Noise in mesoscopic systems

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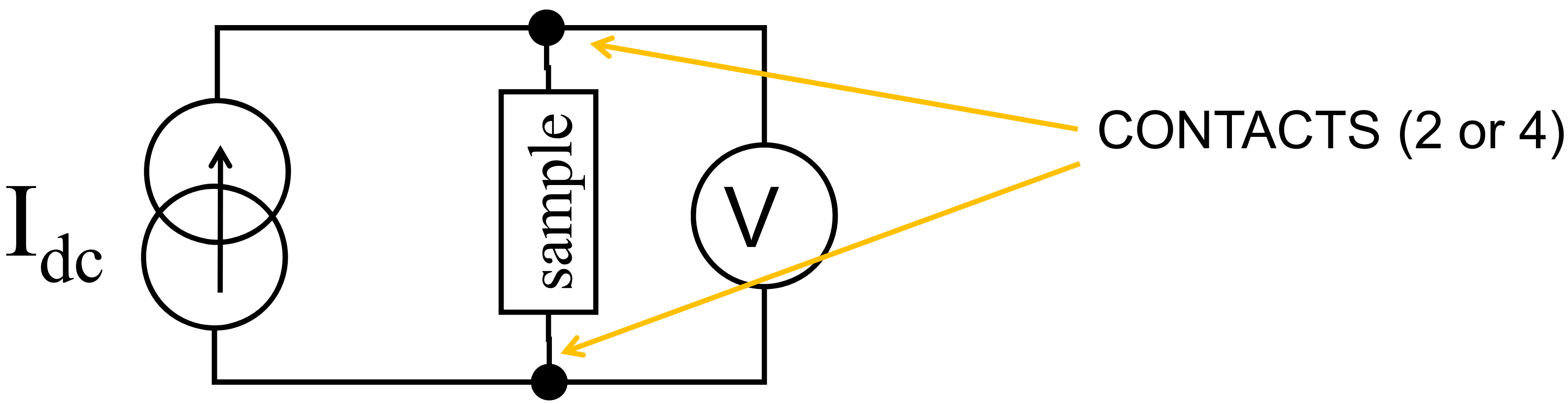
Transport: DC conductance

* STATIC response to a DC voltage

$$I_{dc}(V_{dc}) \rightarrow G(V_{dc}) = \frac{dI_{dc}}{dV_{dc}}$$

- For V_{dc}=0: LINEAR conductance
- Measure of the dissipation

Setup: DC Current source + voltmeter or DC voltage source + ampmeter
OR: low frequency (Hz-kHz) source + lock-in amplifier



Transport: AC conductance

* LINEAR, DYNAMICAL response to an oscillating field

$$V = V_{dc} + \delta V \cos \omega_0 t \rightarrow G(V_{dc}, \omega_0) = \frac{\partial I_{\omega_0}}{\partial V_{\omega_0}}$$

It is a COMPLEX quantity.

* Real part (in-phase response): dissipation

$$\mathrm{Re}[G(V_{dc}, \omega_0)] \xrightarrow[\omega_0 \rightarrow 0]{} G(V_{dc})$$

