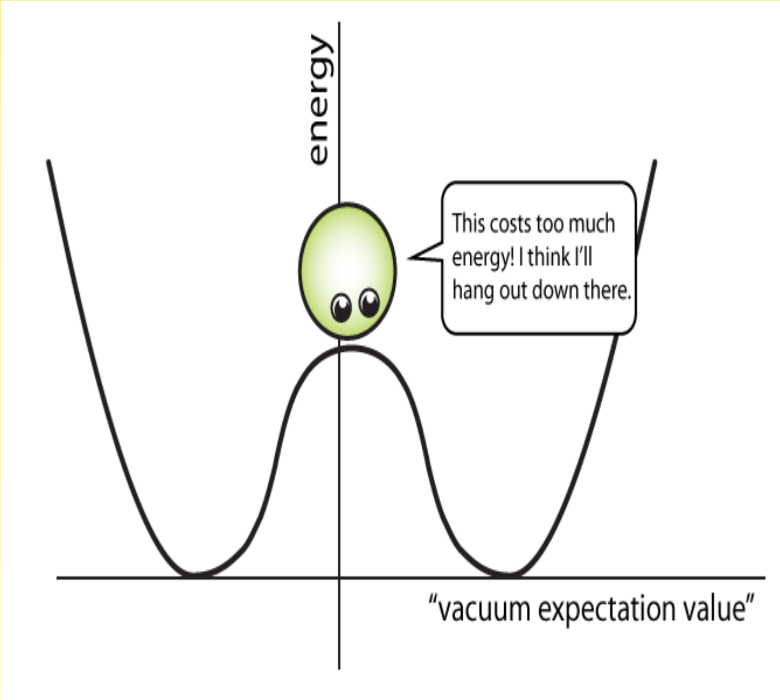
$\mu \to 3e$

$(m_\nu)_{ee} \, (m_\nu)_{e\mu}$

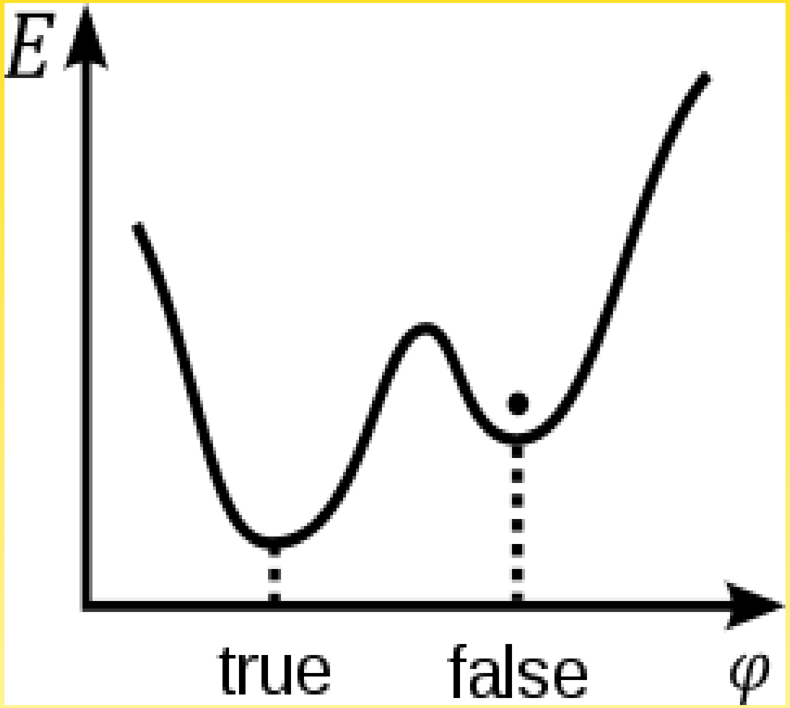
TREE-LEVEL

Phenomenology of heavy singlets: Higgs

Higgs potential is rather flat $\Rightarrow$ corrections are dangerous



$\longrightarrow$

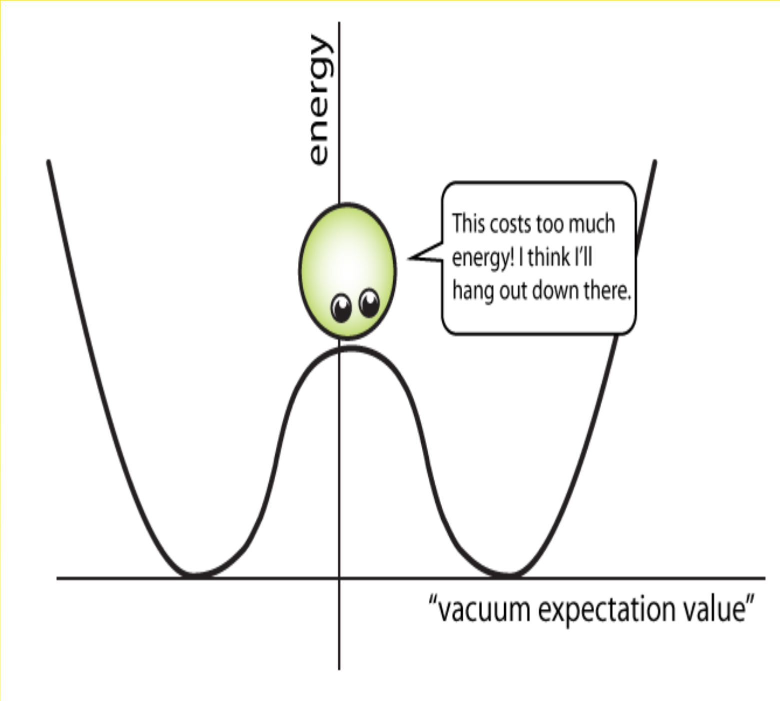


vacuum probably metastable: could tunnel to true vacuum

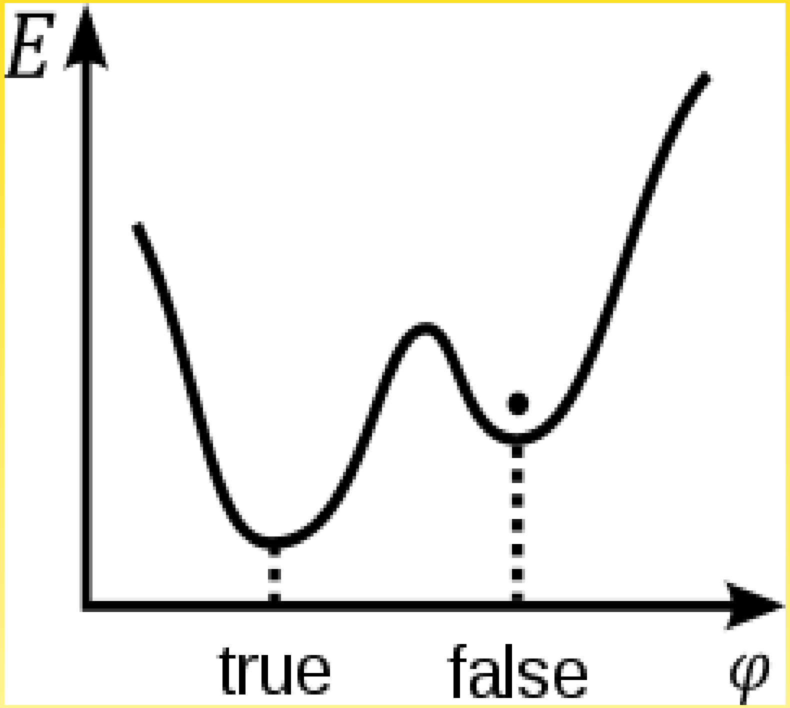
replacing all fields and forces with new fields and forces

Phenomenology of heavy singlets: Higgs

Higgs potential is rather flat $\Rightarrow$ corrections are dangerous



$\longrightarrow$



vacuum probably metastable: could tunnel to true vacuum

replacing all fields and forces with new fields and forces

(not good)

Phenomenology of heavy singlets: Higgs

Higgs coupling λ is driven to negative values by top Yukawa:

$$\beta_\lambda \propto -24\,\mathrm{Tr}\left(Y_u^\dagger Y_u\right)^2 \propto m_t^4 \Rightarrow m_h \geq f(\Lambda)$$

vacuum stability bound

With $m_h = 126$ GeV:

- vacuum probably metastable
- could be $\lambda(M_{\mathrm{Pl}}) = 0$

(Holthausen, Lim, Lindner; Bezrukov *et al.*; Degrassi *et al.*; Masina)

strong dependence on top mass, α_s

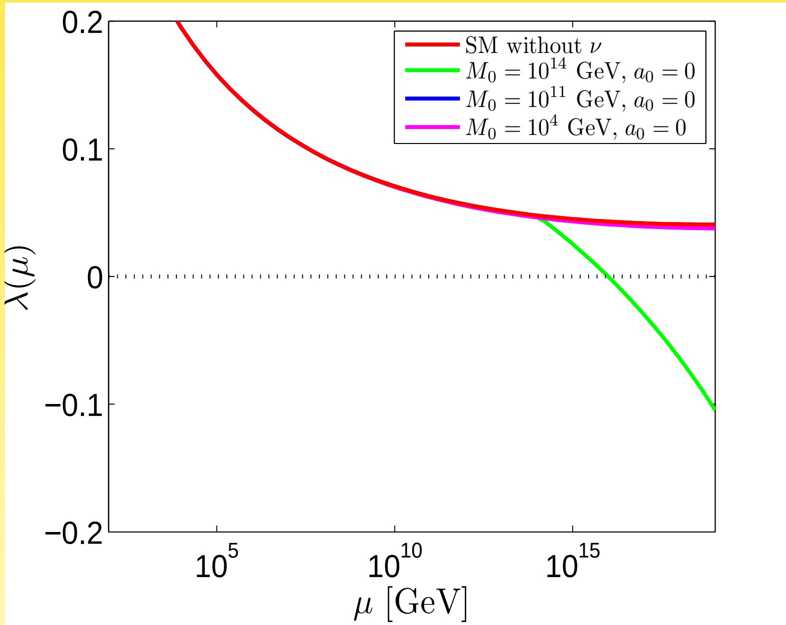
Phenomenology of heavy singlets: Higgs

Lepton-Higgs-N_R vertex: Dirac Yukawa $\bar{\nu}_L\, Y_\nu\, N_R$ contribution

$$\Delta\beta_\lambda = -8\,\mathrm{Tr}\left(Y_\nu^\dagger Y_\nu\right)^2 \propto m_D^4$$

Casas *et al.*; Strumia *et al.*; W.R., Zhang

makes vacuum stability condition worse!

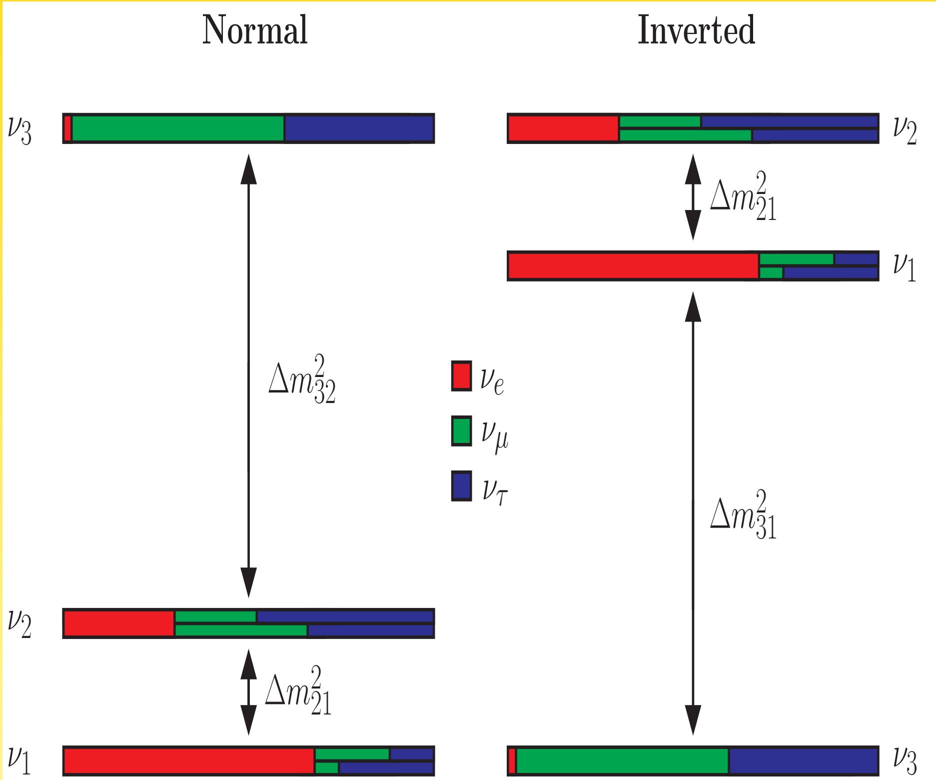


(also effect if tricks are played to produce TeV-scale N_R at colliders)

Contents

III Neutrino Mass

III3) Limits on neutrino mass(es)

