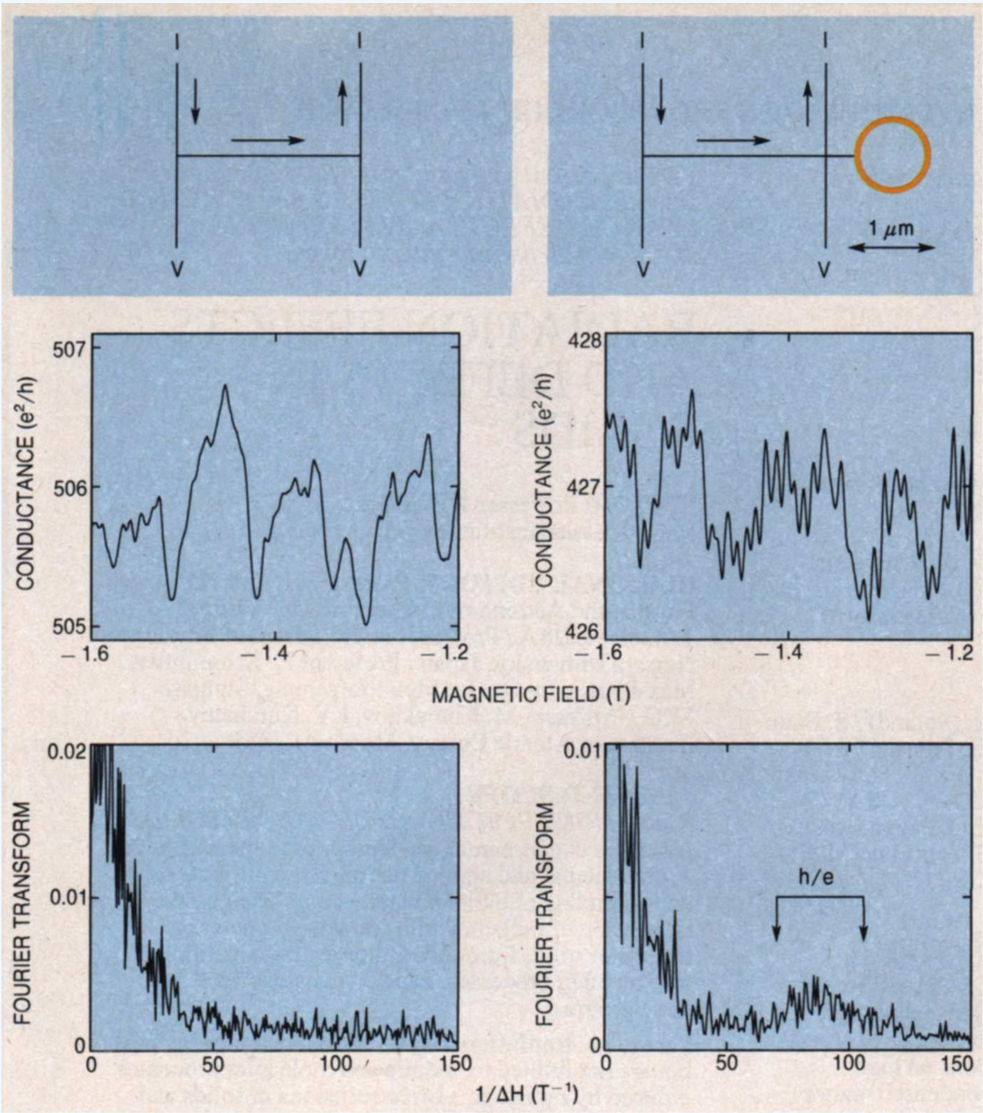
Quantum transport in nanoscale solids

The Landauer approach

Dietmar Weinmann

Institut de Physique et Chimie des Matériaux de Strasbourg

Quantum effects in electron transport



R. A. Webb, S. Washburn,

Physics Today, Dec. 1988

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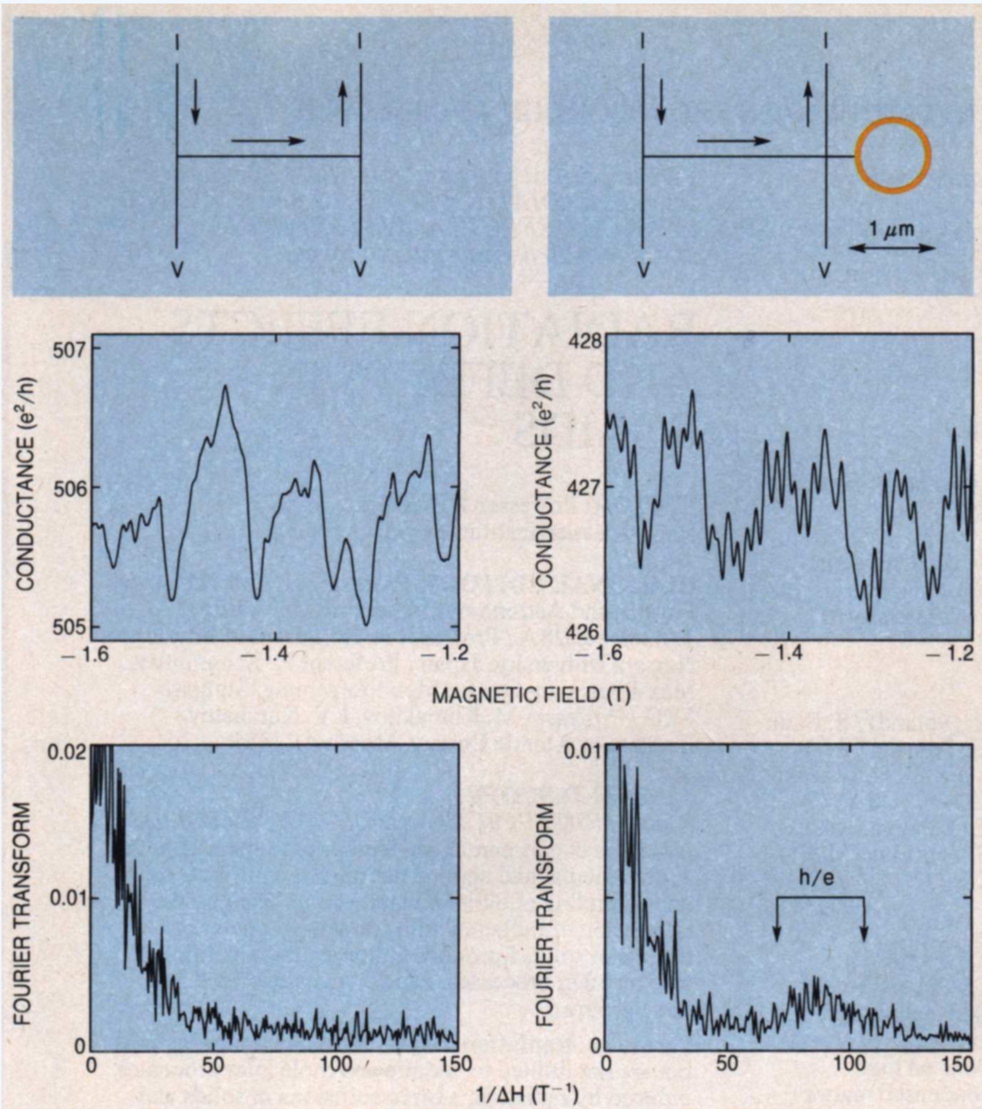

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far from classical

$\Downarrow$

quantum effects in electronic transport

Outline

- Basic concepts
- Quantum conductance, Landauer's approach

Basic concepts

Quantum *versus* classical behavior

microscopic	macroscopic
atoms, molecules	large pieces of matter
quantum	classical

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intermediate:

Nanoscale systems

Quantum *versus* classical behavior

microscopic	macroscopic
atoms, molecules	large pieces of matter
quantum	classical

intermediate:

Nanoscale systems

quantum effects and large number of atoms

quantum and statistical physics

"mesoscopic regime" [B. Reulet, friday]

Quantum behavior

isolated system A:

$$i\hbar \frac{\partial}{\partial t} |\psi_\mathrm{A}(t)\rangle = H_\mathrm{A} |\psi_\mathrm{A}(t)\rangle$$

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$$i\hbar\frac{\partial}{\partial t}|\psi_{\mathrm{A}}(t)\rangle = H_{\mathrm{A}}|\psi_{\mathrm{A}}(t)\rangle$$

time evolution

$$|\psi_{\mathrm{A}}(t)\rangle = \exp(-iH_{\mathrm{A}}t/\hbar)|\psi_{\mathrm{A}}(0)\rangle$$

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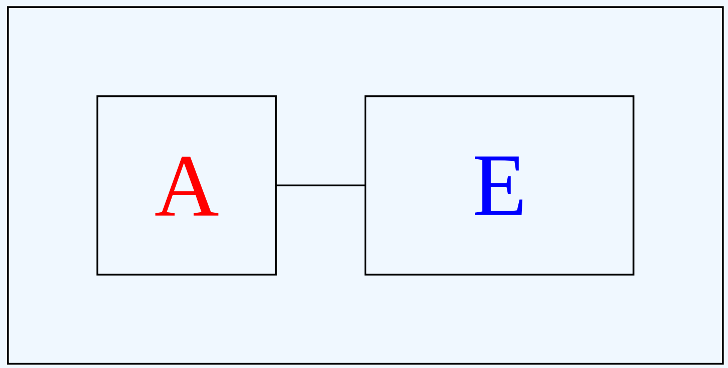
How can classical behavior emerge?

Coupling to an environment

realistic, A coupled to environment E:



combined system $H = H_{\mathrm{A}} + H_{\mathrm{E}} + H_{\mathrm{AE}}$

$$|\psi(t)\rangle = \exp(-\mathrm{i}Ht/\hbar)|\psi(0)\rangle$$

Coupling to an environment

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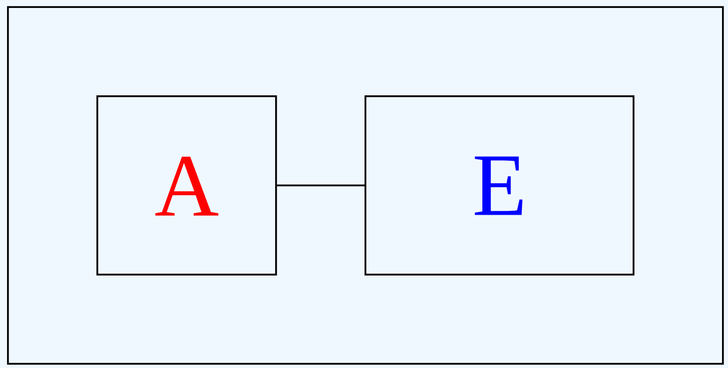


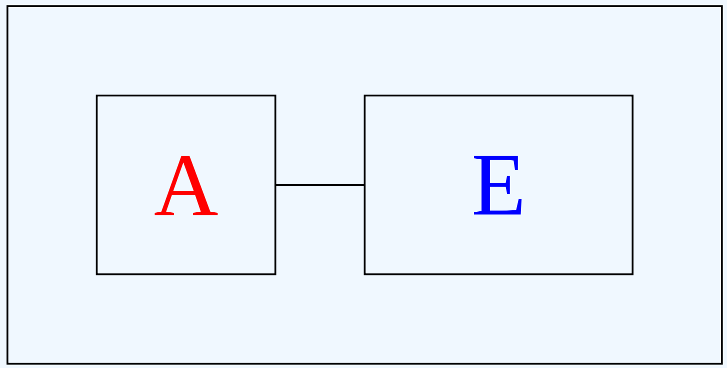
combined system $H = H_{\mathrm{A}} + H_{\mathrm{E}} + H_{\mathrm{AE}}$

$$|\psi(t)\rangle = \exp(-\mathrm{i}Ht/\hbar)|\psi(0)\rangle$$

Do we have $|\psi_{\mathrm{A}}(t)\rangle = \exp(-\mathrm{i}H_{\mathrm{A}}t/\hbar)|\psi_{\mathrm{A}}(0)\rangle$?