* Imaginary part (response in quadrature): reaction, i.e. delay

$$\mathrm{Im}[G(V_{dc}, \omega_0)] \xrightarrow[\omega_0 \rightarrow 0]{} 0$$

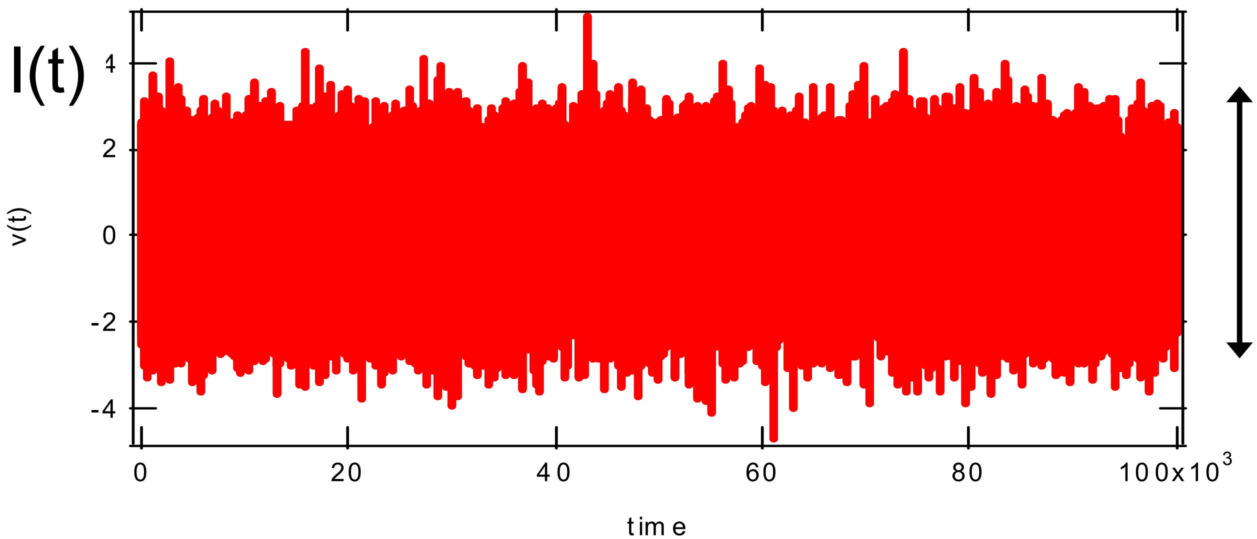
INTERESTING WHEN: $\omega_0 \tau > 1$

Summary

	$\langle \bullet \rangle$	
	$I(V)$	DC characteristics
$\dfrac{\partial \bullet}{\partial V_{\omega_0}}$	$\dfrac{\partial I_{\omega_0}}{\partial V_{\omega_0}}$	AC conductance
$\dfrac{\partial^2 \bullet}{\partial V_{\omega_0} \partial V_{\omega_1}}$	$\dfrac{\partial^2 I_{\omega_0 \pm \omega_1}}{\partial V_{\omega_0} \partial V_{\omega_1}}$	Non-linear AC transport (mixing, rectification)

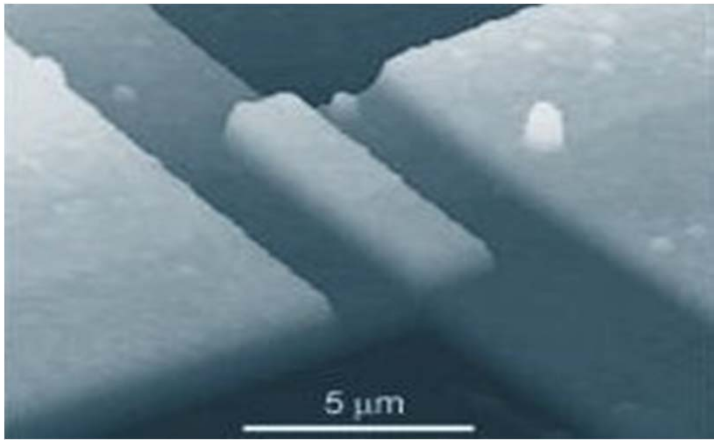
Fluctuations (noise)