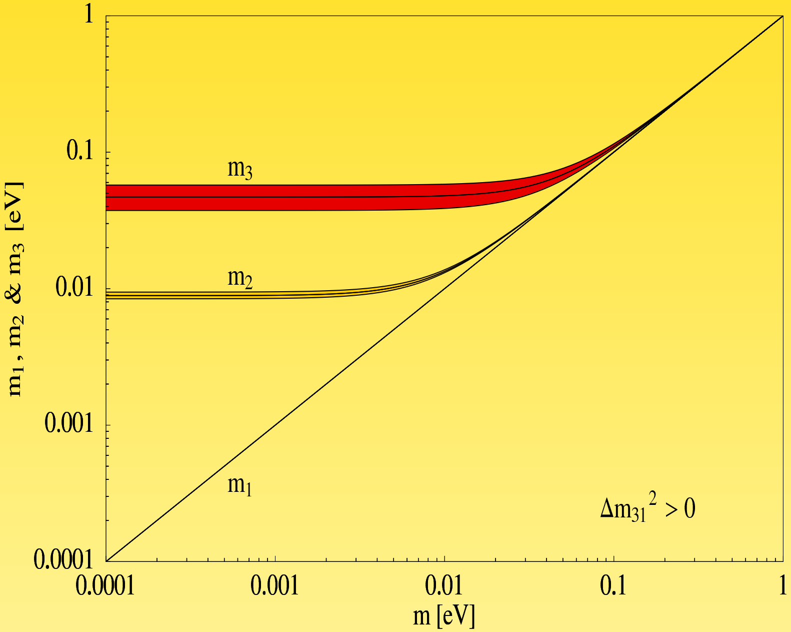


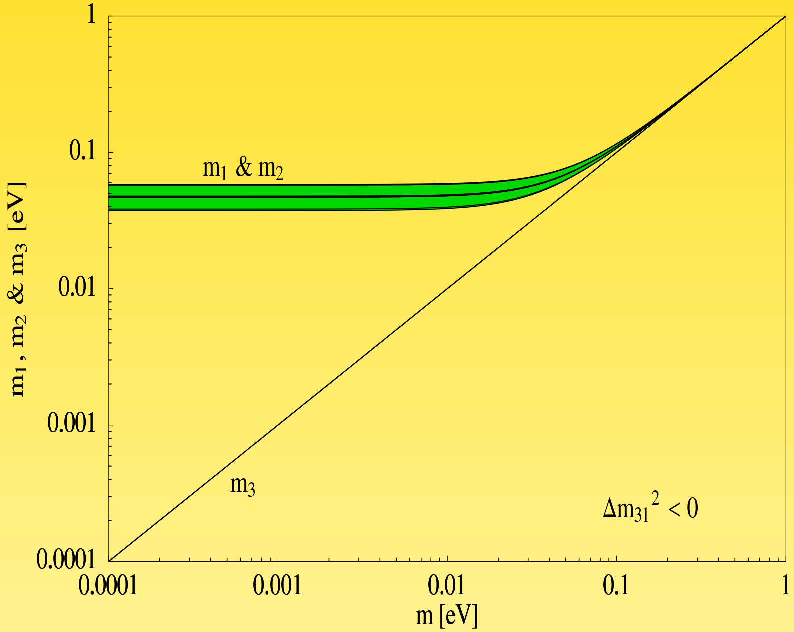
$\Delta m^2_{21} = 8 \times 10^{-5}\ \mathrm{eV}^2$ and $|\Delta m^2_{31}| \simeq |\Delta m^2_{32}| = 2 \times 10^{-3}\ \mathrm{eV}^2$

- normal ordering: $\Delta m^2_{31} > 0$
- inverted ordering: $\Delta m^2_{31} < 0$

Neutrino masses

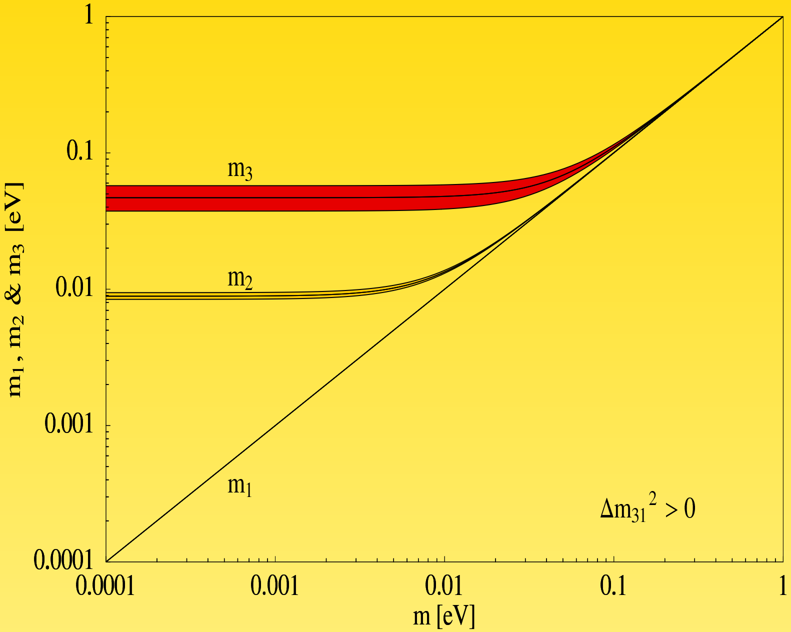
- neutrino masses $\leftrightarrow$ (scale of their origin)$^{-1}$
- neutrino mass ordering $\leftrightarrow$ form of m_ν

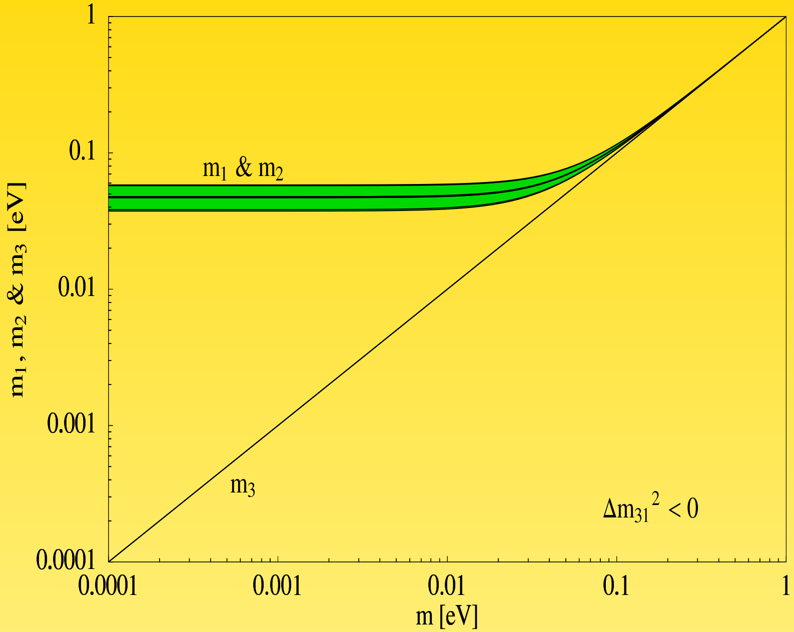




- $m_3^2 \simeq \Delta m_A^2 \gg m_2^2 \simeq \Delta m_\odot^2 \gg m_1^2$: normal hierarchy (NH)
- $m_2^2 \simeq |\Delta m_A^2| \simeq m_1^2 \gg m_3^2$: inverted hierarchy (IH)
- $m_3^2 \simeq m_2^2 \simeq m_1^2 \equiv m_0^2 \gg \Delta m_A^2$: quasi-degeneracy (QD)

Neutrino masses





Neutrino mass hierarchy is moderate!

$$\text{NH}: \qquad \frac{m_2}{m_3} \geq \sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_\text{A}}} \simeq \frac{1}{5}$$

$$\text{IH}: \qquad \frac{m_1}{m_2} \gtrsim 1 - \frac{1}{2}\,\frac{\Delta m^2_\odot}{\Delta m^2_\text{A}} \simeq 0.98$$

Upper limits:

- direct searches (Kurie plot of ^{3}H)

$$m(\nu_e) = \sqrt{\sum |U_{ei}|^2 \, m_i^2} \leq 2.3 \text{ eV} \text{ at 95 \% C.L. (Mainz, Troitsk)}$$

- cosmology (hot dark matter)

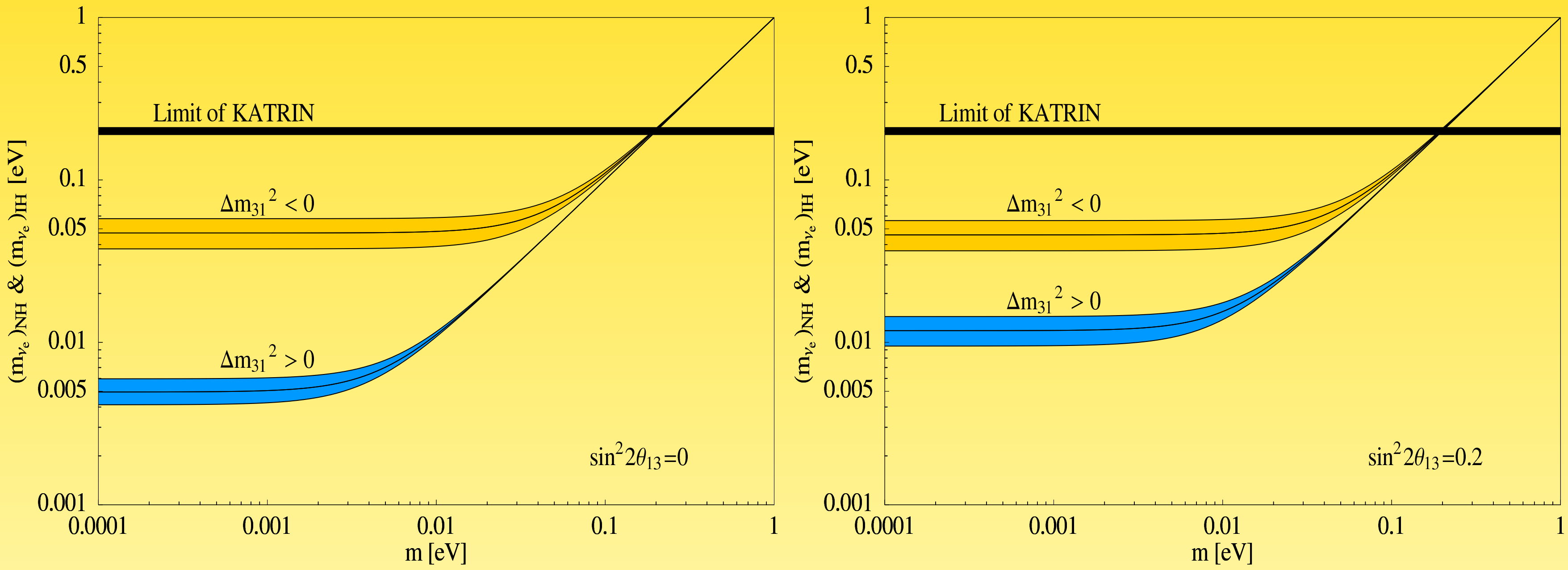
$$\Sigma = \sum m_i \lesssim \text{eV}$$

- neutrino-less double beta decay ($0\nu\beta\beta$)

$$|m_{ee}| = \left| \sum U_{ei}^2 \, m_i \right| \lesssim \text{eV}$$

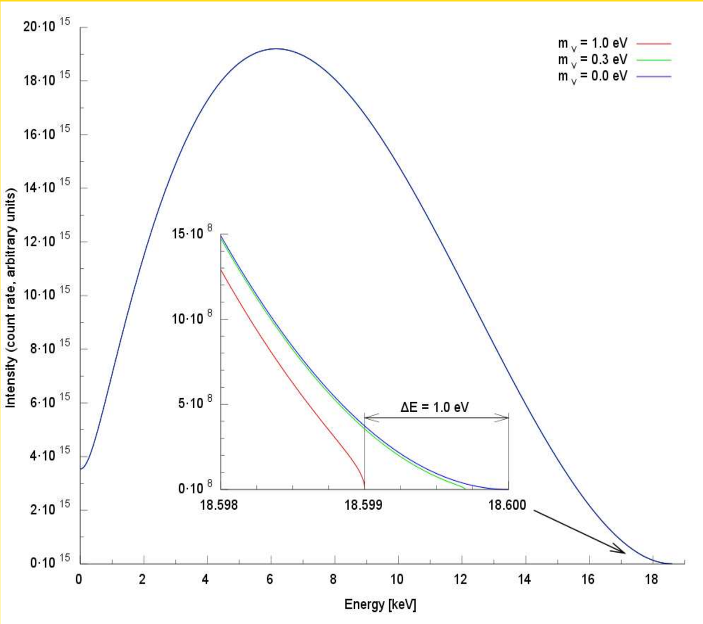
$$\underline{m(\nu_e) = \sqrt{\sum |U_{ei}|^2\, m_i^2}}$$