Coupling to an environment

realistic, A coupled to environment E:



combined system $H = H_{\mathrm{A}} + H_{\mathrm{E}} + H_{\mathrm{AE}}$

$$|\psi(t)\rangle = \exp(-\mathrm{i}Ht/\hbar)|\psi(0)\rangle$$

Do we have $\cancel{|\psi_{\mathrm{A}}(t)\rangle = \exp(-\mathrm{i}H_{\mathrm{A}}t/\hbar)|\psi_{\mathrm{A}}(0)\rangle}$?

Coupling leads to decoherence on a timescale τ_ϕ

Phase coherence time/length

Quantum effects are suppressed due to decoherence

$\rightsquigarrow$ phase coherence time τ_ϕ

Electrons move $\Rightarrow$ phase coherence length L_ϕ

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Electrons move $\Rightarrow$ phase coherence length L_ϕ

ballistic motion (v_F Fermi velocity): $L_\phi = v_\mathrm{F}\tau_\phi$

diffusive motion (D diffusion constant): $L_\phi = \sqrt{D\tau_\phi}$

Phase coherence time/length

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Electrons move $\Rightarrow$ phase coherence length L_ϕ

ballistic motion (v_F Fermi velocity): $L_\phi = v_\mathrm{F}\tau_\phi$

diffusive motion (D diffusion constant): $L_\phi = \sqrt{D\tau_\phi}$

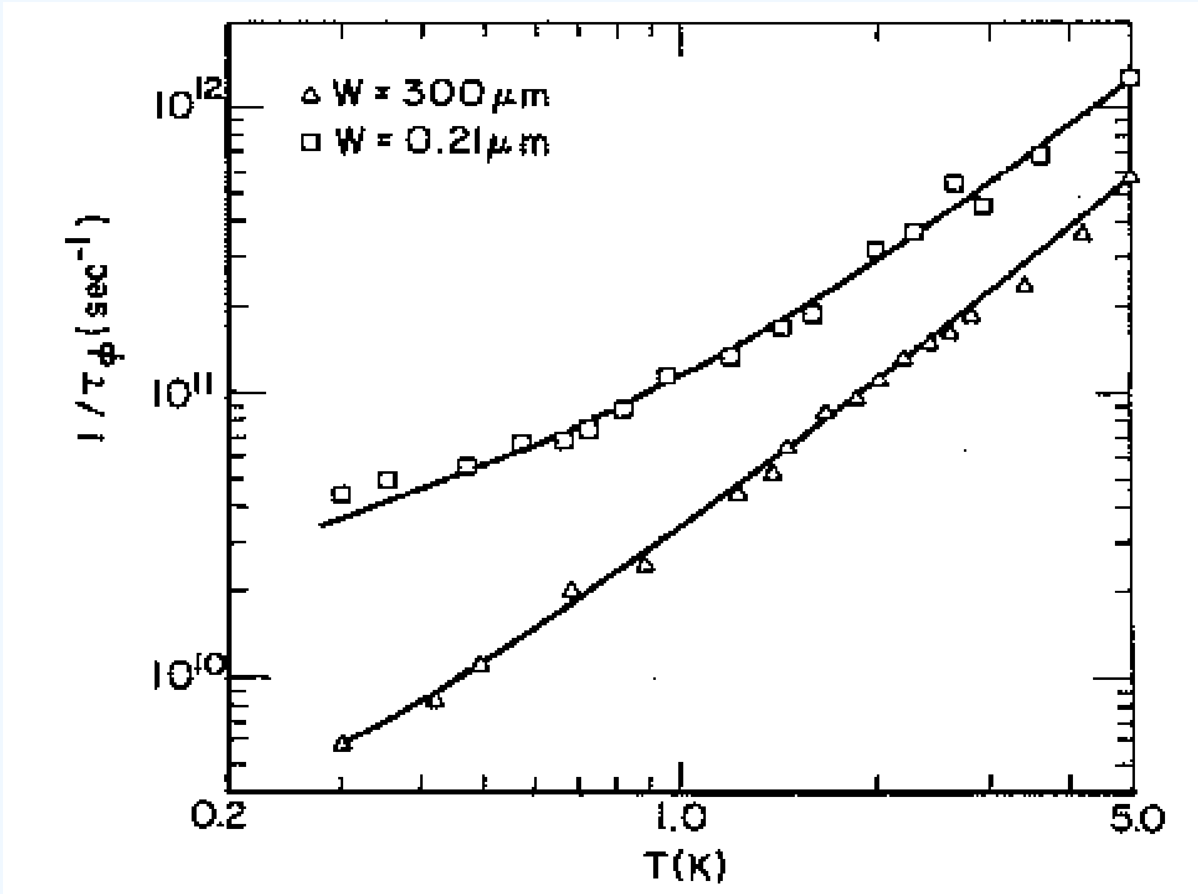
Quantum effects relevant when:

$$\boxed{L_\phi \gtrsim L}$$

(L: size of the sample or other relevant length scale)

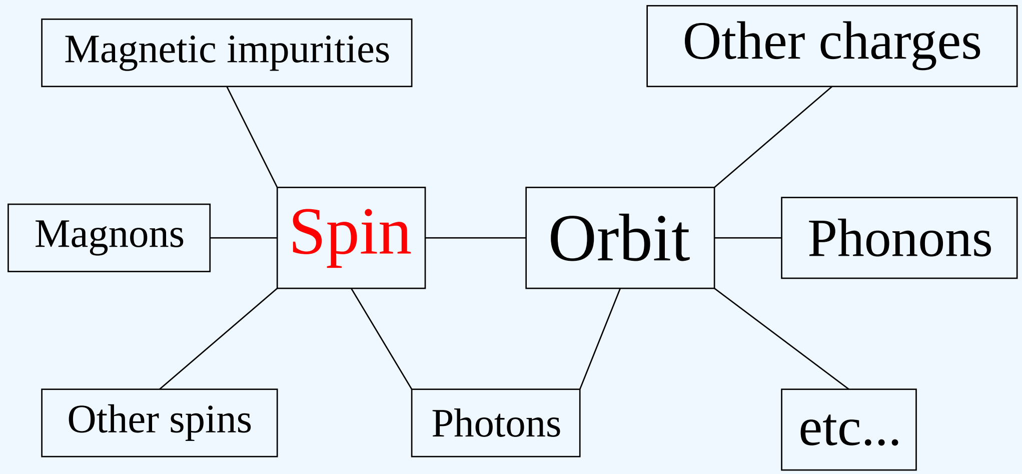
Phase coherence time

Experiment in GaAs [Choi *et al.*, Phys. Rev. B ’87]



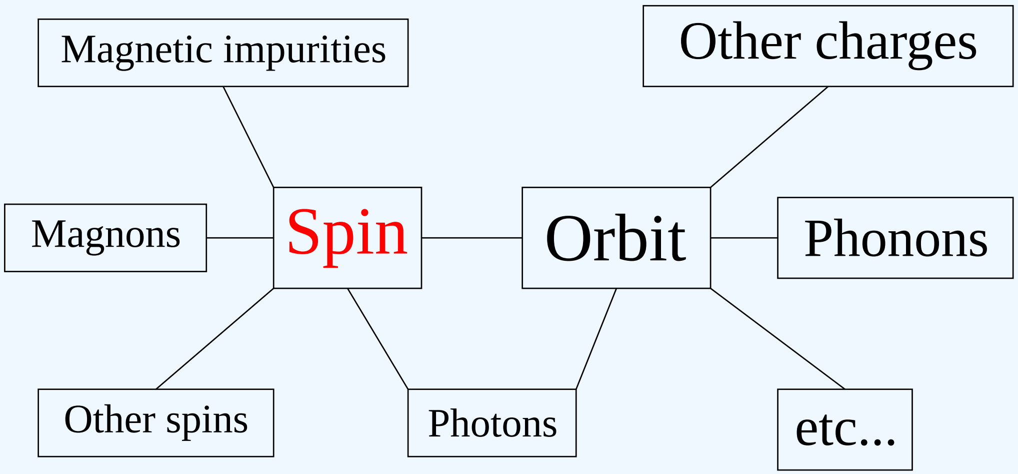
$\rightsquigarrow L_\phi \approx 1 - 10\,\mu$m at $T \sim 1$ K

Spin coherence



Spin is often weakly coupled to other degrees of freedom

Spin coherence



Spin is often weakly coupled to other degrees of freedom

$\rightsquigarrow$ Spin coherence time $\tau_{\phi,\mathrm{S}} \gg \tau_{\phi,\mathrm{O}}$

Example: $\tau_{\phi,\mathrm{S}} > 100\,\mathrm{ns}$ in GaAs (Kikkawa&Awschalom, PRL '98)

$\Rightarrow$ Quantum computing, Spin electronics, ...

[Coupling to mechanical degrees of freedom: R. Leturq, thursday]

Quantum conductance

Landauer's approach

Conductance quantization

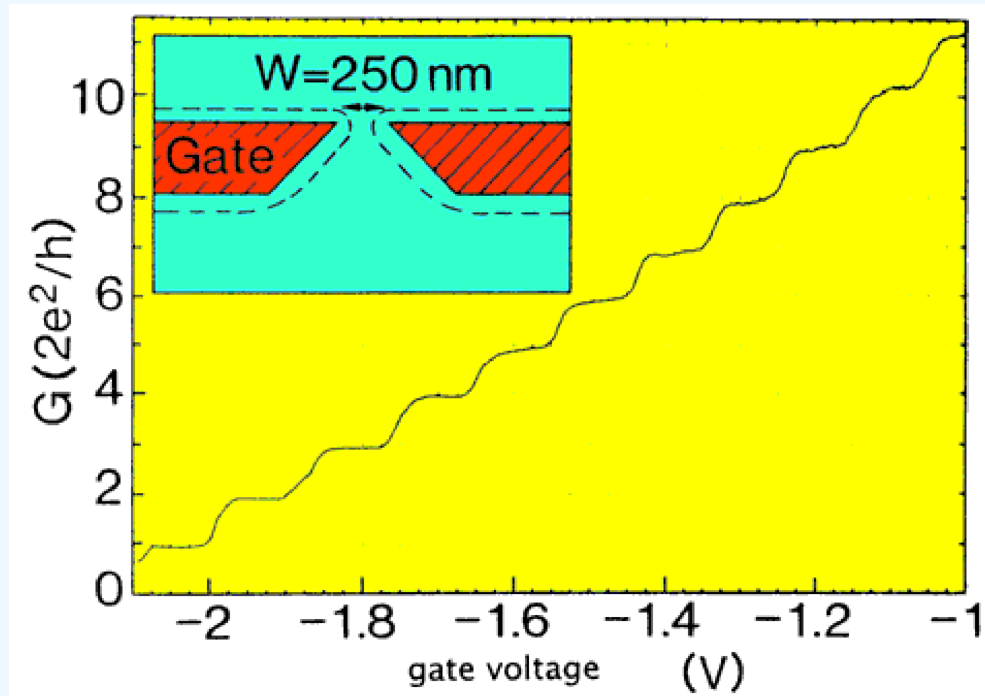
Point contact

D. A. Wharam et al.

J. Phys. C **21**, L209 (1988)

B. J. van Wees et al.

Phys. Rev. Lett. **60**, 848 (1988)