Record current vs. Time and calculate the variance

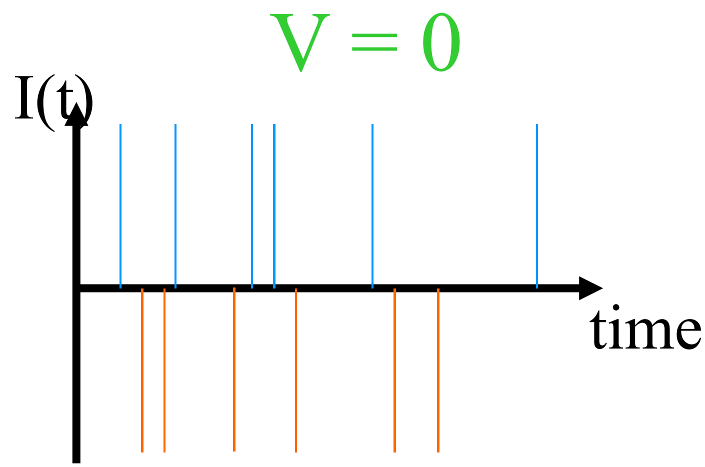


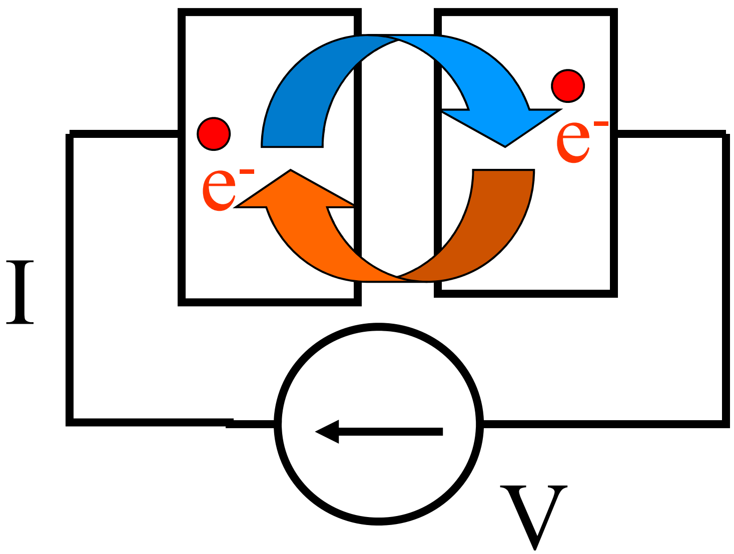
$$\langle \delta I^2 \rangle$$

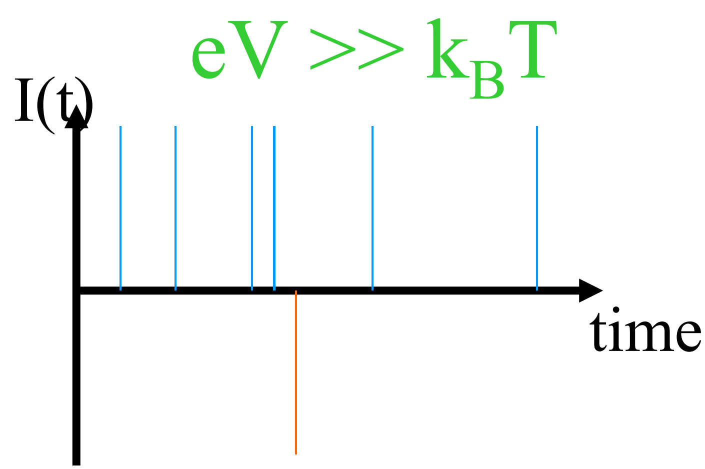
Example: the tunnel junction



L. Spietz, Yale Univ.



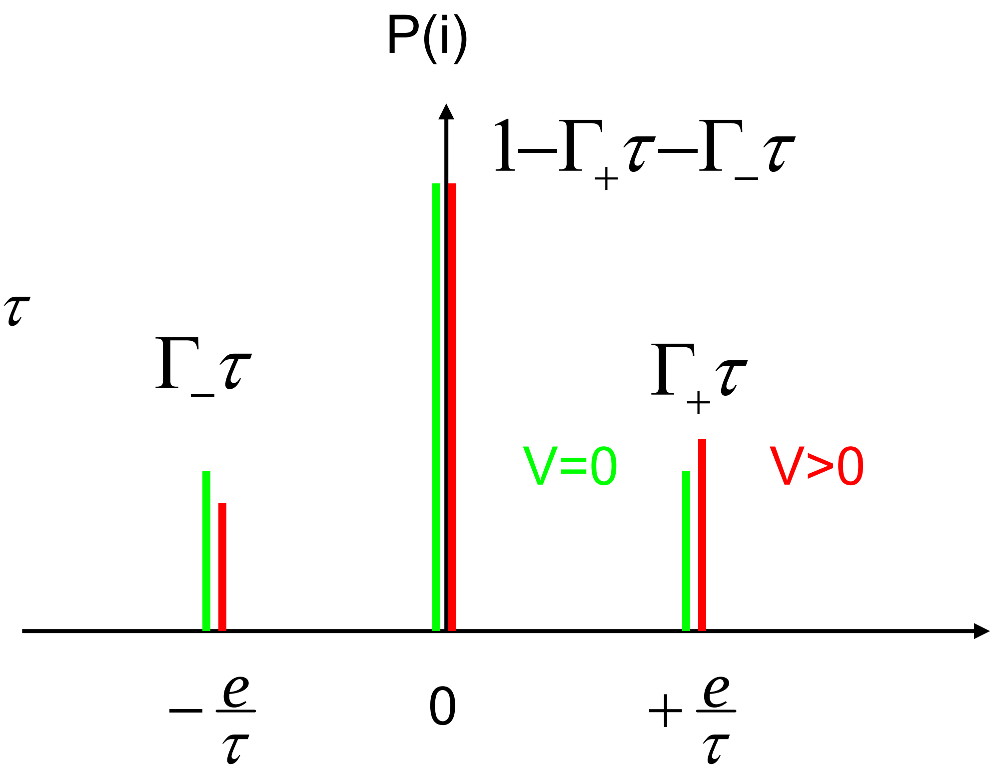




Statistics of the Current in a tunnel junction

Each time τ :

$$P(I) = \begin{cases} P(+1e) = \Gamma_+ \tau \\ P(-1e) = \Gamma_- \tau \\ P(0e) = 1 - \Gamma_+ \tau - \Gamma_- \tau \end{cases}$$