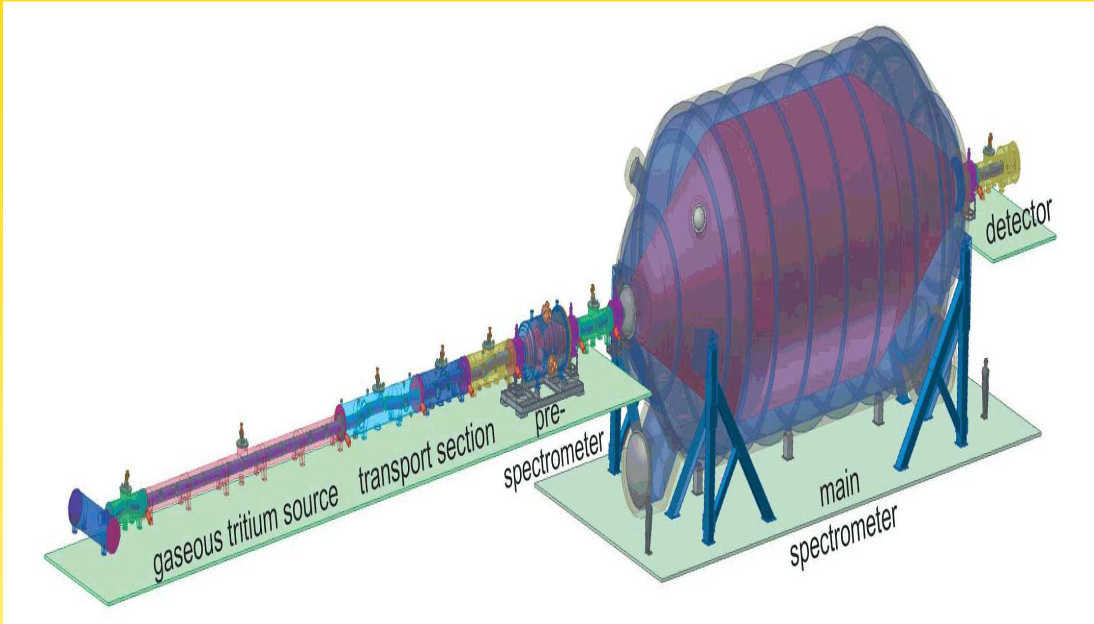
$$m(\nu_e)^{\rm NH} \simeq \sqrt{s_{12}^2\, c_{13}^2\, \Delta m_\odot^2 + s_{13}^2\, \Delta m_{\rm A}^2} \ll m(\nu_e)^{\rm IH} \simeq \sqrt{c_{13}^2\, \Delta m_{\rm A}^2}$$



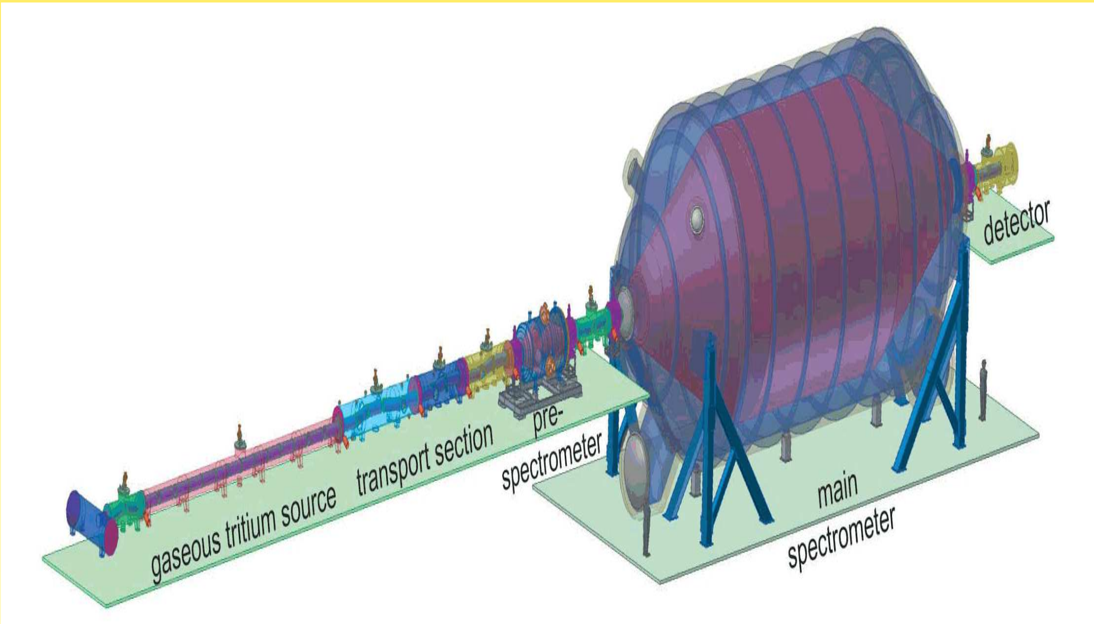
- almost independent on mixing angles
- difference of normal and inverted shows up well below KATRIN limit

KATRIN

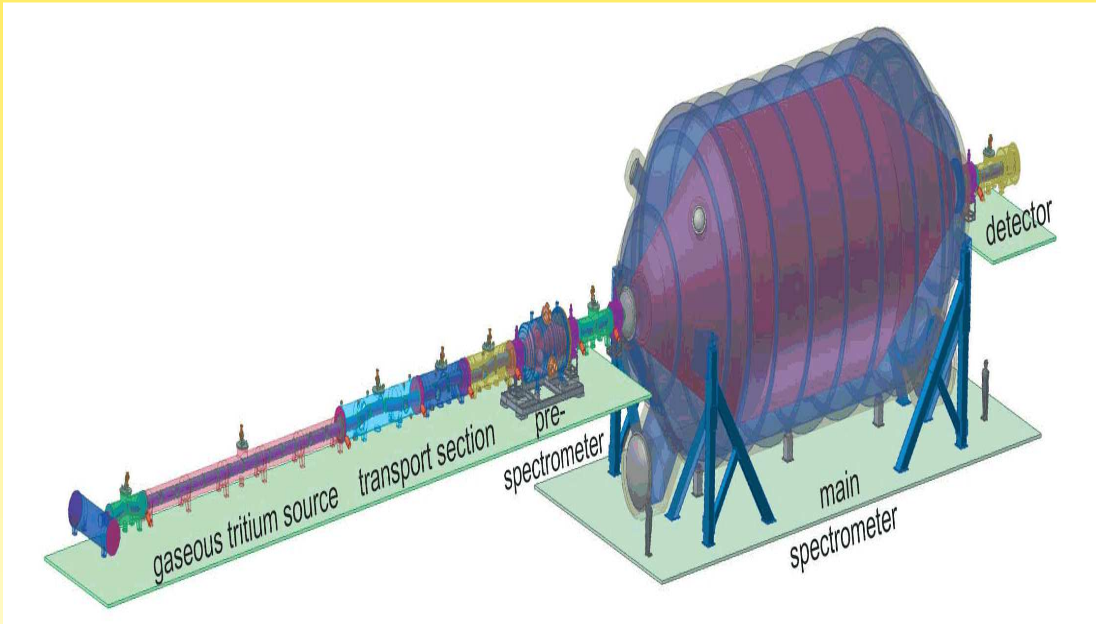




KATRIN
Karlsruhe Tritium Neutrino Experiment


gaseous tritium source
transport section
pre-
spectrometer
main
spectrometer
detector


ν


gaseous tritium source
transport section
pre-
spectrometer
main
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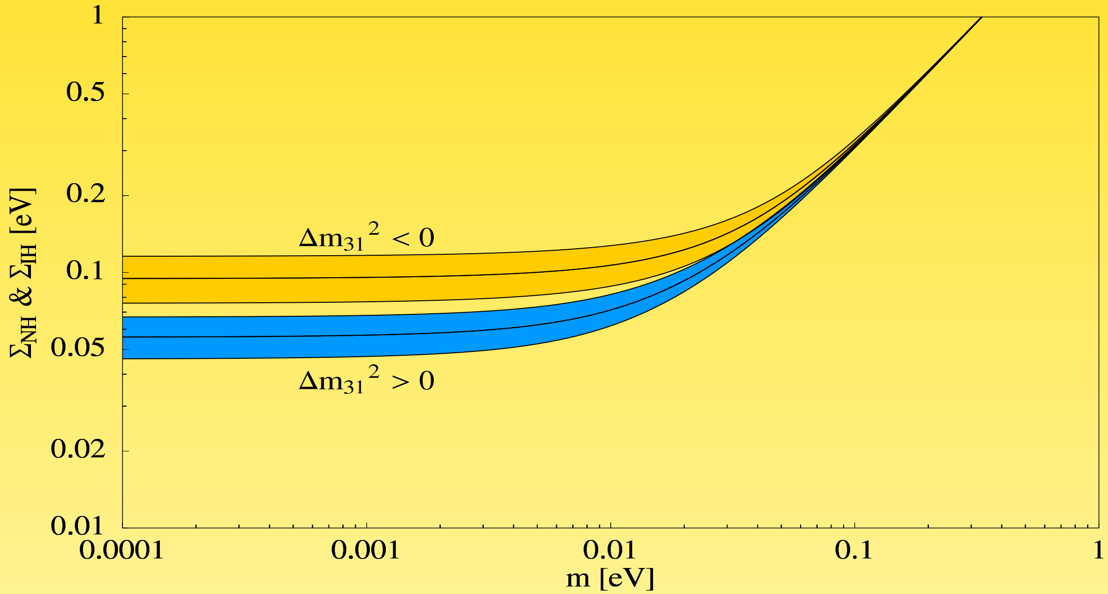
detector


778
2006/10/06 11:12


UNITED KINGDOM
IRELAND
Dublin
Belfast
London
Celtic Sea
Bay of Biscay
FRANCE
Paris
GERMANY
Berlin
POLAND
Warsaw
UKRAINE
BELARUS
ROMANIA
Bucharest
Black Sea
BULGARIA
TURKEY
GREECE
ITALY
Rome
SPAIN
Madrid
PORTUGAL
Lisbon
Mediterranean Sea
ALGERIA
MOROCCO
TUNISIA
Tyrrhenian Sea
Adriatic Sea
Ionian Sea
Aegean Sea
Vienna
Budapest
Belgrade
Bratislava
Constanța
Rotterdam
Istanbul
Scale 1: 19,500,000
Lambert Conformal Conic Projection, standard parallels 40°N and 56°N
300 Kilometers
300 Miles

$\Sigma = \sum m_i$

$$\Sigma^{\rm NH} \simeq \sqrt{\Delta m^2_{\rm A}} < \Sigma^{\rm IH} \simeq 2\sqrt{\Delta m^2_{\rm A}}$$



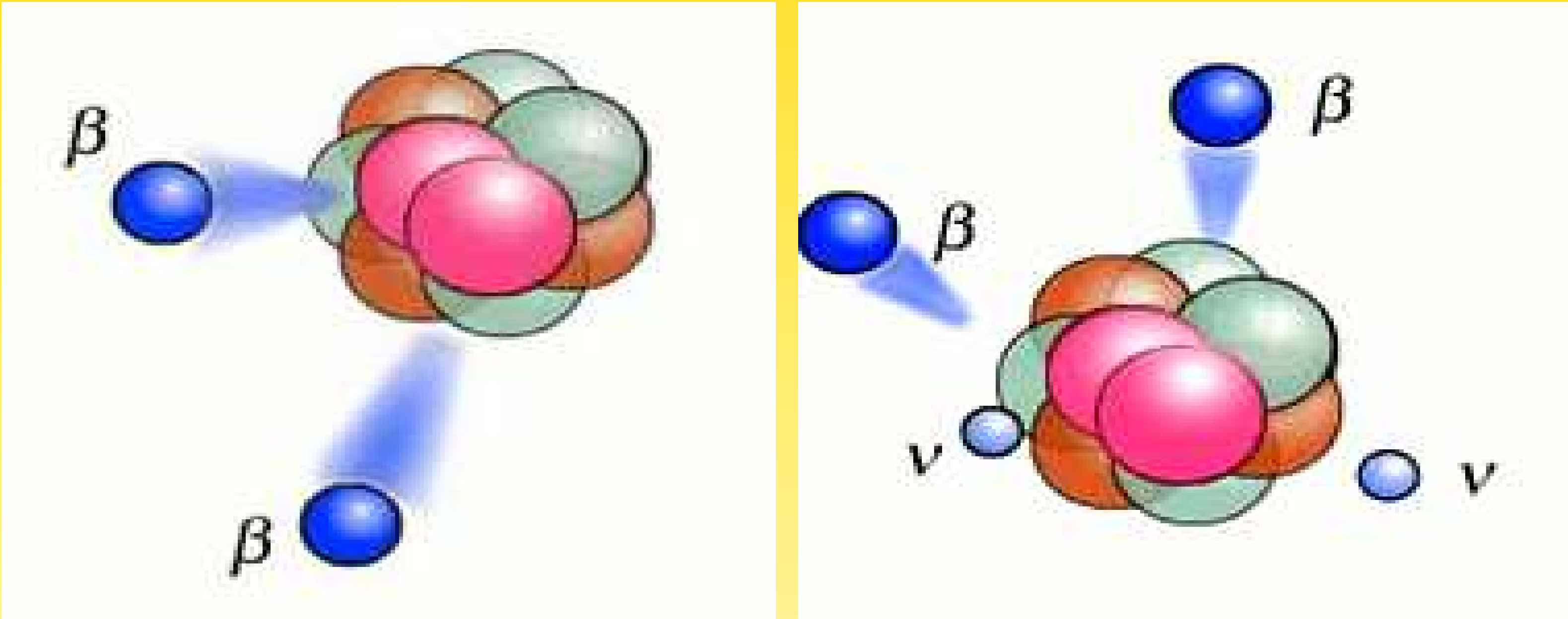
- independent on mixing angles
- systematics?