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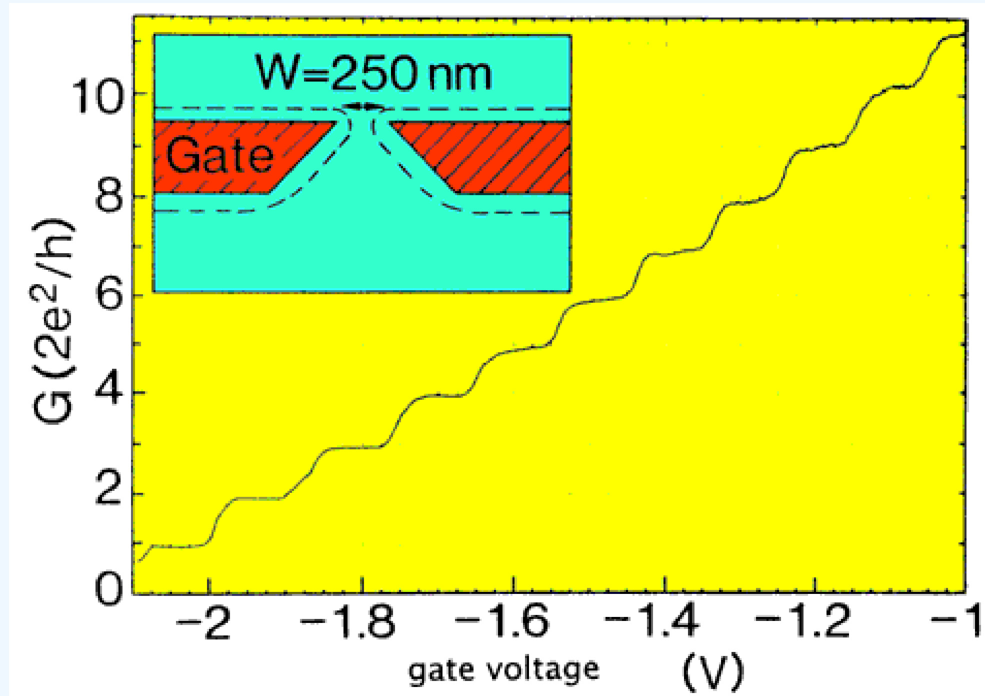

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steps of $2e^2/h$ in the conductance!

Conductance and conductivity

	conductance	conductivity
Ohm's law	$I = GV$	$\vec{j} = \sigma \vec{E}$
	sample	material
	global	local

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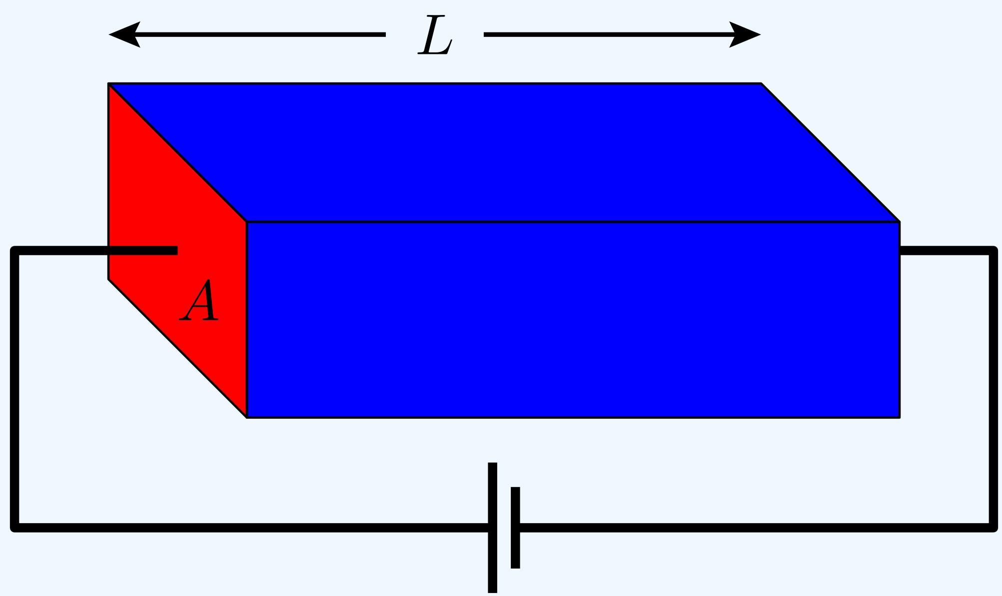
classical:

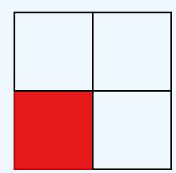
$$G = \sigma \frac{A}{L}$$

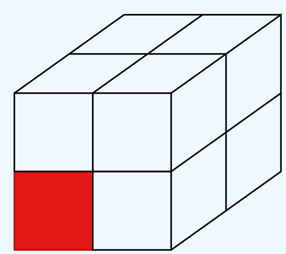


Scaling of the conductance

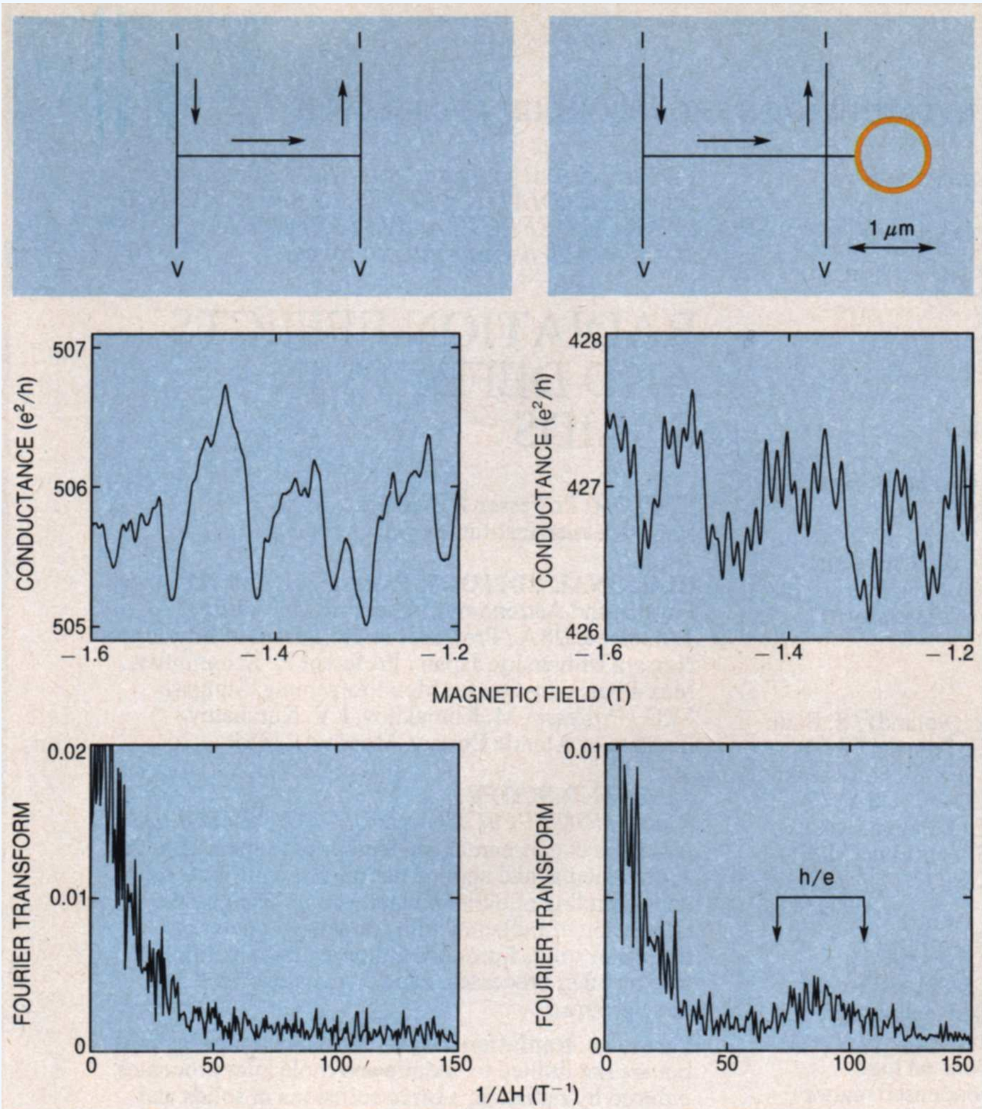
$$G = \sigma \frac{A}{L} \qquad\qquad A \sim L^{d-1}$$