$$\langle I \rangle = \left(+\frac{e}{\tau}\right)\Gamma_+ \tau + \left(-\frac{e}{\tau}\right)\Gamma_- \tau = e\left(\Gamma_+ - \Gamma_-\right) = GV$$

Current fluctuations in a tunnel junction at low frequency

$$\langle \delta I^2 \rangle = \left(\frac{e}{\tau}\right)^2 (\Gamma_+\tau + \Gamma_-\tau) = 2eIB\coth\left(\frac{eV}{2k_BT}\right)$$ B=bandwidth

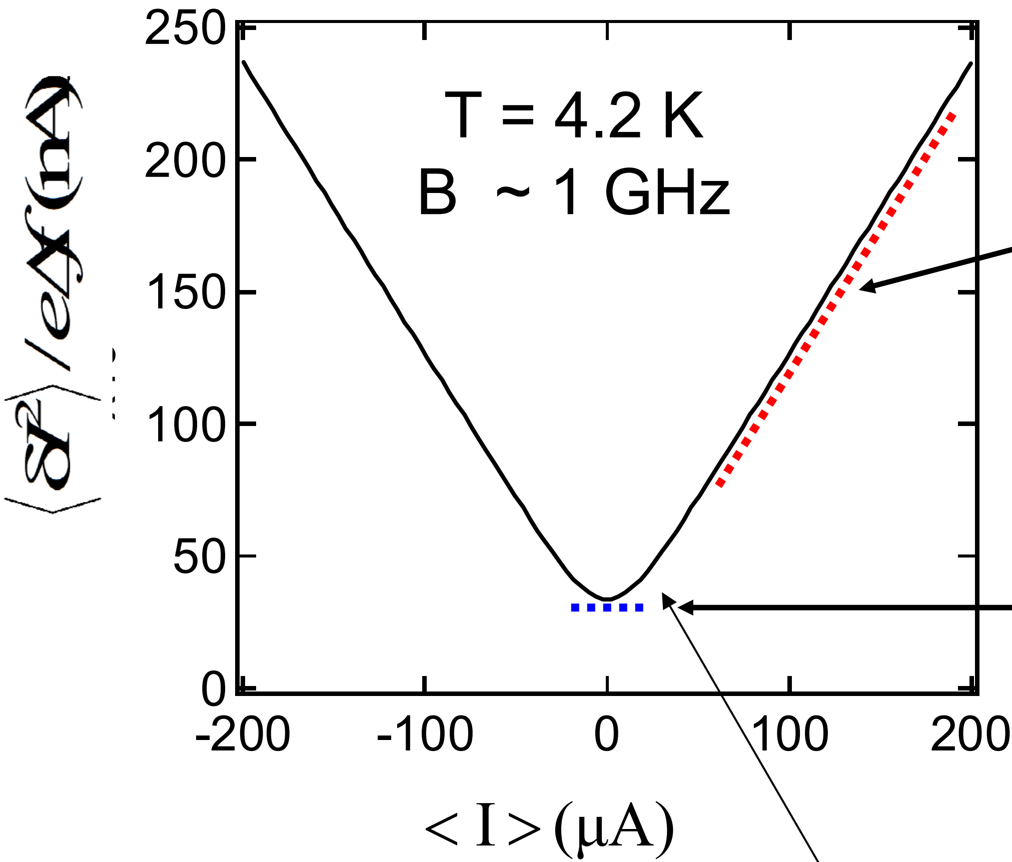
$$S_2 = \begin{cases} 4k_BTG & \text{if } eV << k_BT \\ 2eI & \text{if } eV >> k_BT \end{cases}$$