Contents

III Neutrino Mass

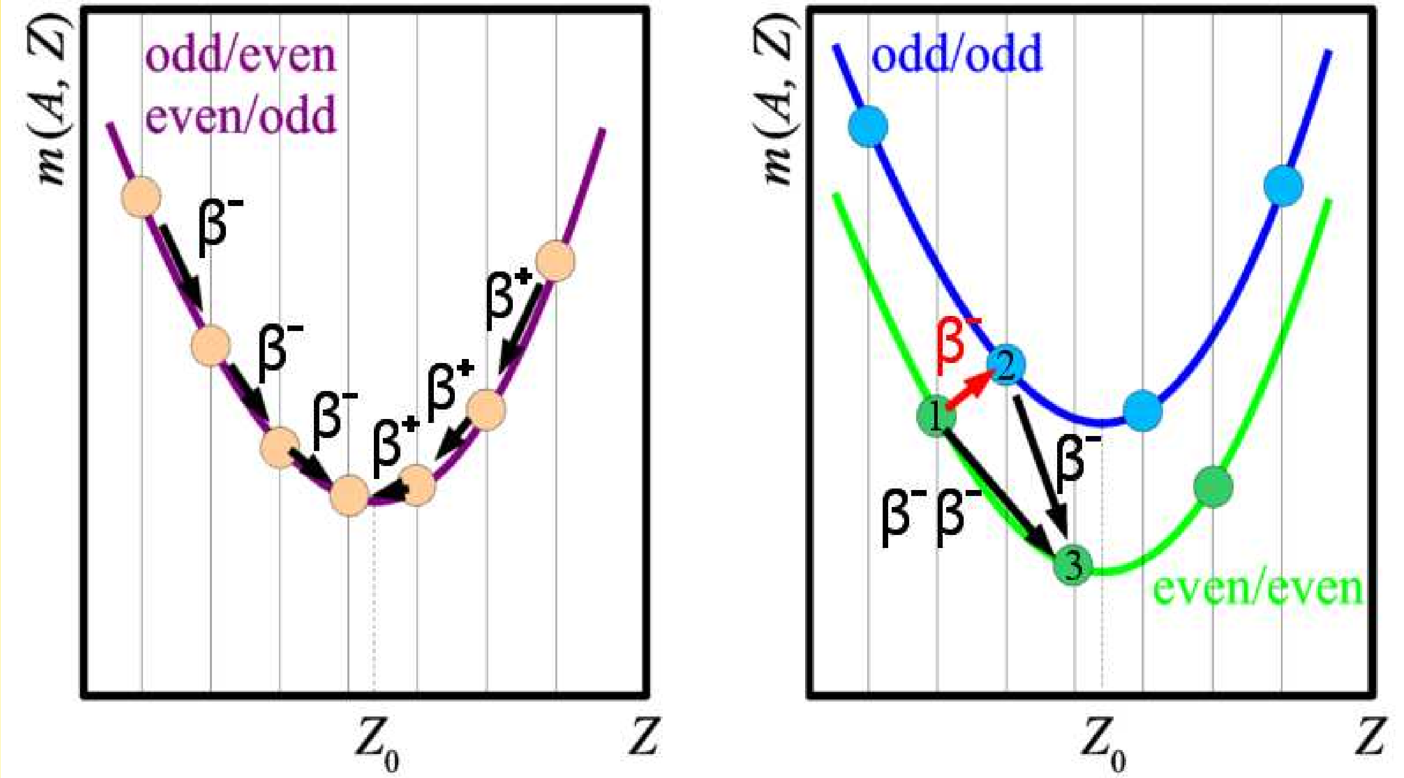
III4) Neutrinoless double beta decay

$$(A, Z) \to (A, Z+2) + 2\,e^- \quad (0\nu\beta\beta)$$



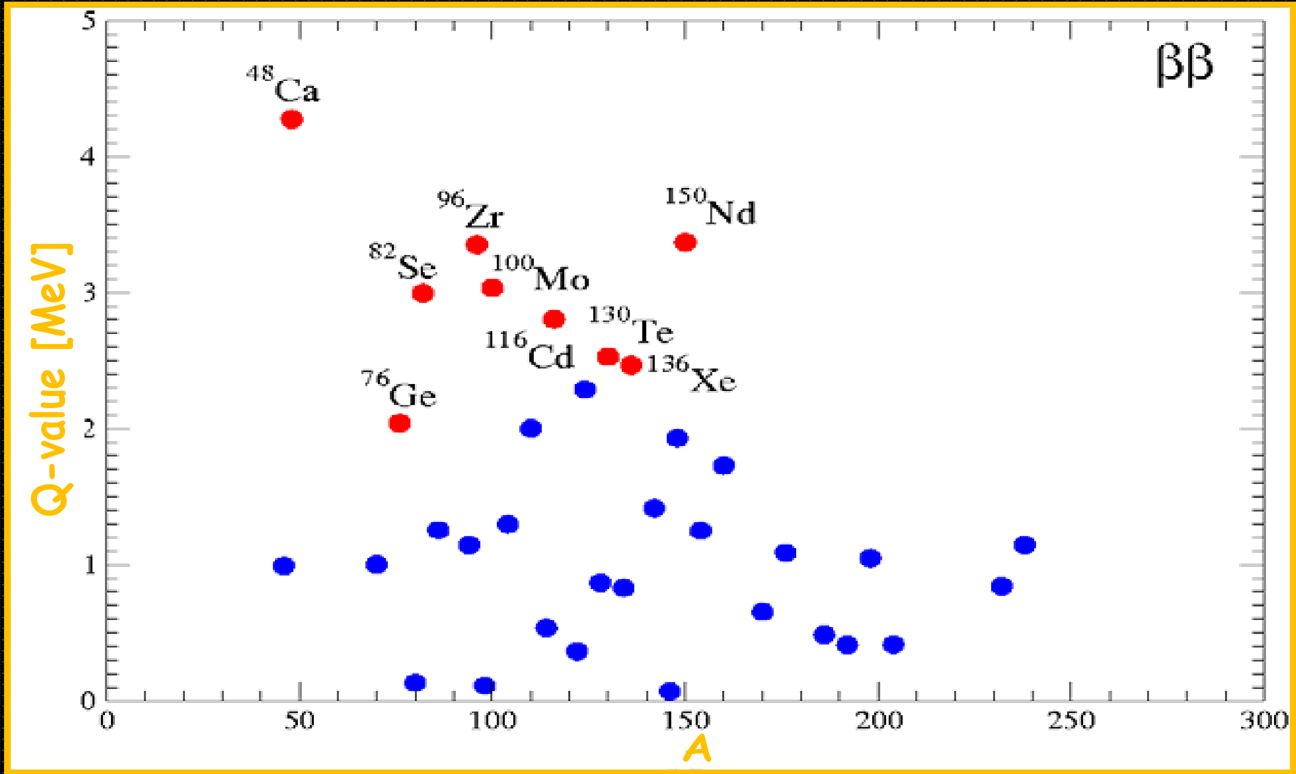
- second order in weak interaction: $\Gamma \propto G_F^4 \Rightarrow$ rare!
- not to be confused with $(A, Z) \to (A, Z+2) + 2\,e^- + 2\,\bar{\nu}_e \quad (2\nu\beta\beta)$ (which occurs more often but is still rare)

Need to forbid single β decay:



- $E_{\text{Bindung}} = E_{\text{Volumen}} - E_{\text{Oberfläche}} - E_{\text{Coulomb}} - E_{\text{Symmetrie}} \pm E_{\text{Paarbildung}}$
- $\Rightarrow$ even/even $\rightarrow$ even/even
- either direct ($0\nu\beta\beta$) or two simultaneous decays with virtual (energetically forbidden) intermediate state ($2\nu\beta\beta$)

How many nuclei in this condition?



Slide by A. Giuliani

Upcoming/running experiments: exciting time!!

best limit was from 2001...

Name	Isotope	source = detector; calorimetric with			source ≠ detector
		high energy res.	low energy res.	event topology	event topology
AMoRE	^{100}Mo	✓	–	–	–
CANDLES	^{48}Ca	–	✓	–	–
COBRA	^{116}Cd	–	–	✓	–
CUORE	^{130}Te	✓	–	–	–
DCBA	^{150}Nd	–	–	–	✓
EXO	^{136}Xe	–	–	✓	–
GERDA	^{76}Ge	✓	–	–	–
KamLAND-Zen	^{136}Xe	–	✓	–	–
LUCIFER	^{82}Se	✓	–	–	–
MAJORANA	^{76}Ge	✓	–	–	–
MOON	^{100}Mo	–	–	–	✓
NEXT	^{136}Xe	–	–	✓	–
SNO+	^{130}Te	–	✓	–	–
SuperNEMO	^{82}Se	–	–	–	✓
XMASS	^{136}Xe	–	✓	–	–

Experiment	Isotope	Mass of Isotope [kg]	Sensitivity $T^{0\nu}_{1/2}$ [yrs]	Status	Start of data-taking
GERDA	^{76}Ge	18	3×10^{25}	running	$\sim$ 2011
		40	2×10^{26}	in progress	$\sim$ 2012
		1000	6×10^{27}	R&D	$\sim$ 2015
CUORE	^{130}Te	200	$6.5 \times 10^{26*}$	in progress	$\sim$ 2013
			$2.1 \times 10^{26**}$		
MAJORANA	^{76}Ge	30-60	$(1-2) \times 10^{26}$	in progress	$\sim$ 2013
		1000	6×10^{27}	R&D	$\sim$ 2015
EXO	^{136}Xe	200	6.4×10^{25}	in progress	$\sim$ 2011
		1000	8×10^{26}	R&D	$\sim$ 2015
SuperNEMO	^{82}Se	100-200	$(1-2) \times 10^{26}$	R&D	$\sim$ 2013-15
KamLAND-Zen	^{136}Xe	400	4×10^{26}	in progress	$\sim$ 2011
		1000	10^{27}	R&D	$\sim$ 2013-15
SNO+	^{150}Nd	56	4.5×10^{24}	in progress	$\sim$ 2012
		500	3×10^{25}	R&D	$\sim$ 2015

Interpretation of Experiments

Master formula:

$$\Gamma^{0\nu} = G_x(Q, Z)\,|\mathcal{M}_x(A, Z)\,\eta_x|^2$$

- $G_x(Q, Z)$: phase space factor
- $\mathcal{M}_x(A, Z)$: nuclear physics
- η_x: particle physics

Interpretation of Experiments

Master formula:

$$\Gamma^{0\nu} = G_x(Q,Z)\,|\mathcal{M}_x(A,Z)\,\eta_x|^2$$

- $G_x(Q,Z)$: phase space factor; calculable
- $\mathcal{M}_x(A,Z)$: nuclear physics; problematic
- η_x: particle physics; interesting

3 Reasons for Multi-isotope determination

1.) credibility

2.) test NME calculation

$$\frac{T^{0\nu}_{1/2}(A_1,Z_1)}{T^{0\nu}_{1/2}(A_2,Z_2)} = \frac{G(Q_2,Z_2)}{G(Q_1,Z_1)} \frac{|\mathcal{M}(A_2,Z_2)|^2}{|\mathcal{M}(A_1,Z_1)|^2}$$

systematic errors drop out, ratio sensitive to NME model

3.) test mechanism

$$\frac{T^{0\nu}_{1/2}(A_1,Z_1)}{T^{0\nu}_{1/2}(A_2,Z_2)} = \frac{G_x(Q_2,Z_2)}{G_x(Q_1,Z_1)} \frac{|\mathcal{M}_x(A_2,Z_2)|^2}{|\mathcal{M}_x(A_1,Z_1)|^2}$$

particle physics drops out, ratio of NMEs sensitive to mechanism

Why should we probe Lepton Number Violation?