1d  $G \sim 1/L$

2d  $G_\square$

3d  $G \sim L$

Non-locality of quantum transport



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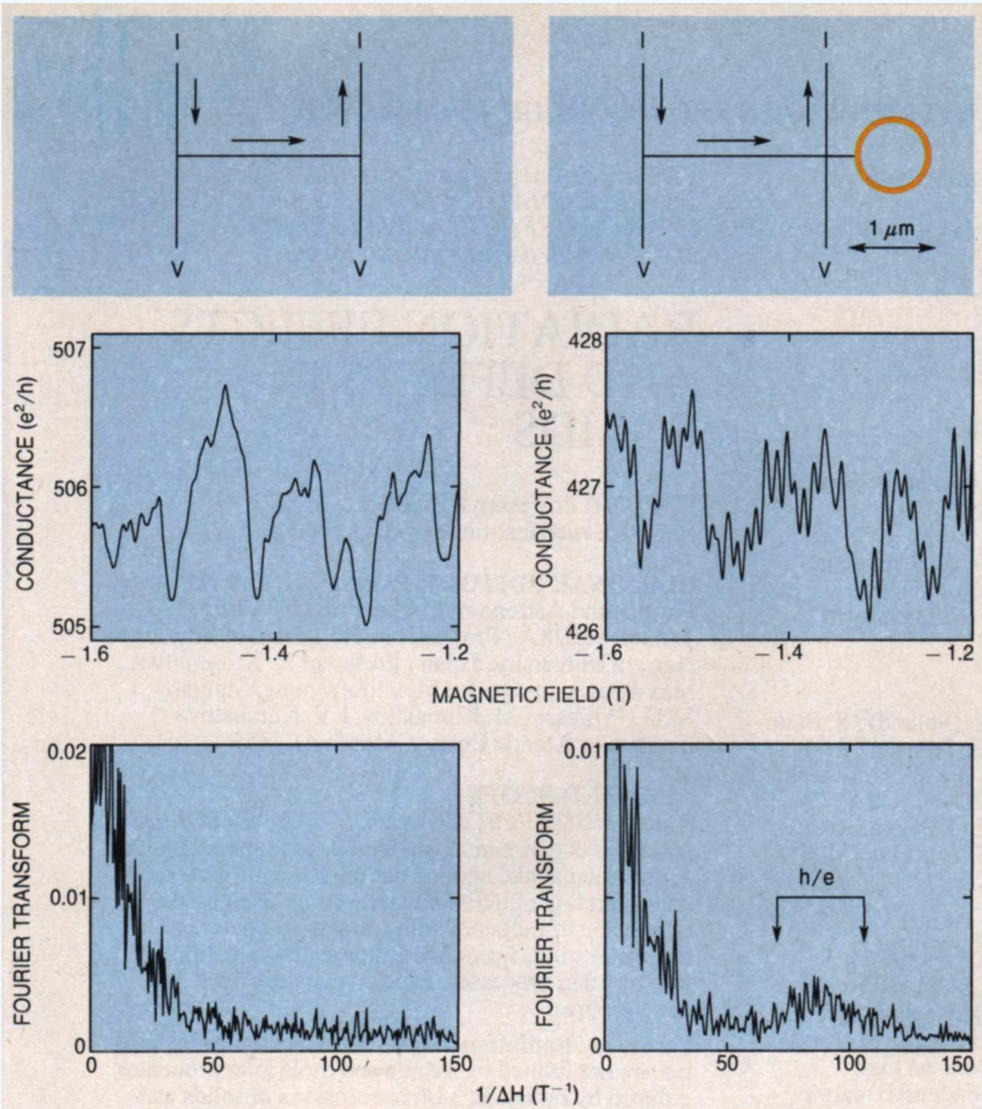

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quantum transport is
non-local

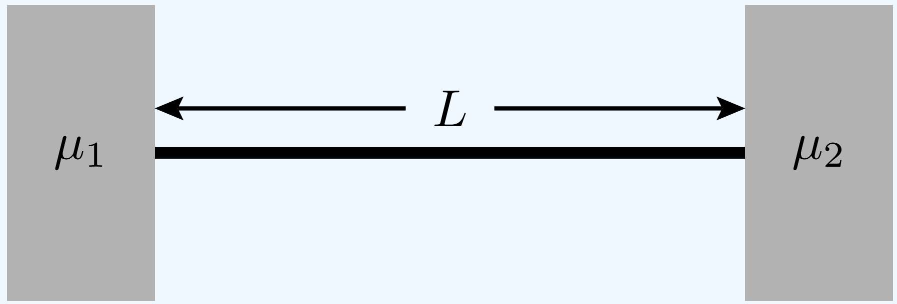
$\Downarrow$

$$G \neq \sigma \frac{A}{L}$$

$\Downarrow$

Conductance has
to be considered

Electrons in a perfect 1d wire



1d wavefunctions

$$\psi_k(x) \propto \exp(ikx)$$

$k > 0$: forward propagation with $\hbar v = \partial E / \partial k$

Current carried by a state: $I_k = ev/L$

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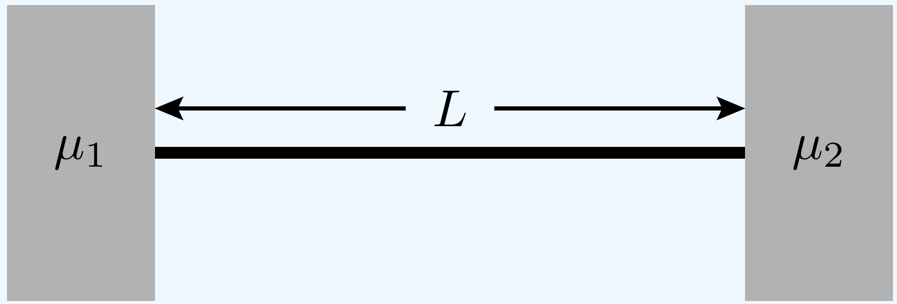

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Density of states: $\rho_+ = L/hv$

Total current carried by $k > 0$ states:

$$I_+ = \int_0^\infty \mathrm{d}E \, f_{\mu_1} I_k \rho_+$$

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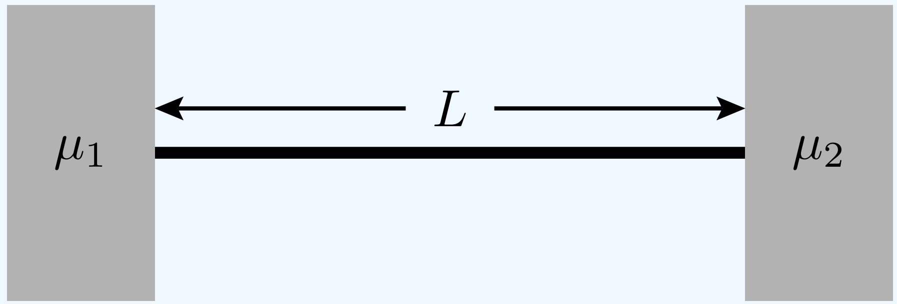

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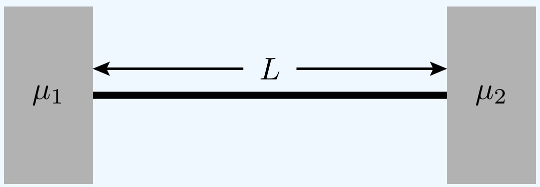
$$I_+ = \int_0^\infty \mathrm{d}E \, f_{\mu_1} I_k \rho_+ = \frac{e}{h} \int_0^\infty \mathrm{d}E \, f_{\mu_1}$$

Quantum conductance of a perfect 1d wire



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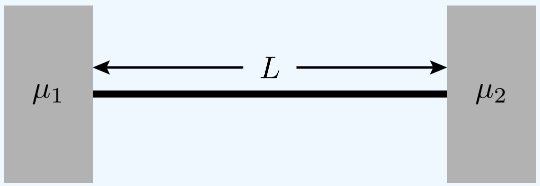

$$I_+ = \frac{e}{h}\int_0^\infty \mathrm{d}E\, f_{\mu_1} \qquad I_- = -\frac{e}{h}\int_0^\infty \mathrm{d}E\, f_{\mu_2}$$

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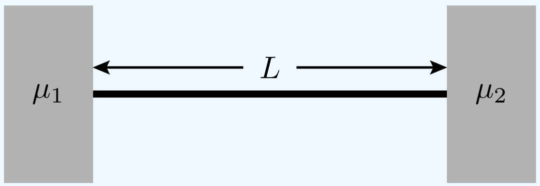

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Net current:

$$I = I_+ + I_- = \frac{e}{h}\int_0^\infty \mathrm{d}E\,(f_{\mu_1} - f_{\mu_2})$$

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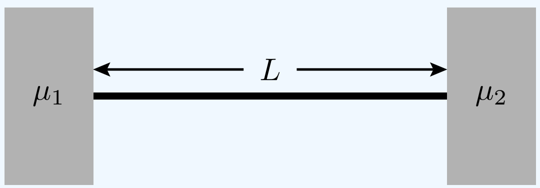

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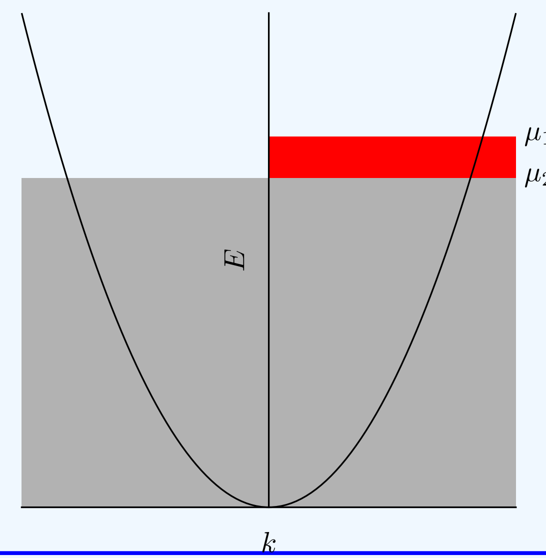
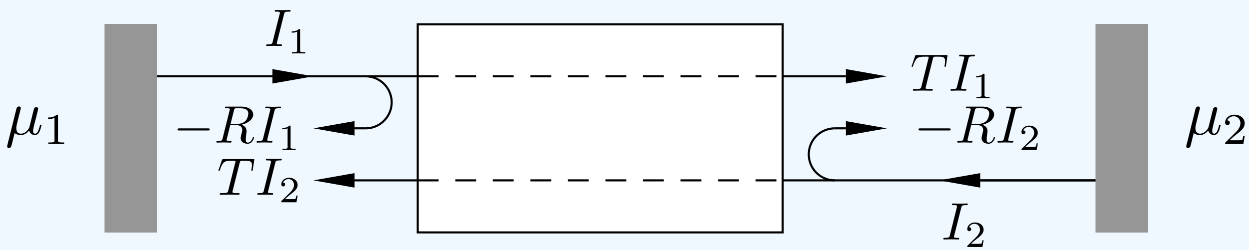
Low temperature $f_\mu \to \Theta(\mu - E)$:

$$I = \frac{e}{h}\underbrace{(\mu_1 - \mu_2)}_{eV} = \frac{e^2}{h}V$$

$$G = \frac{I}{V} = \frac{e^2}{h}$$



Landauer: Conductance from scattering



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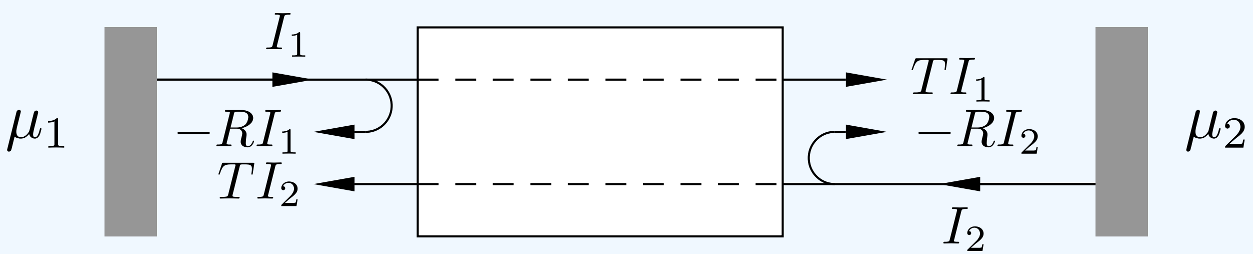

$$I = TI_1 - TI_2 = \frac{e}{h} T \underbrace{(\mu_1 - \mu_2)}_{eV}$$

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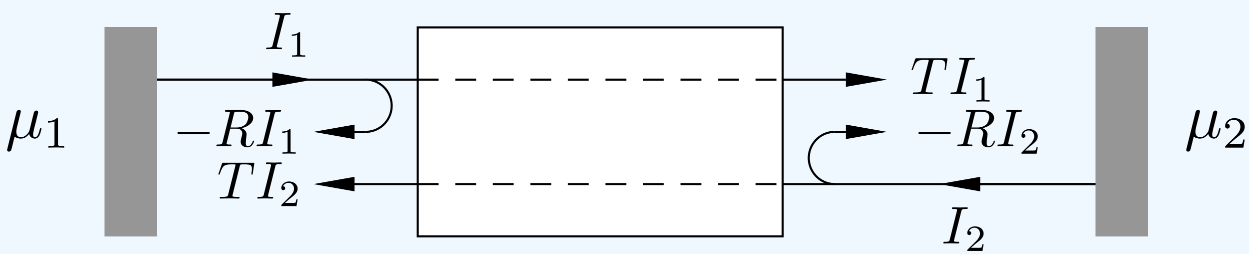