Equilibrium (Johnson) noise: macroscopic, fluctuation-dissipation theorem

Noise spectral density in A²/Hz

Shot noise: discreteness of charge

Experiment (ω=0,T=4.2K)



Poissonian statistics

$$\left\langle \delta I^2 \right\rangle = 2eIB$$

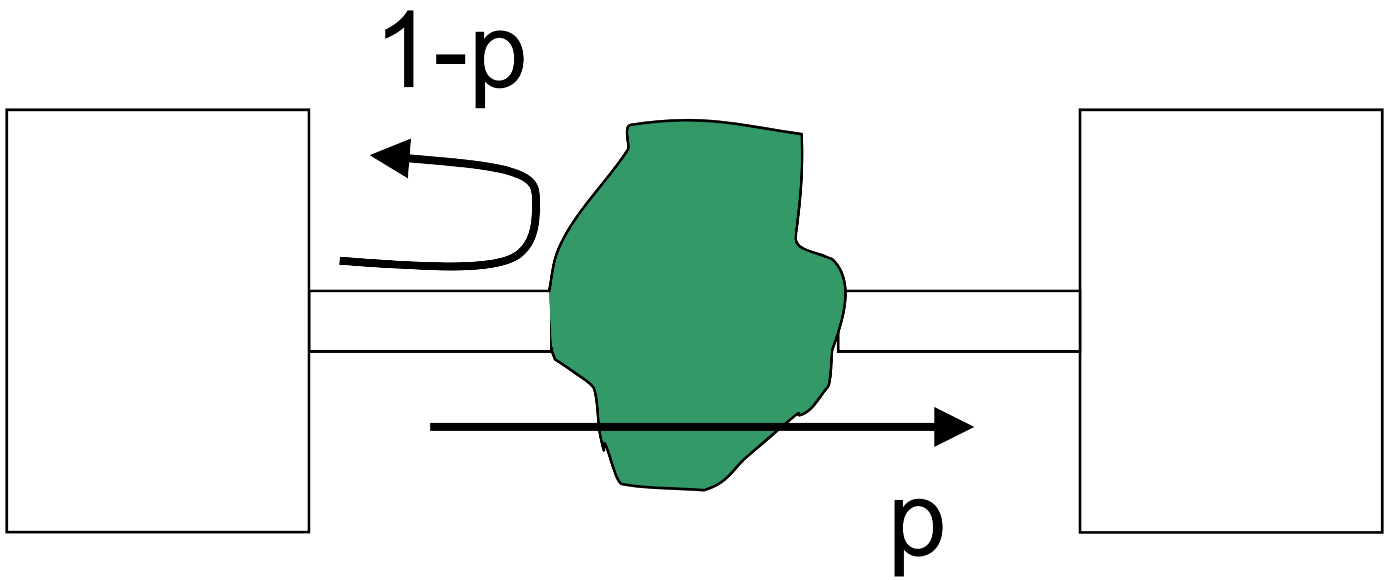
Equilibrium noise (fluctuation-dissipation theorem)