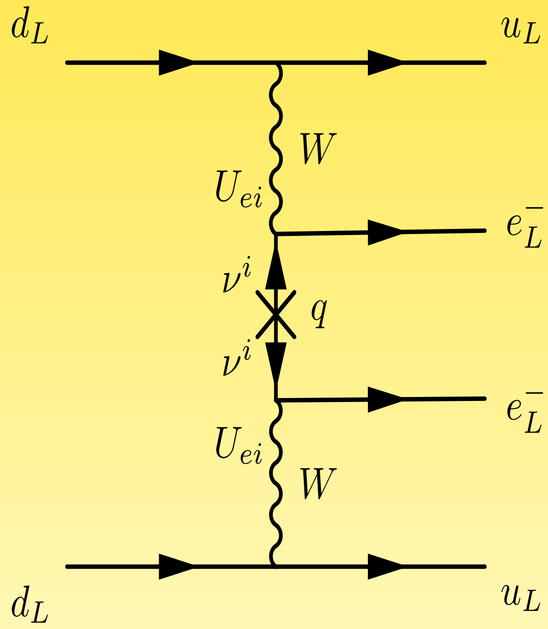
- L and B *accidentally* conserved in SM
- effective theory: $\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda}\,\mathcal{L}_{\rm LNV} + \frac{1}{\Lambda^2}\,\mathcal{L}_{\rm LFV,\,BNV,\,LNV} + \ldots$
- baryogenesis: B is violated
- B, L often connected in GUTs
- GUTs have seesaw and Majorana neutrinos
- (chiral anomalies: $\partial_\mu J^\mu_{B,L} = c\, G_{\mu\nu}\, \tilde{G}^{\mu\nu} \neq 0$ with $J^B_\mu = \sum \overline{q_i}\, \gamma_\mu\, q_i$ and $J^L_\mu = \sum \overline{\ell_i}\, \gamma_\mu\, \ell_i$)

$\Rightarrow$ Lepton Number Violation as important as Baryon Number Violation

($0\nu\beta\beta$ is much more than a neutrino mass experiment)

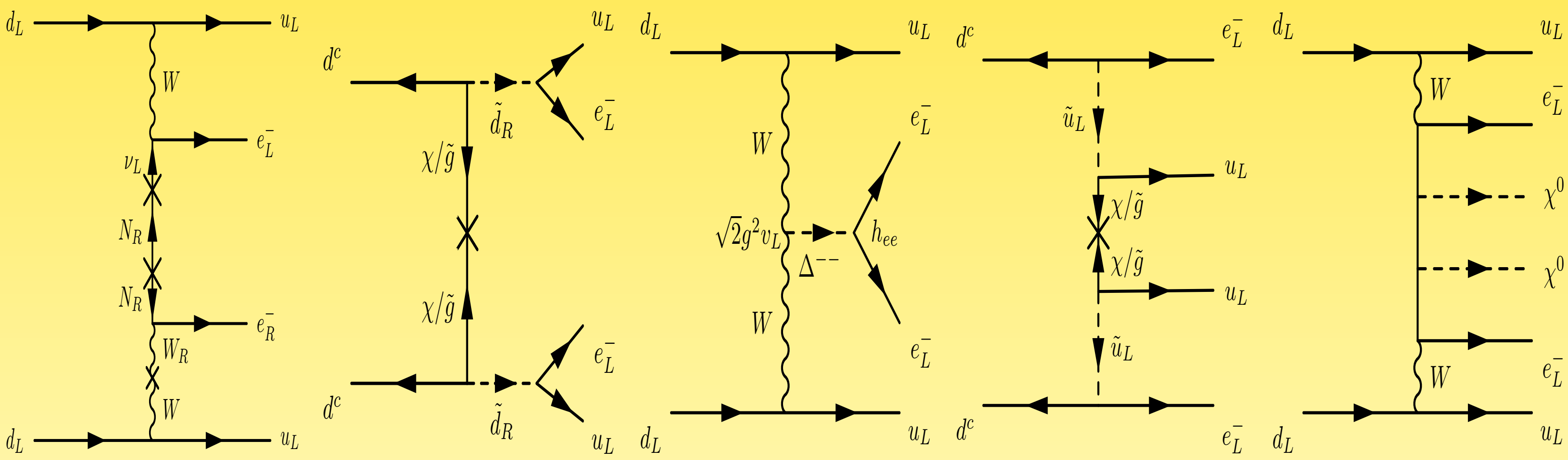
Standard Interpretation

Neutrinoless Double Beta Decay is mediated by light and massive Majorana neutrinos (the ones which oscillate) and all other mechanisms potentially leading to $0\nu\beta\beta$ give negligible or no contribution

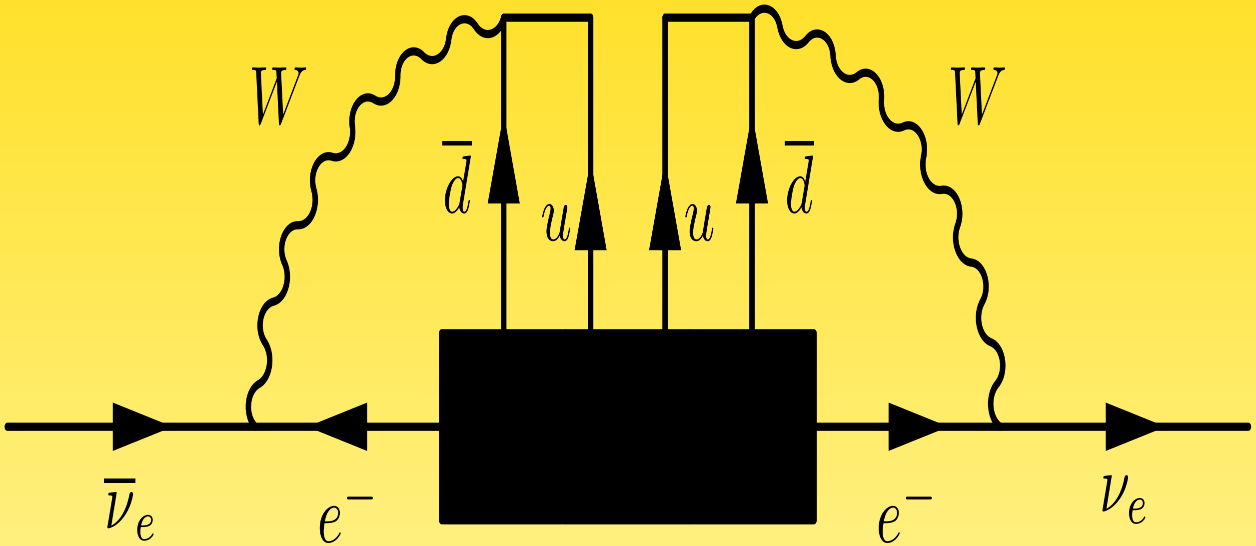


Non-Standard Interpretations:

There is at least one other mechanism leading to Neutrinoless Double Beta Decay and its contribution is at least of the same order as the light neutrino exchange mechanism



Schechter-Valle theorem: no matter what process, neutrinos are Majorana:



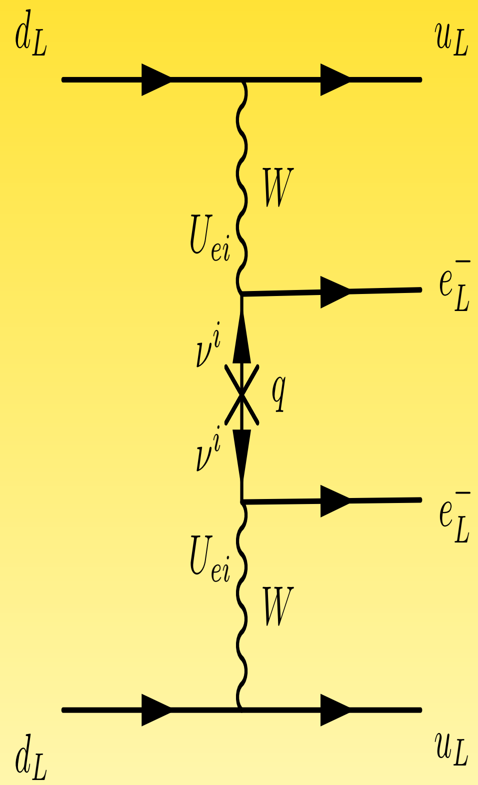
Blackbox diagram is 4 loop:

$$m_\nu^{\mathrm{BB}} \sim \frac{G_F^2}{(16\pi^2)^4}\mathrm{MeV}^5 \sim 10^{-25}\,\mathrm{eV}$$

mechanism	physics parameter	current limit	test
light neutrino exchange	$\left\|U_{ei}^2\, m_i\right\|$	0.5 eV	oscillations, cosmology, neutrino mass
heavy neutrino exchange	$\left\|\frac{S_{ei}^2}{M_i}\right\|$	2×10^{-8} GeV^{-1}	LFV, collider
heavy neutrino and RHC	$\left\|\frac{V_{ei}^2}{M_i\, M_{W_R}^4}\right\|$	4×10^{-16} GeV^{-5}	flavor, collider
Higgs triplet and RHC	$\left\|\frac{(M_R)_{ee}}{m_{\Delta_R}^2\, M_{W_R}^4}\right\|$	10^{-15} GeV^{-1}	flavor, collider e^- distribution
λ-mechanism with RHC	$\left\|\frac{U_{ei}\, \tilde{S}_{ei}}{M_{W_R}^2}\right\|$	1.4×10^{-10} GeV^{-2}	flavor, collider, e^- distribution
η-mechanism with RHC	$\tan\zeta\, \left\|U_{ei}\, \tilde{S}_{ei}\right\|$	6×10^{-9}	flavor, collider, e^- distribution
short-range $\not{R}$	$\frac{\left\|\lambda'^2_{111}\right\|}{\Lambda_{SUSY}^5}$ $\Lambda_{SUSY} = f(m_{\tilde{g}}, m_{\tilde{u}_L}, m_{\tilde{d}_R}, m_{\chi_i})$	7×10^{-18} GeV^{-5}	collider, flavor
long-range $\not{R}$	$\left\|\sin 2\theta^b\, \lambda'_{131}\, \lambda'_{113} \left(\frac{1}{m_{\tilde{b}_1}^2} - \frac{1}{m_{\tilde{b}_2}^2}\right)\right\|$ $\sim \frac{G_F}{q}\, m_b \frac{\left\|\lambda'_{131}\, \lambda'_{113}\right\|}{\Lambda_{SUSY}^3}$	2×10^{-13} GeV^{-2} 1×10^{-14} GeV^{-3}	flavor, collider
Majorons	$\|\langle g_\chi \rangle\|$ or $\|\langle g_\chi \rangle\|^2$	$10^{-4} \ldots 1$	spectrum, cosmology

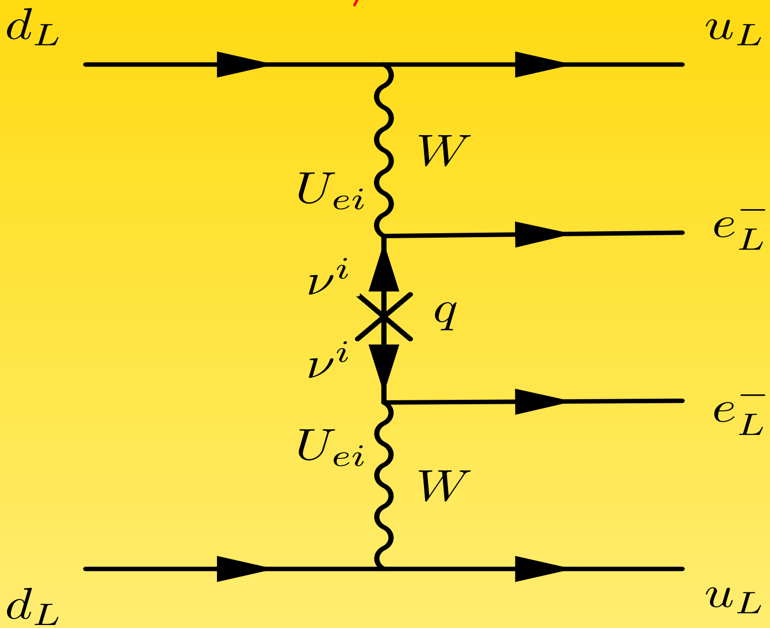
Lepton Number Violation and Majorana Neutrinos: Neutrinoless Double Beta Decay