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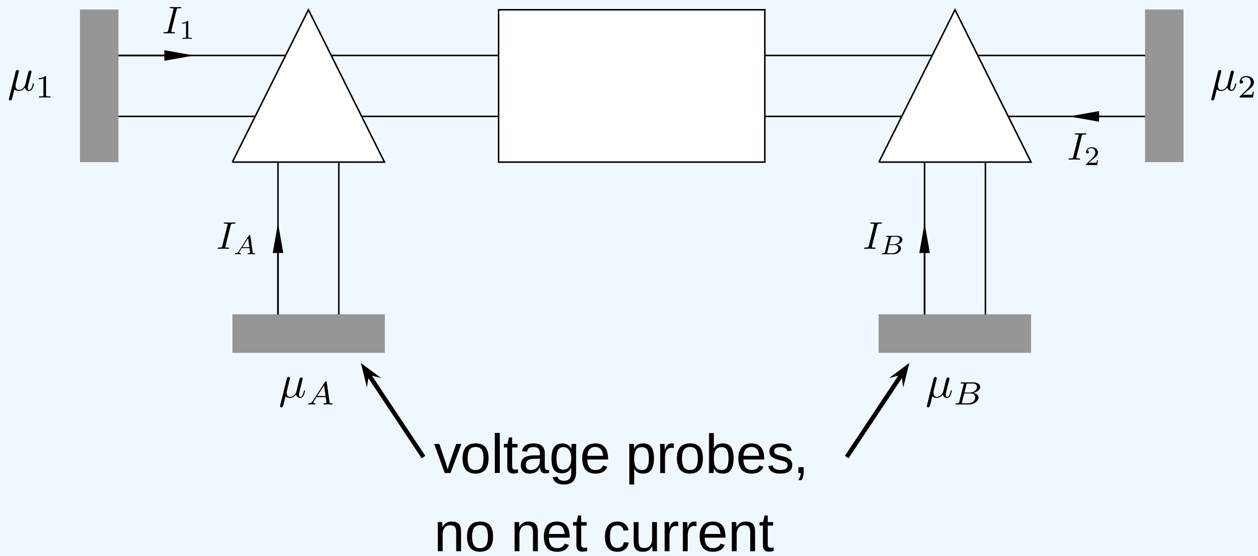
→ two terminal conductance

$$\boxed{G_{2t} = \frac{e^2}{h} T}$$

finite for $T = 1$!

Four terminal conductance

[H.-L. Engquist, P. W. Anderson, Phys. Rev. B **24**, 1151 (1981)]



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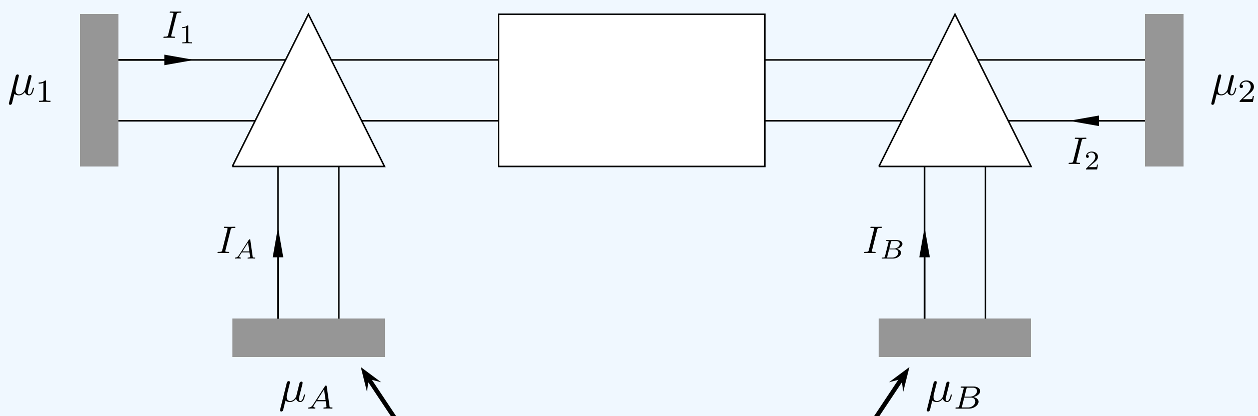

voltage probes,
no net current

four terminal conductance $$G_{4t} = \frac{eI}{\mu_A - \mu_B}$$

$$\rightarrow \boxed{G_{4t} = \frac{e^2}{h}\frac{T}{1-T}}$$

$T \to 1$: infinite conductance

Contact resistance

difference between two and four terminal resistance:
contact resistance

$$\left(\frac{1}{G_{2t}} - \frac{1}{G_{4t}}\right) = \frac{h}{e^2}\left(\frac{1}{T} - \frac{1-T}{T}\right) = \frac{h}{e^2}$$

Contact resistance

difference between two and four terminal resistance: contact resistance

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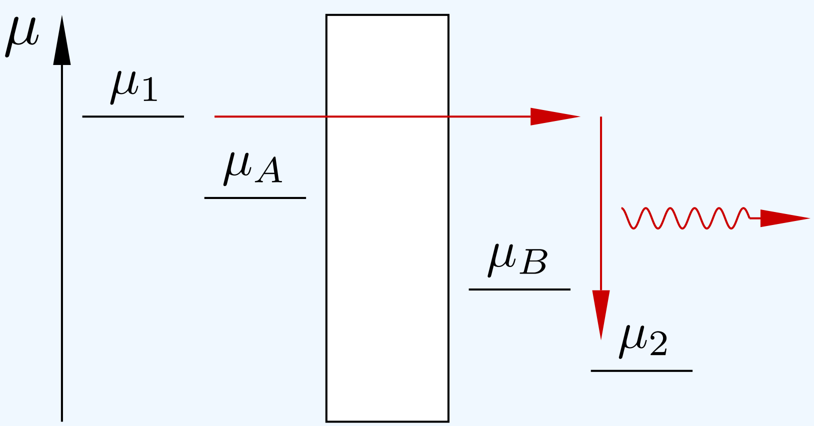
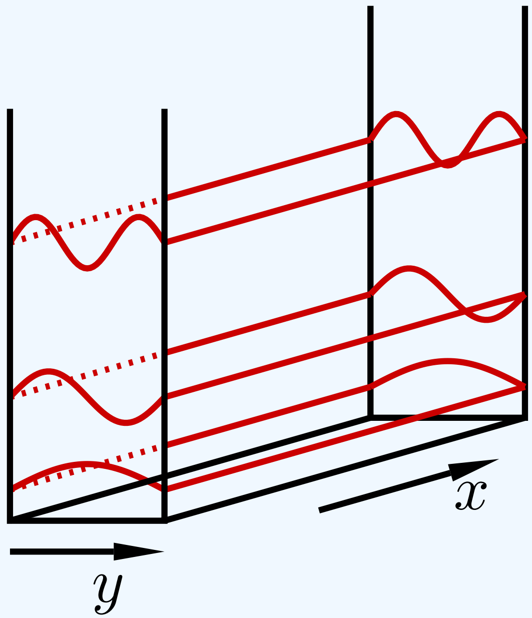


equilibration in the reservoirs leads to dissipation

Electrons in a quasi-1d clean quantum wire



$$H = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + V(y)$$

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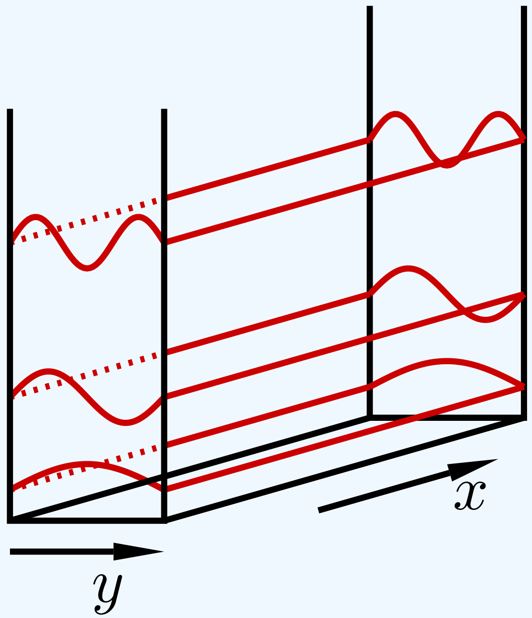


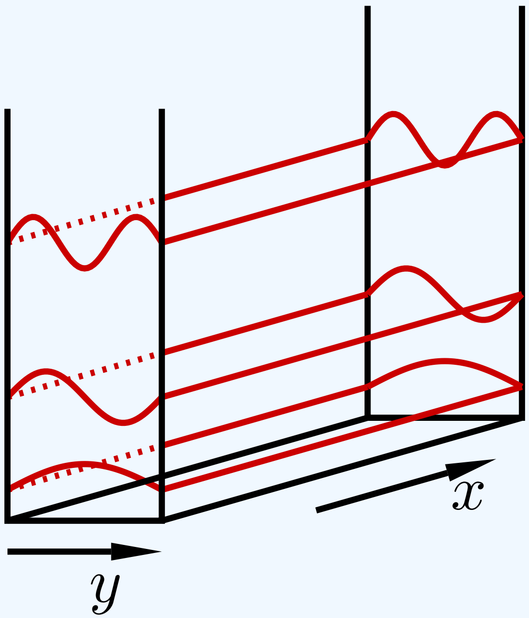
$$H = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + V(y)$$

separable! $H = H_x + H_y$

$$\rightsquigarrow \quad \psi(x,y) \propto \chi_n(y)\exp(ikx)$$

$$E_{n,k} = E_n + \frac{\hbar^2 k^2}{2m}$$

Electrons in a quasi-1d clean quantum wire



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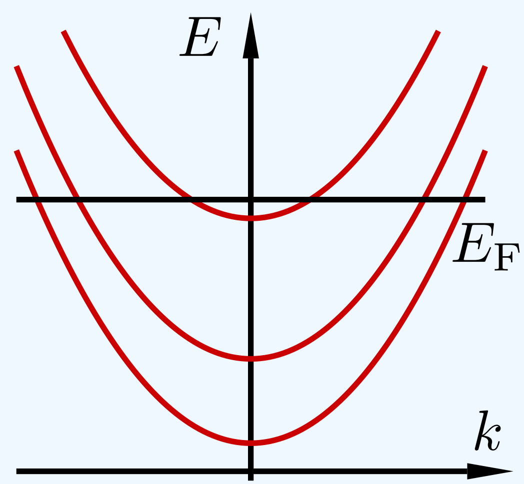
“conduction channels”

conductance per open channel:

$$\boxed{e^2/h}$$



Conductance of a many-channel scatterer

Generalization of the 2-terminal conductance:

$$G_{2t} = \frac{e^2}{h} \sum_{n,m} T_{n,m}$$

$$n \quad \longrightarrow \quad T_{n,m} \quad \longrightarrow \quad m$$

sum runs over occupied channels $E_n, E_m < E_F$

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Generalizations to

finite voltage, temperature $\rightarrow$ textbooks

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interactions: Peter Schmitteckert, wednesday

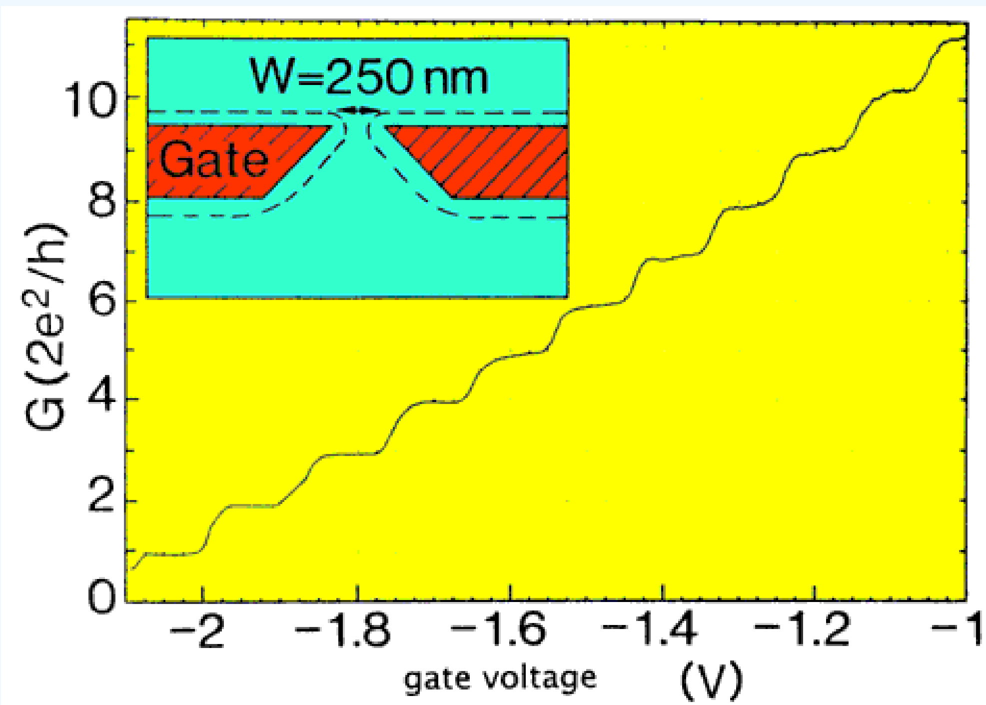
Conductance quantization

Point contacts:

$$T_{n,m} \approx \delta_{n,m}$$

$$\to G_{2t} \approx \frac{e^2}{h} N_c$$

N_c: # of occupied channels



van Wees *et al.*
Phys. Rev. Lett '88

www.ilorentz.org

Summary

Quantum transport:

- quantum effects: low temperatures; small lengthscales relevant in nanoscale systems

Summary

Quantum transport:

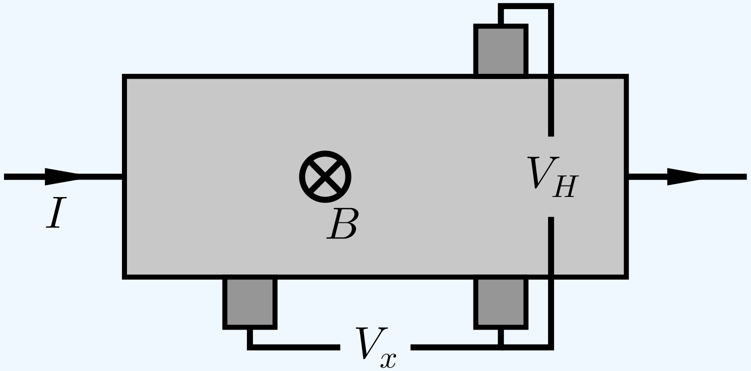
- quantum effects: low temperatures; small lengthscales

  relevant in nanoscale systems

- Landauer formalism:

  Quantum conductance $\longleftrightarrow$ transmission

Hall Effect



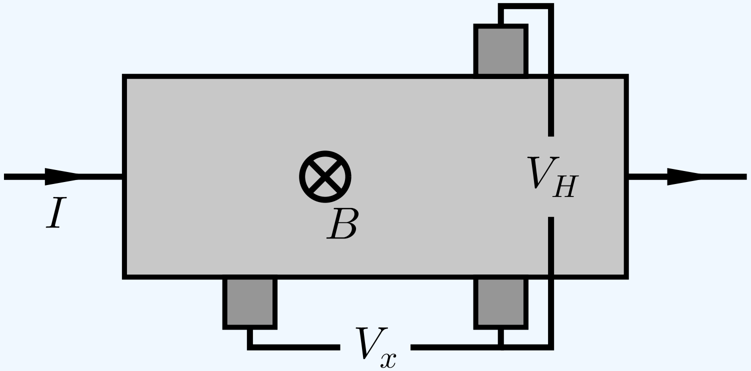
Classical: Hall 1879

$V_x = IR$ independent of B

$V_H = \frac{B}{en_s} I$ linear in B

$\rightarrow \quad R_H = \frac{V_H}{I} = \frac{B}{en_s}$ Drude

Hall Effect



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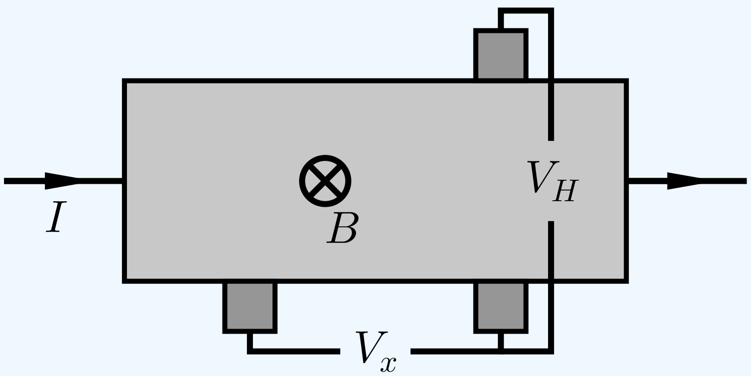
strong magnetic field, clean sample:

cyclotron orbits $r_c = mv_{\mathrm{F}}/eB$; $\omega_c = eB/m$

Hall Effect



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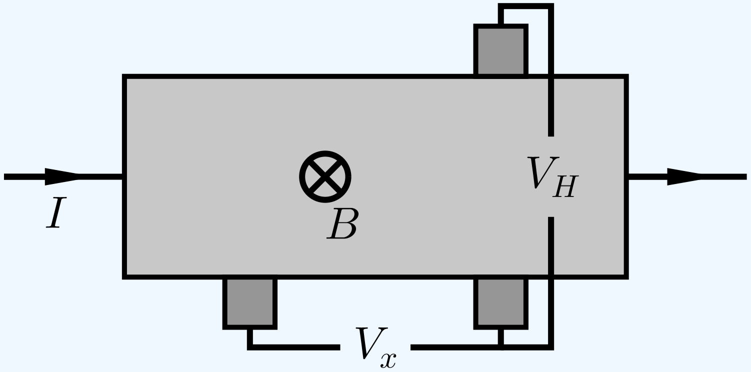
Quantum mechanics ($L_\phi > r_c$):

quantized motion $\rightarrow$ Landau levels $E = E_0 + (n + 1/2)\hbar\omega_c$

Hall Effect



Classical: Hall 1879

$V_x = IR$ independent of B

$V_H = \frac{B}{en_s} I$ linear in B

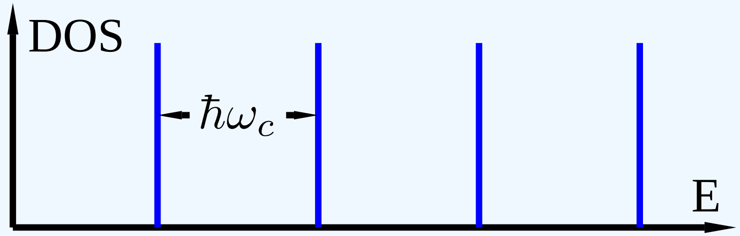
$\rightarrow \quad R_H = \frac{V_H}{I} = \frac{B}{en_s}$ Drude

strong magnetic field, clean sample:

cyclotron orbits $r_c = mv_{\mathrm{F}}/eB$; $\omega_c = eB/m$

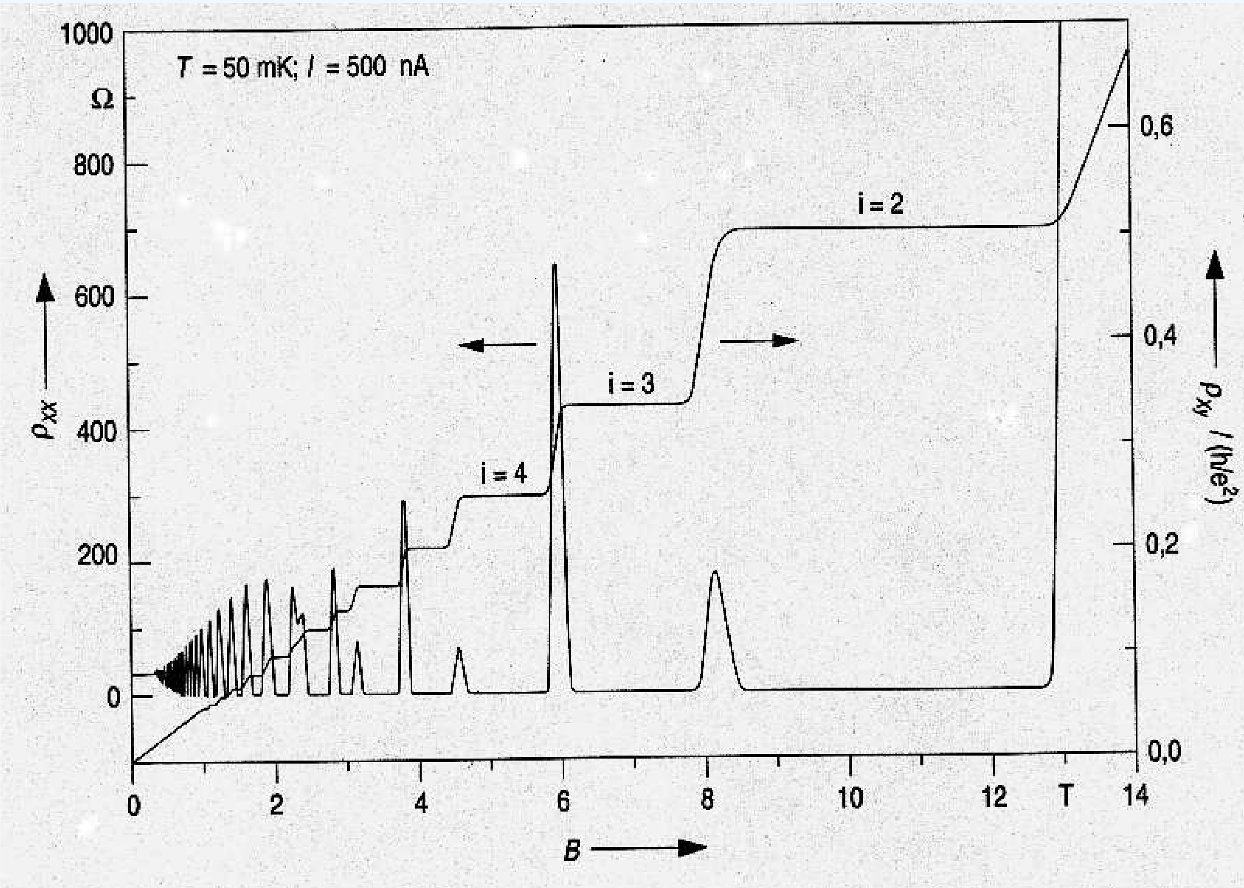
Quantum mechanics ($L_\phi > r_c$):