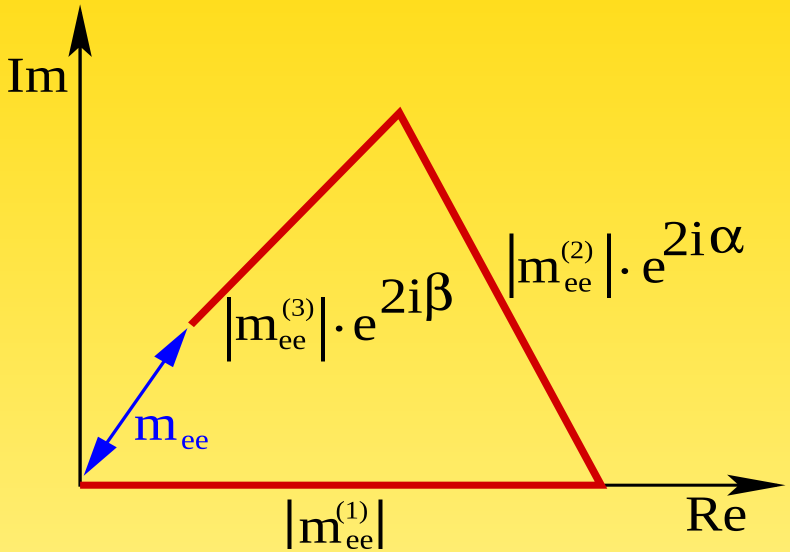
$$nn \to pp\, e^- e^-$$



has two requirements: $\nu = \nu^c$ AND $m_\nu \neq 0$

$\Delta L \neq 0$: Neutrinoless Double Beta Decay





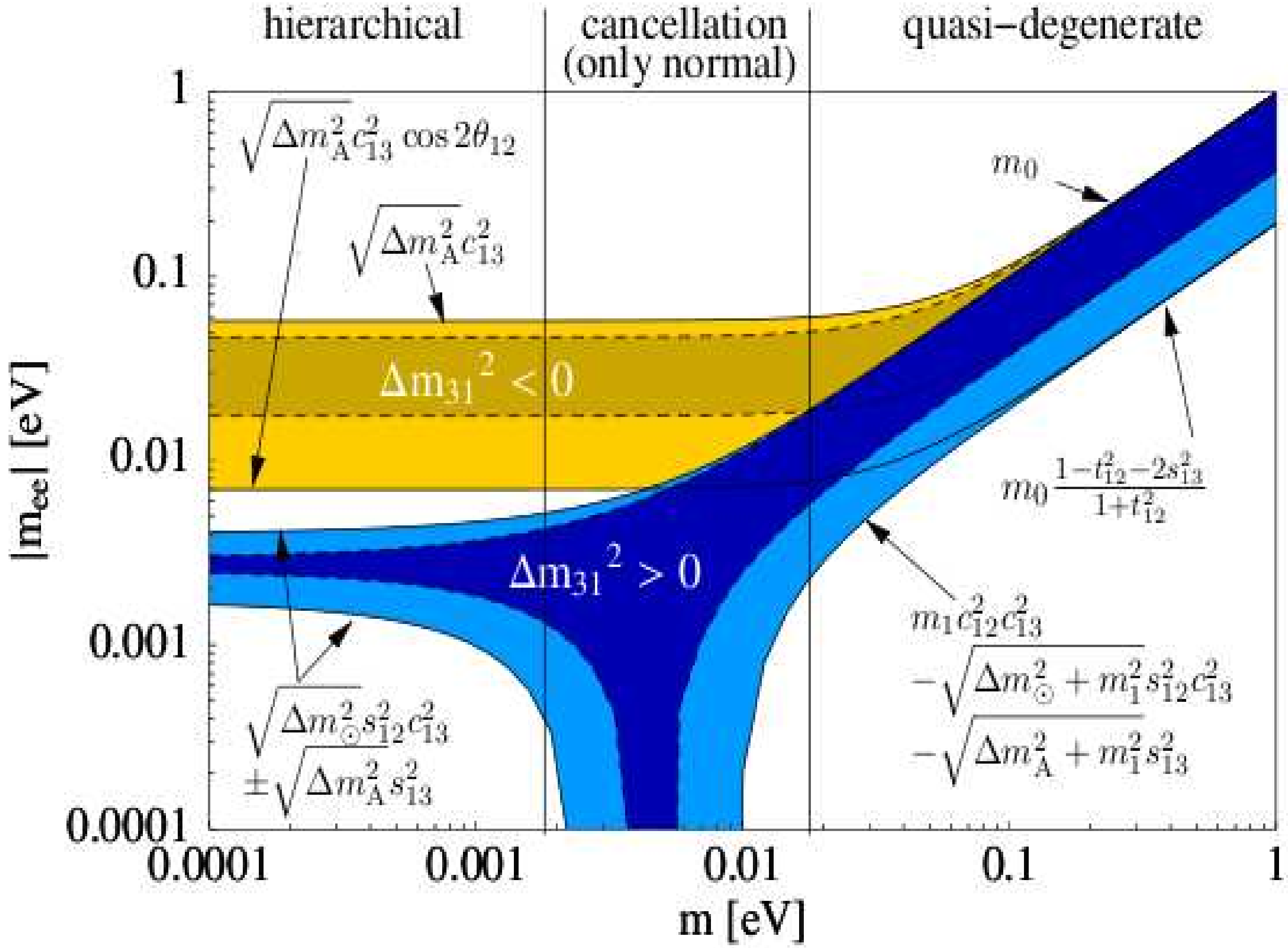
Amplitude proportional to coherent sum (<u>"effective mass"</u>):

$$|m_{ee}| \equiv \left|\sum U_{ei}^2 \, m_i\right| = \left||U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, m_2 \, e^{2i\alpha} + |U_{e3}|^2 \, m_3 \, e^{2i\beta}\right|$$

$$= f\left(\theta_{12}, m_i, |U_{e3}|, \mathrm{sgn}(\Delta m_{\mathrm{A}}^2), \alpha, \beta\right)$$

7 out of 9 parameters of m_ν!

The usual plot

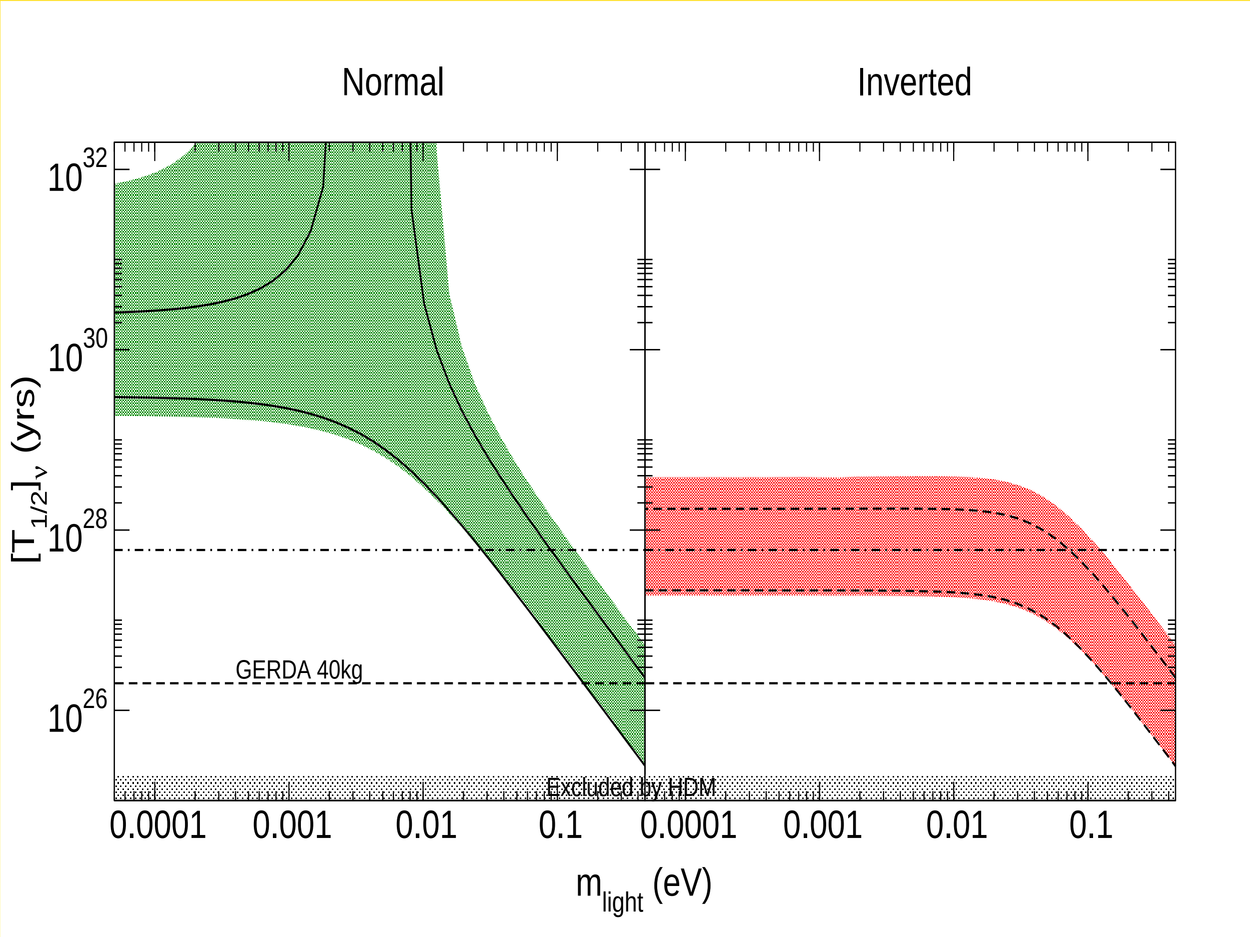


Crucial points

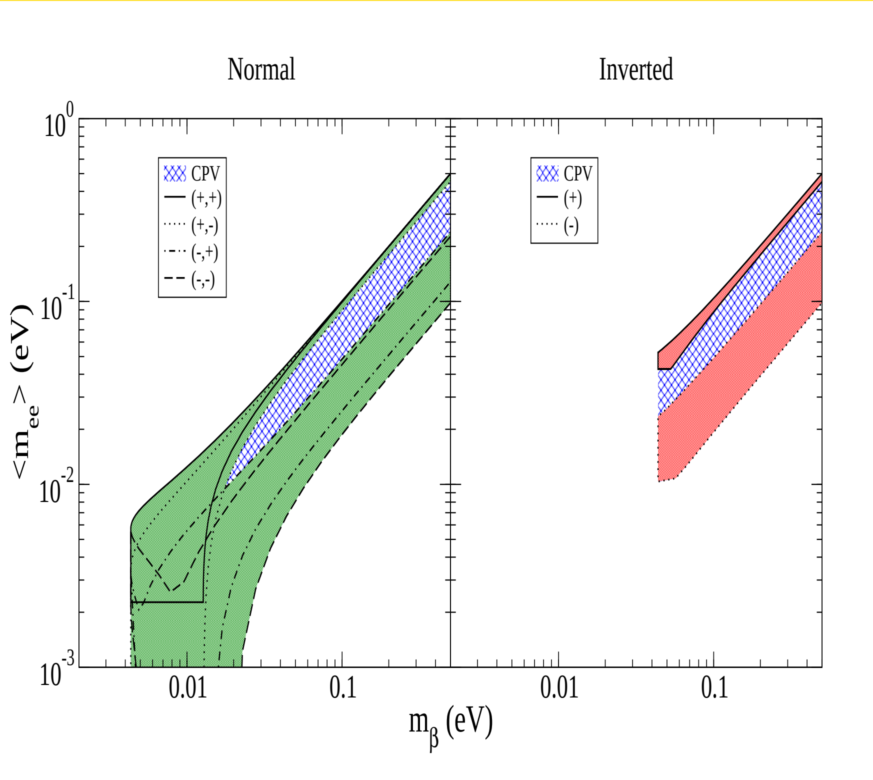
- NH: can be zero!
- IH: cannot be zero! $|m_{ee}|^{\rm inh}_{\rm min} = \sqrt{\Delta m^2_{\rm A}}\, c_{13}^2\, \cos 2\theta_{12}$
- gap between $|m_{ee}|^{\rm inh}_{\rm min}$ and $|m_{ee}|^{\rm nor}_{\rm max}$
- QD: cannot be zero! $|m_{ee}|^{\rm QD}_{\rm min} = m_0\, c_{13}^2\, \cos 2\theta_{12}$
- QD: cannot distinguish between normal and inverted ordering

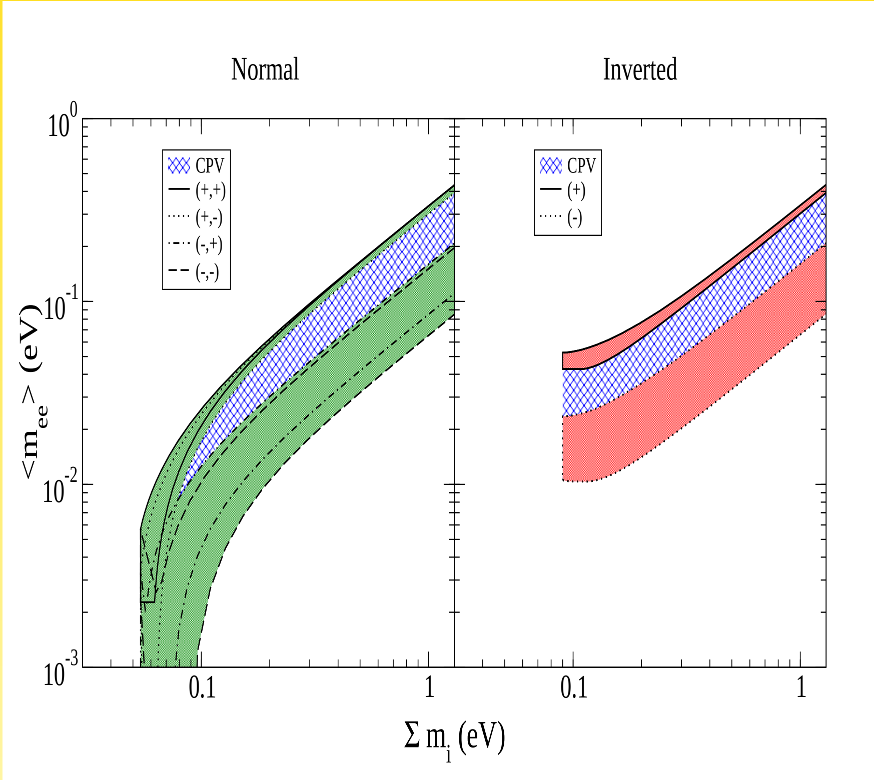
The usual plot

(life-time instead of $|m_{ee}|$)

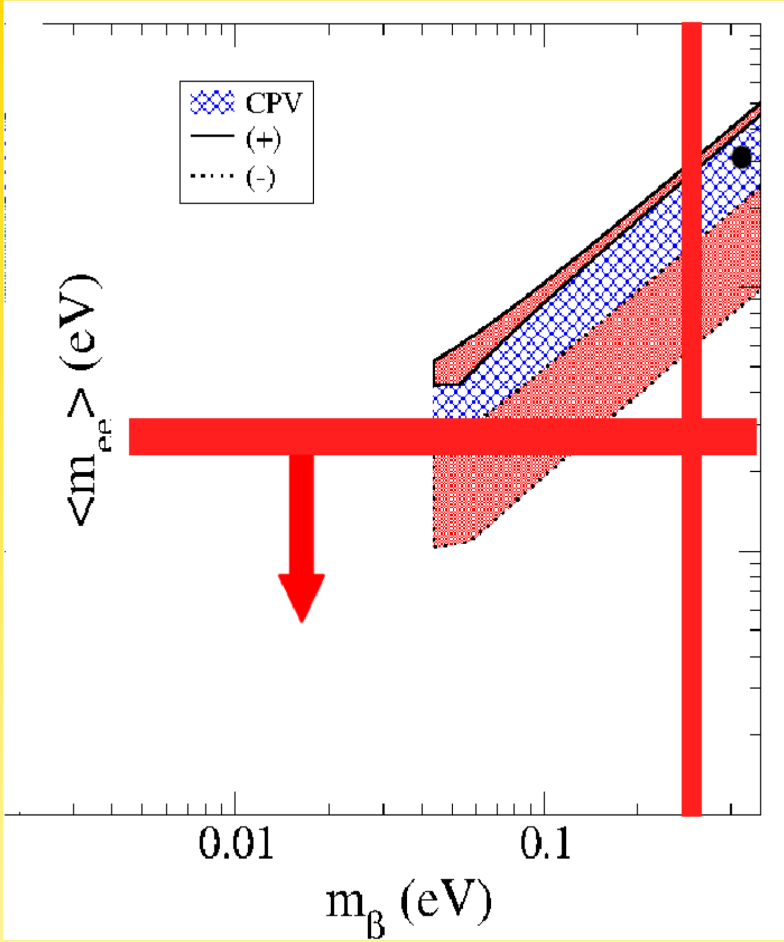


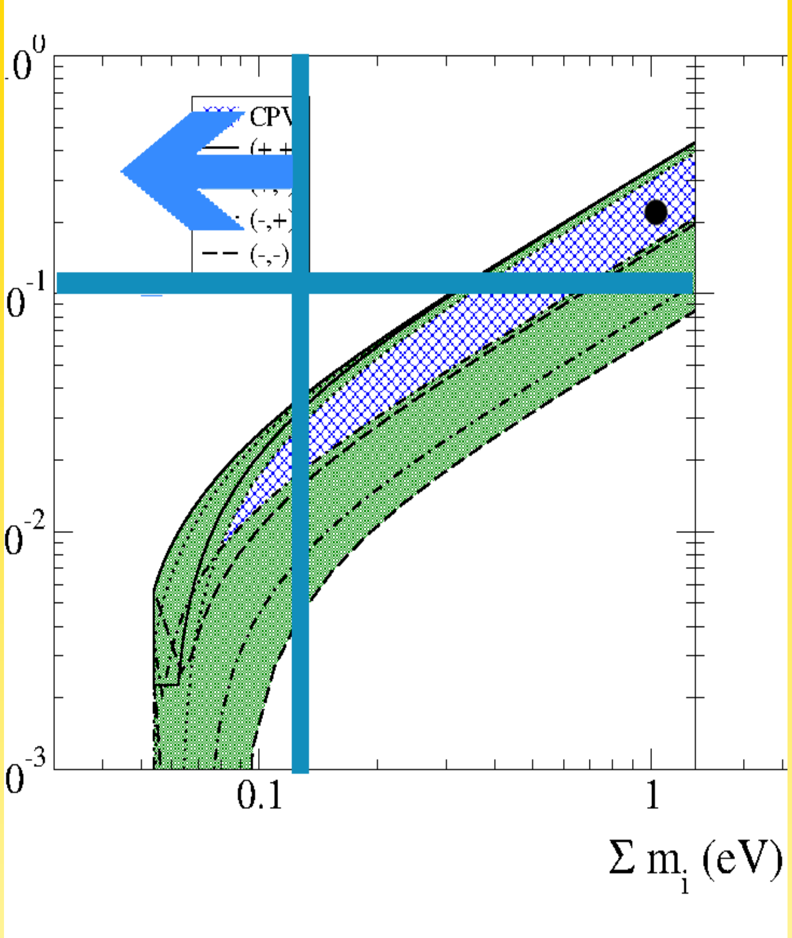
Plot against other observables





Complementarity of $|m_{ee}| = U_{ei}^2\, m_i$, $m_\beta = \sqrt{|U_{ei}|^2\, m_i^2}$ and $\Sigma = \sum m_i$





CP violation!

Dirac neutrinos!

Something else does $0\nu\beta\beta$!

Neutrino Mass

$$m(\text{heaviest}) \gtrsim \sqrt{|m_3^2 - m_1^2|} \simeq 0.05 \text{ eV}$$

3 **complementary** methods to measure neutrino mass:

Method	observable	now [eV]	near [eV]	far [eV]	pro	con
Kurie	$\sqrt{\sum \lvert U_{ei}\rvert^2 m_i^2}$	2.3	0.2	0.1	model-indep.; theo. clean	final?; worst
Cosmo.	$\sum m_i$	0.7	0.3	0.05	best; NH/IH	systemat.; model-dep.
$0\nu\beta\beta$	$\lvert \sum U_{ei}^2 m_i \rvert$	0.3	0.1	0.05	fundament.; NH/IH	model-dep.; theo. dirty

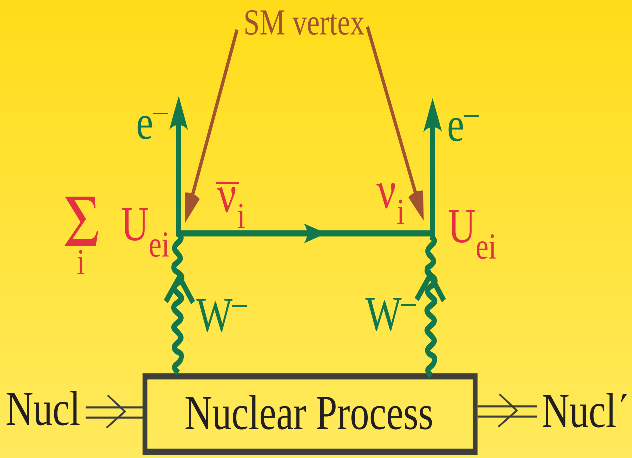
Which mass ordering with which life-time?

	Σ	m_β	$\|m_{ee}\|$
NH	$\sqrt{\Delta m_{\rm A}^2}$	$\sqrt{\Delta m_\odot^2 + \|U_{e3}\|^2 \Delta m_{\rm A}^2}$	$\left\|\sqrt{\Delta m_\odot^2} + \|U_{e3}\|^2 \sqrt{\Delta m_{\rm A}^2} e^{2i(\alpha-\beta)}\right\|$
	$\simeq 0.05$ eV	$\simeq 0.01$ eV	~ 0.003 eV $\Rightarrow T_{1/2}^{0\nu} \gtrsim 10^{28-29}$ yrs
IH	$2\sqrt{\Delta m_{\rm A}^2}$	$\sqrt{\Delta m_{\rm A}^2}$	$\sqrt{\Delta m_{\rm A}^2}\sqrt{1-\sin^2 2\theta_{12}\sin^2\alpha}$
	$\simeq 0.1$ eV	$\simeq 0.05$ eV	~ 0.03 eV $\Rightarrow T_{1/2}^{0\nu} \gtrsim 10^{26-27}$ yrs
QD	$3m_0$	m_0	$m_0\sqrt{1-\sin^2 2\theta_{12}\sin^2\alpha}$
			$\gtrsim 0.1$ eV $\Rightarrow T_{1/2}^{0\nu} \gtrsim 10^{25-26}$ yrs

(for 10^{26} yrs you need 10^{26} atoms, which are 10^3 mols, which are 100 kg)

From life-time to particle physics: Nuclear Matrix Elements

From life-time to particle physics: Nuclear Matrix Elements



- 2 point-like Fermi vertices; "long-range" neutrino exchange; momentum exchange $q \simeq 1/r \simeq 0.1$ GeV
- NME $\leftrightarrow$ overlap of decaying nucleons...
- different approaches (QRPA, NSM, IBM, GCM, pHFB) imply uncertainty
- plus uncertainty due to model details
- plus convention issues (Cowell, PRC **73**; Smolnikov, Grabmayr, PRC **81**; Dueck, W.R., Zuber, PRD **83**)

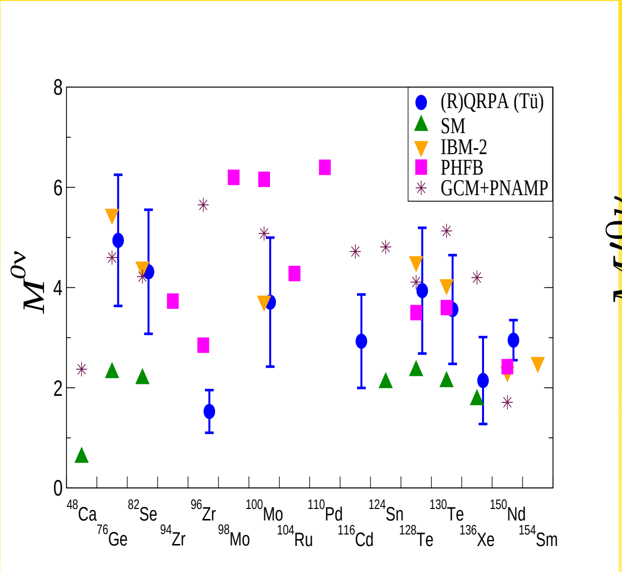
typical model for NME: set of single particle states with a number of possible wave function configurations; solve $\mathcal{H}$ in a mean background field

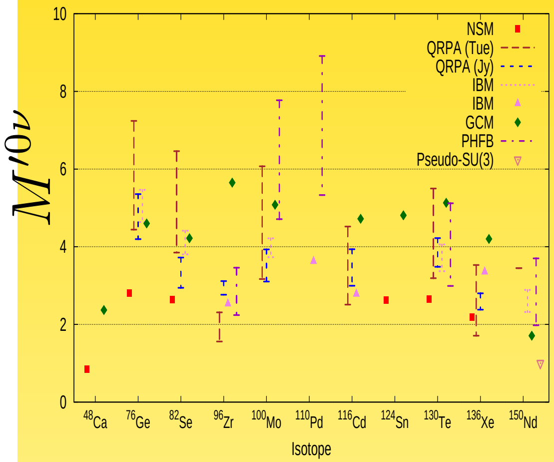
- Quasi-particle Random Phase Approximation (QRPA) **(many single particle states, few configurations)**
- Nuclear Shell Model (NSM) **(many configurations, few single particle states)**
- Interacting Boson Model (IBM) **(many single particle states, few configurations)** **(many single particle states, few configurations)**
- Generating Coordinate Method (GCM) **(many single particle states, few configurations)**
- projected Hartree-Fock-Bogoliubov model (pHFB)