quantized motion $\rightarrow$ Landau levels $E = E_0 + (n + 1/2)\hbar\omega_c$



degenerate $N_D = 2SB/\phi_0$

Quantum Hall Effect

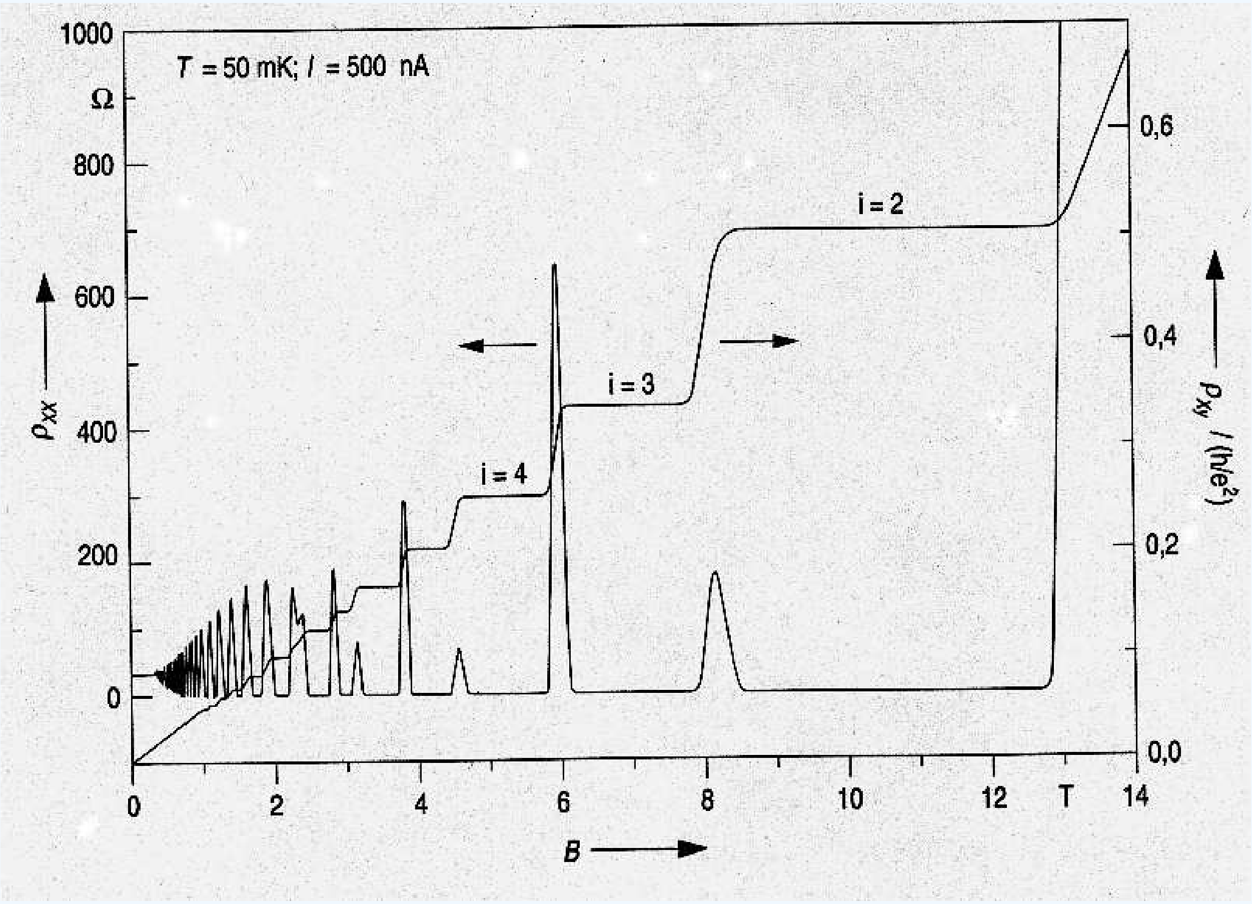


v. Klitzing *et al.*

Phys. Rev. Lett. '80

more recent measurement from www.ptb.de

Quantum Hall Effect



v. Klitzing *et al.*

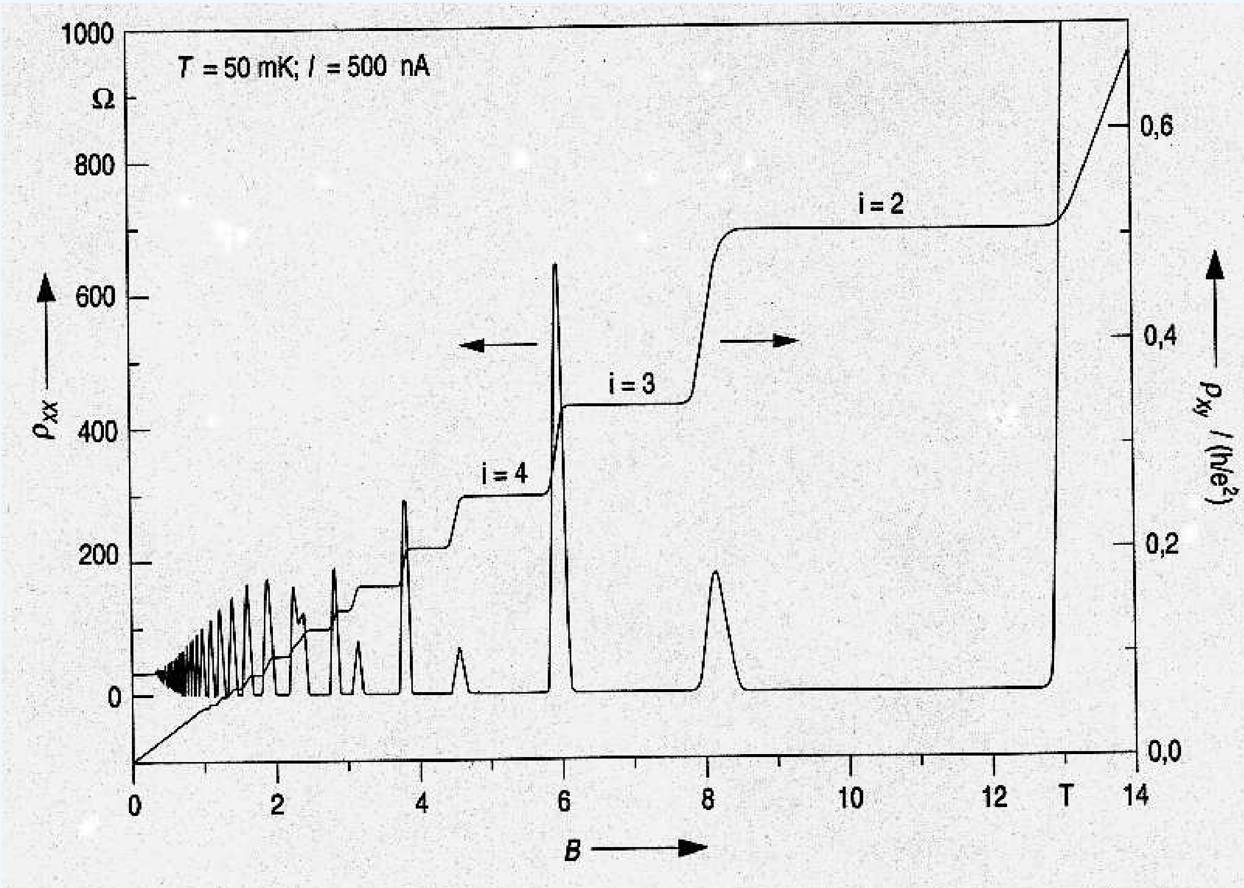
Phys. Rev. Lett. '80

E_{F} between Landau levels:

$$R_x = 0$$

more recent measurement from www.ptb.de

Quantum Hall Effect



more recent measurement from www.ptb.de

v. Klitzing *et al.*

Phys. Rev. Lett. '80

E_{F} between Landau levels:

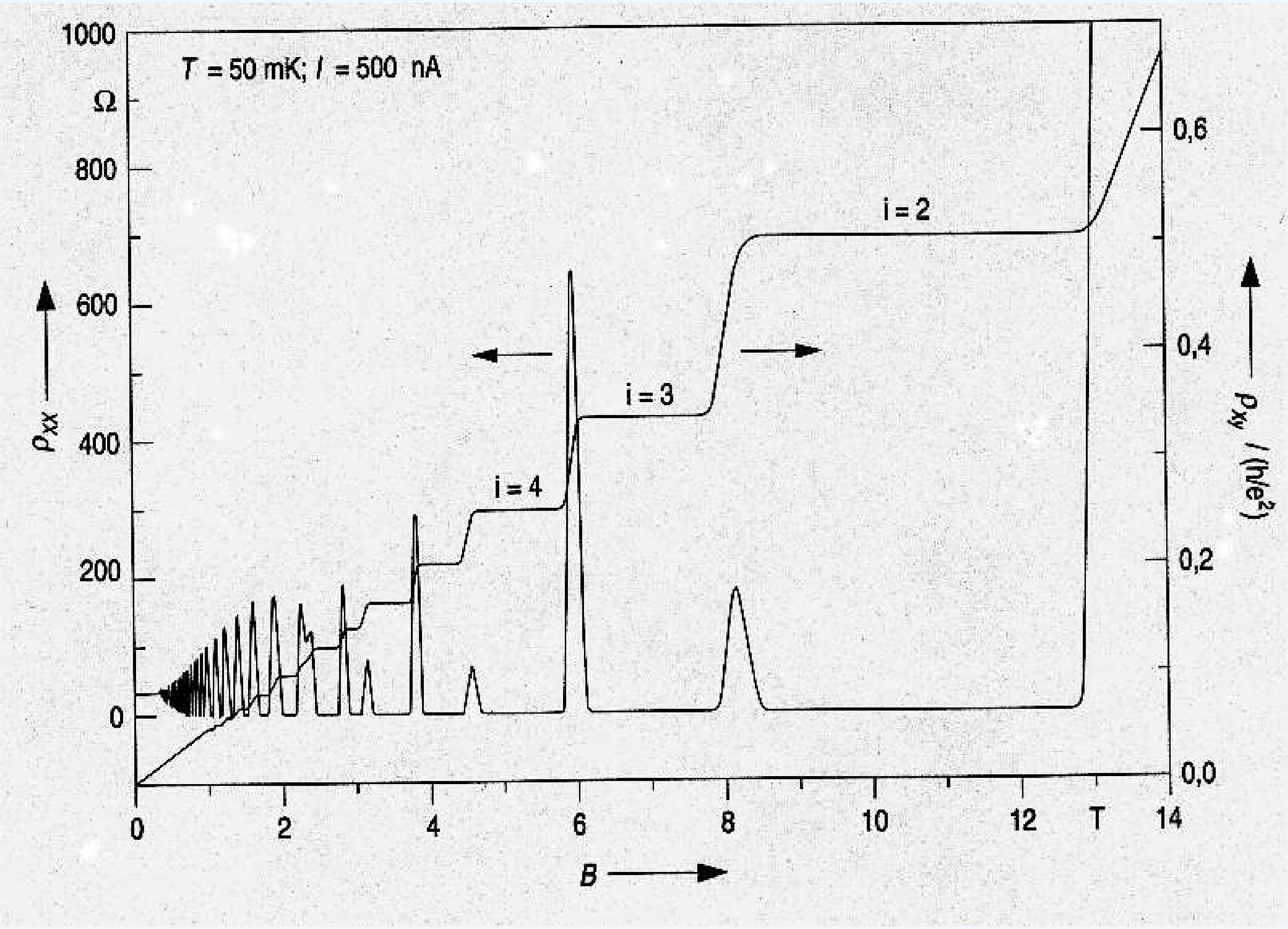
$$R_x = 0$$

$$R_H = \frac{h}{e^2}\frac{1}{n}$$

quantized!