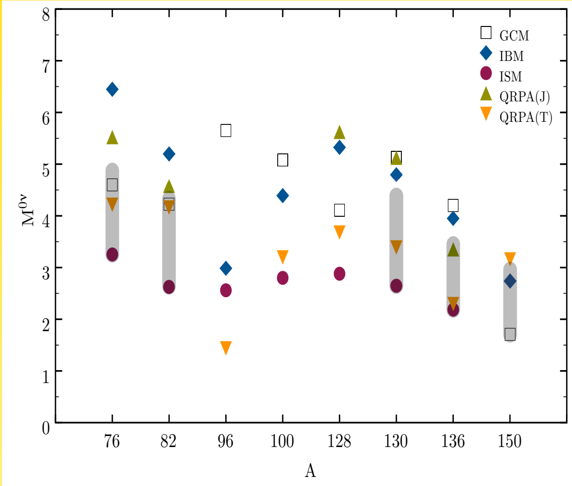
tends to overestimate NMEs

tends to underestimate NMEs

From life-time to particle physics: Nuclear Matrix Elements

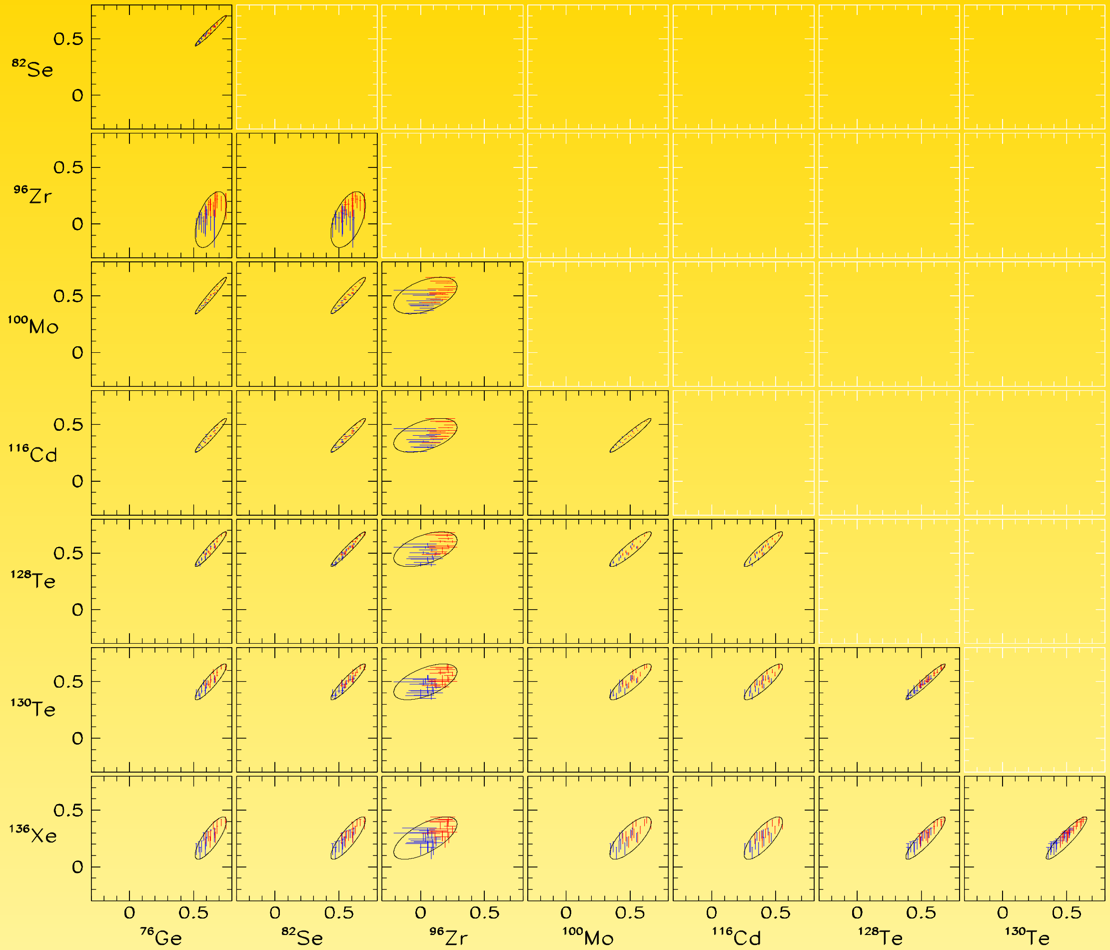






Faessler, 1104.3700 Dueck, W.R., Zuber, PRD **83** Gomez-Cadenas *et al.*, 1109.5515

to better estimate error range: correlations need to be understood



Faessler, Fogli *et al.*, PRD **79**

ellipse major axis: SRC (blue, red) and g_A

ellipse minor axis: g_{pp}

Recent Results

- ^{76}Ge:
  - GERDA: $T_{1/2} > 2.1 \times 10^{25}$ yrs
  - GERDA + IGEX + HDM: $T_{1/2} > 3.0 \times 10^{25}$ yrs
- ^{136}Xe:
  - EXO-200: $T_{1/2} > 1.1 \times 10^{25}$ yrs **(first run with less exposure:** $T_{1/2} > 1.6 \times 10^{25}$ **yrs...)**
  - KamLAND-Zen: $T_{1/2} > 2.6 \times 10^{25}$ yrs

Xe-limit is stronger than Ge-limit when:

$$T_{\rm Xe} > T_{\rm Ge} \, \frac{G_{\rm Ge}}{G_{\rm Xe}} \left| \frac{\mathcal{M}_{\rm Ge}}{\mathcal{M}_{\rm Xe}} \right|^2 \text{ yrs}$$

Xe vs. Ge: Limits on $|m_{ee}|$

NME	^{76}Ge		^{136}Xe	
	GERDA	comb	KLZ	comb
EDF(U)	**0.32**	**0.27**	**0.13**	–
ISM(U)	**0.52**	**0.44**	**0.24**	–
IBM-2	**0.27**	**0.23**	**0.16**	–
pnQRPA(U)	**0.28**	**0.24**	**0.17**	–
SRQRPA-B	**0.25**	**0.21**	**0.15**	–
SRQRPA-A	**0.31**	**0.26**	**0.23**	–
QRPA-A	*0.28*	*0.24*	*0.25*	–
SkM-HFB-QRPA	*0.29*	*0.24*	*0.28*	–

Bhupal Dev, Goswami, Mitra, W.R., PRD **88**

Might not be massive neutrinos...

Note: *standard amplitude* for light Majorana neutrino exchange:

$$\mathcal{A}_{\rm l} \simeq G_F^2 \, \frac{|m_{ee}|}{q^2} \simeq 7 \times 10^{-18} \left(\frac{|m_{ee}|}{0.5 \text{ eV}} \right) \text{GeV}^{-5} \simeq 2.7 \text{ TeV}^{-5}$$

if new heavy particles are exchanged:

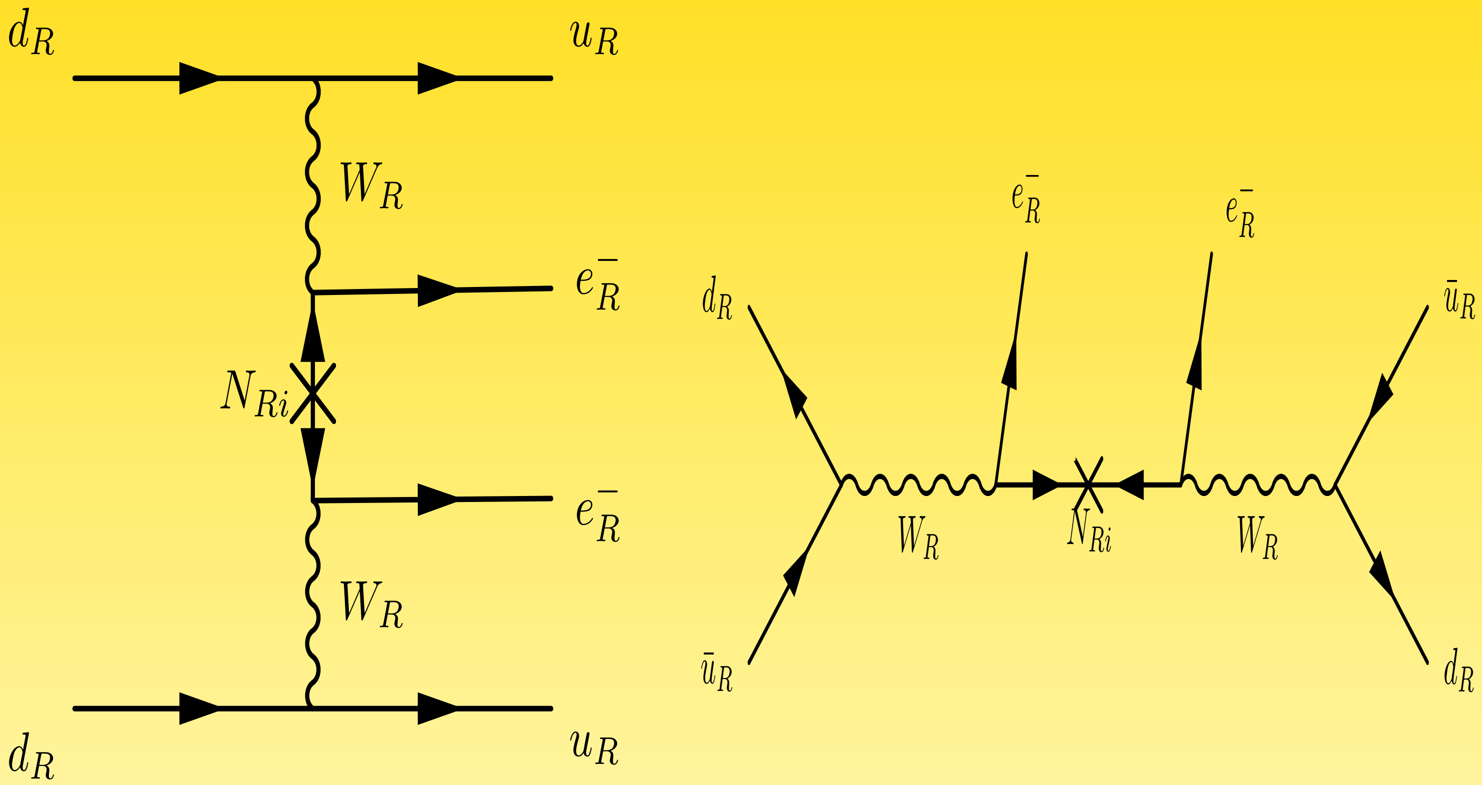
$$\mathcal{A}_{\rm h} \simeq \frac{c}{M^5}$$

$\Rightarrow$ for $0\nu\beta\beta$ holds:

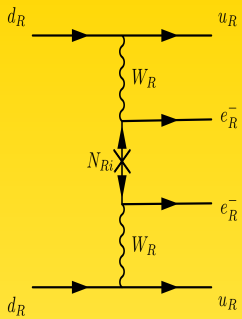
$$\boxed{1 \text{ eV} = 1 \text{ TeV}}$$

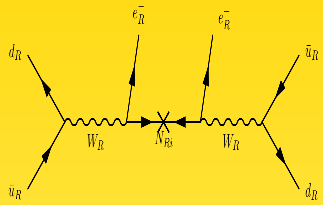
$\Rightarrow$ Phenomenology in colliders, LFV

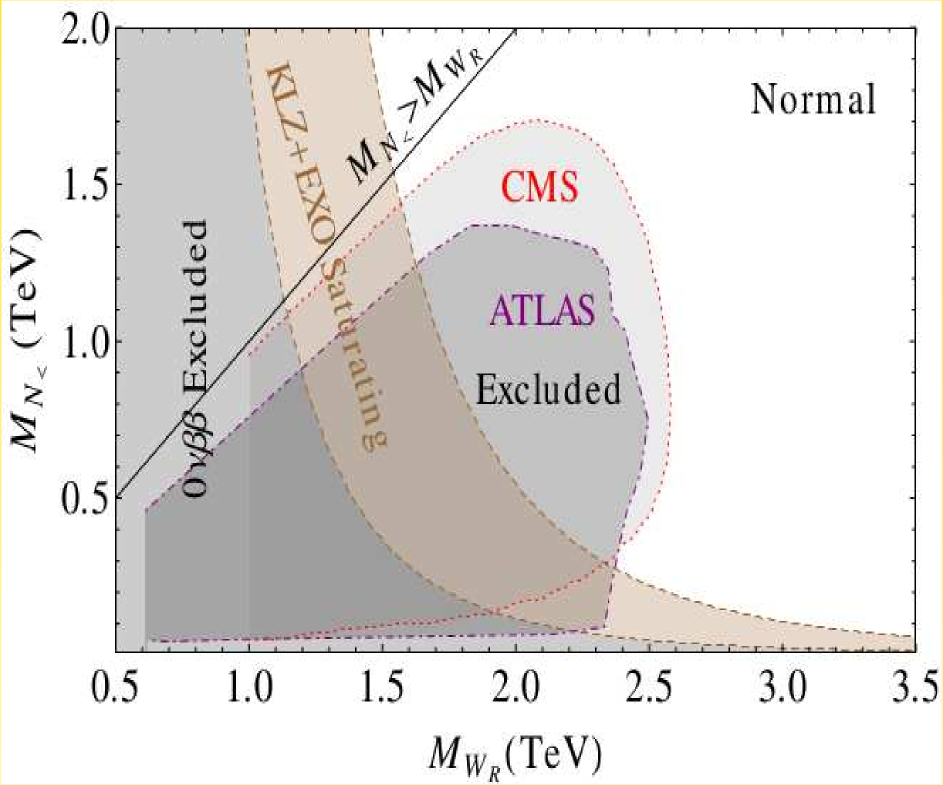
Left-right symmetry



Senjanovic, Keung, 1983; Tello *et al.*; Goswami *et al.*







Bhupal Dev, Goswami, Mitra, W.R., Phys. Rev. **D88**

Topics not covered

- Future Neutrino Experiments
- Flavor Symmetries
- Leptogenesis
- Neutrinos and Dark Matter
- Neutrinos in Cosmology
- Neutrinos in Astrophysics
- Neutrinos in Cosmic Rays
- Sterile Neutrinos
- ...

At last, let me mention the quarter final :-)

Summary