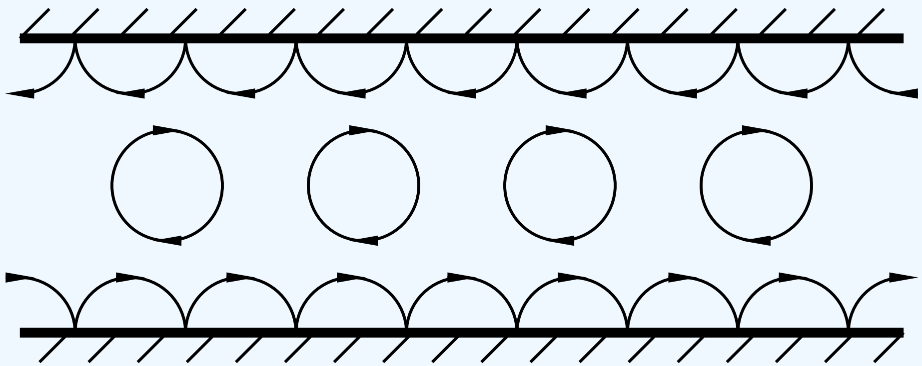
Application: Resistance standard

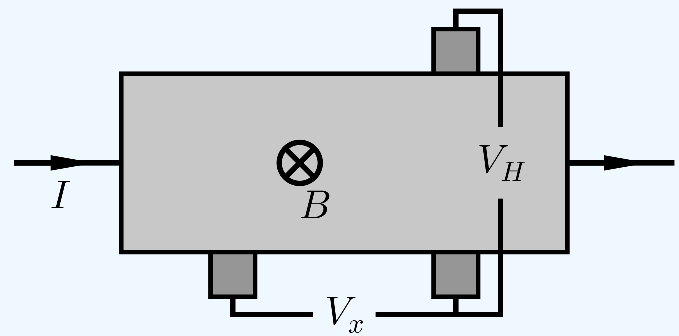
Quantum Hall Effect



Edge channels

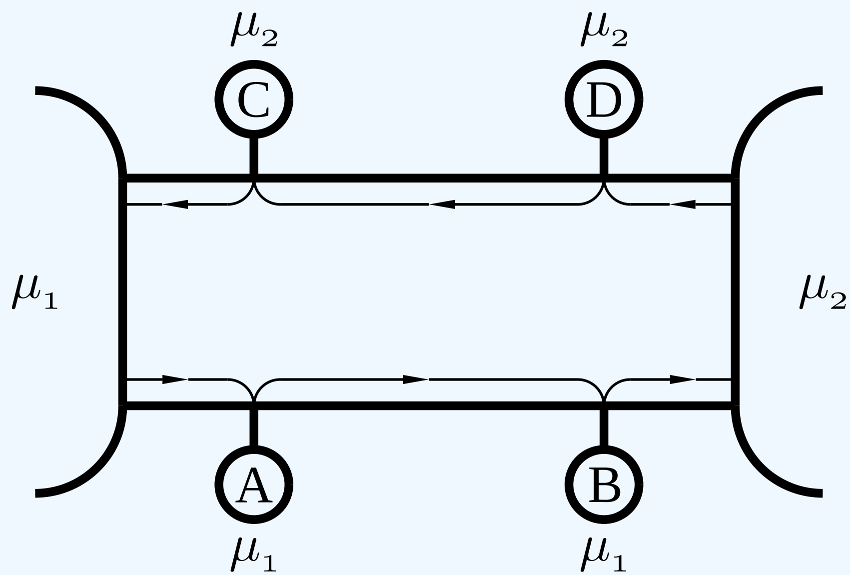
2d, strong magnetic field



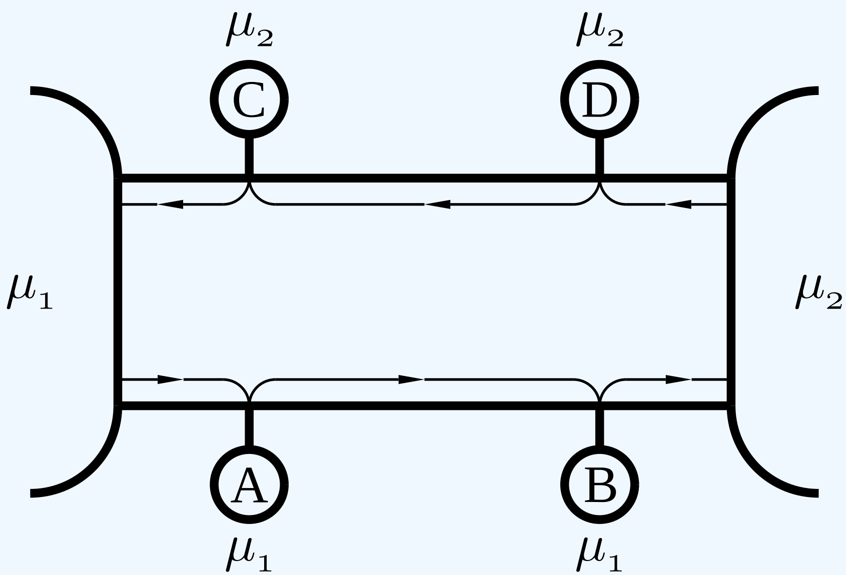


“skipping orbits” $\longrightarrow$ edge channels (1 per Landau level)

Quantum Hall Effect



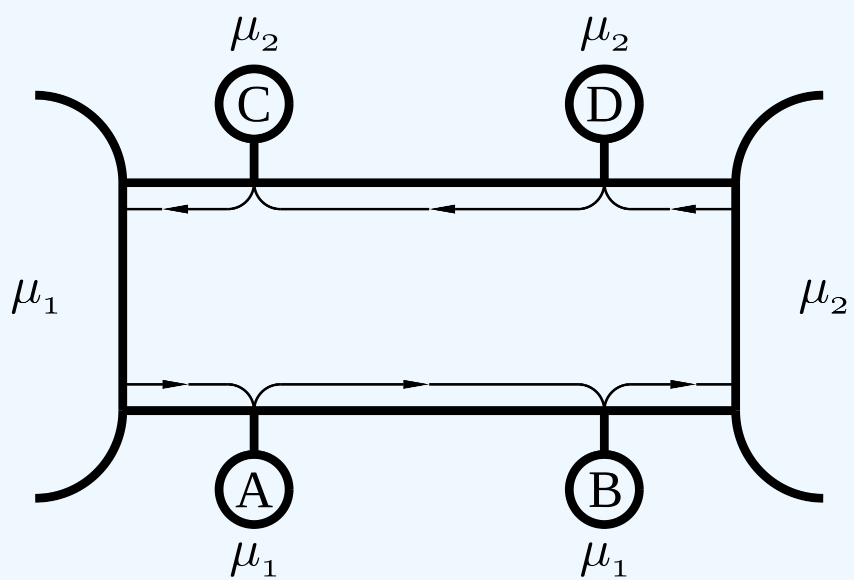
Quantum Hall Effect



$$T_{n,m} = \delta_{n,m}$$

spatial separation of
forward and backward channels

Quantum Hall Effect



$$I = 2\frac{e^2}{h} N_{\mathrm{LL}} \frac{\mu_1 - \mu_2}{e}$$

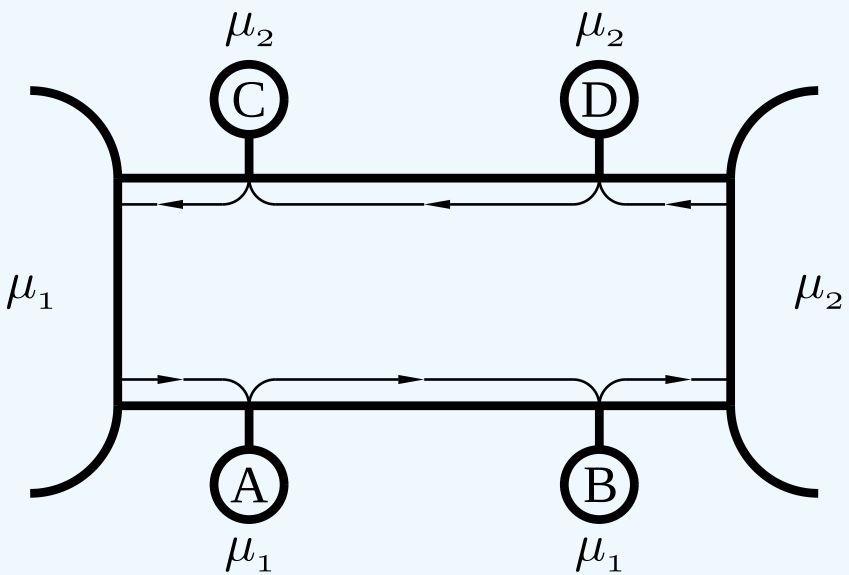
$$V_x = V_A - V_B = 0$$

$$\rightarrow R_x = V_x / I = 0$$

$$T_{n,m} = \delta_{n,m}$$

spatial separation of
forward and backward channels

Quantum Hall Effect



$$T_{n,m} = \delta_{n,m}$$

spatial separation of
forward and backward channels

$$I = 2\frac{e^2}{h} N_{\mathrm{LL}} \frac{\mu_1 - \mu_2}{e}$$

$$V_x = V_A - V_B = 0$$

$$\rightarrow R_x = V_x / I = 0$$

$$V_H = V_A - V_C = (\mu_1 - \mu_2)/e$$

$$\rightarrow R_H = V_H / I = \frac{h}{2e^2} \frac{1}{N_{\mathrm{LL}}}$$