$$\left\langle \delta I^2 \right\rangle = 4k_B TGB$$

crossover

$$eV = k_B T$$

Transport in mesoscopic systems

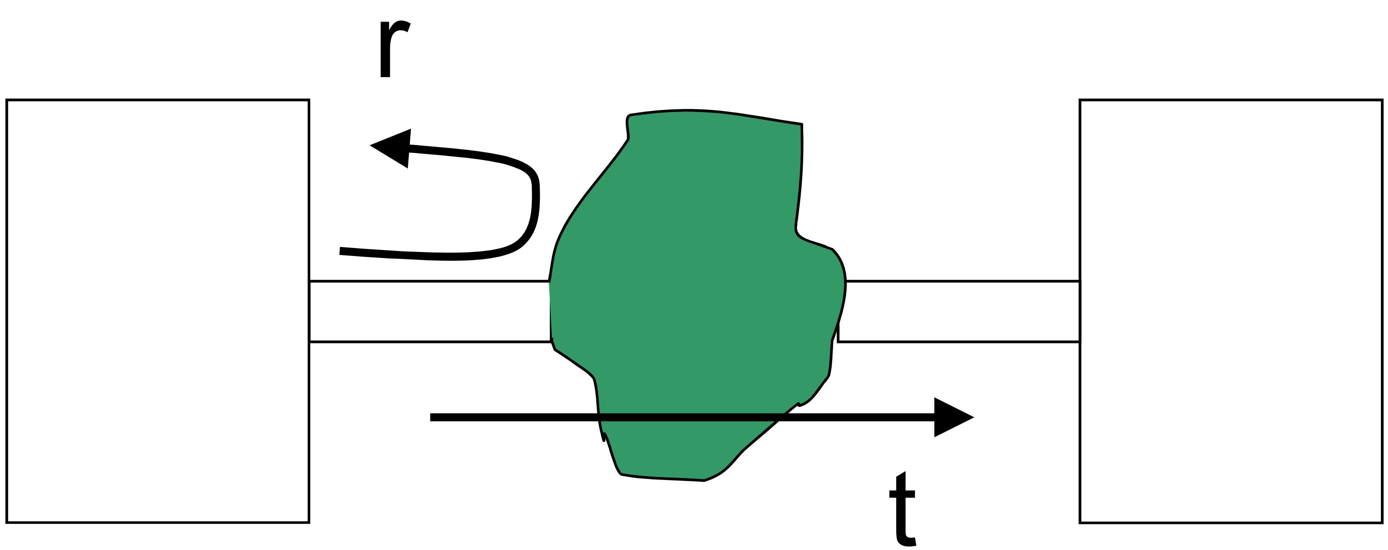


-The electrons cross the sample with a certain probability p≠0,1

- The incoming stream of electrons has thermal fluctuations

So the current fluctuates !

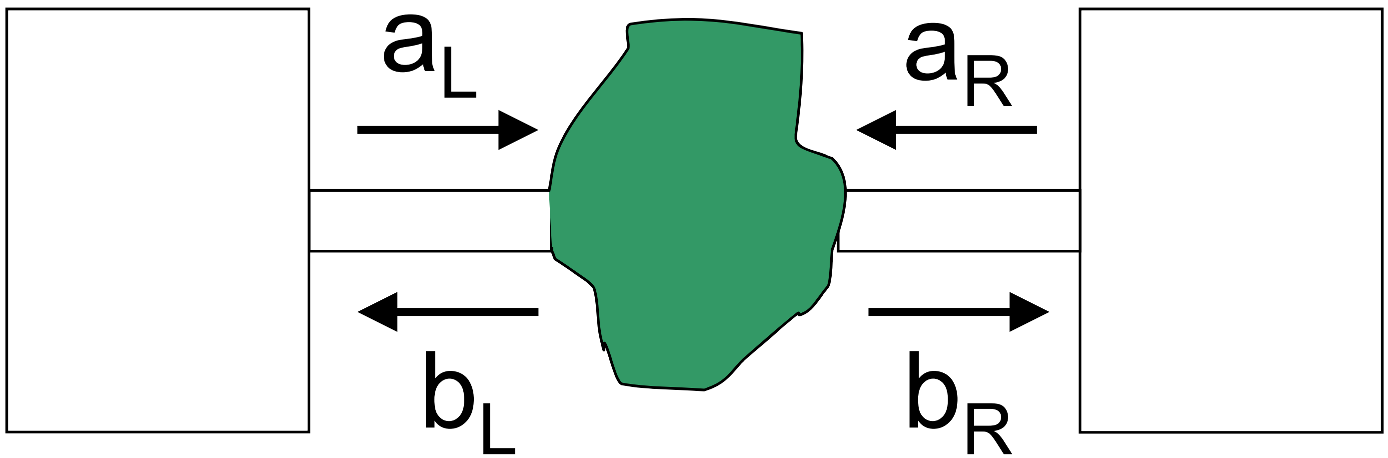
The scattering approach: waves



Incoming waves are partially reflected and transmitted

$$p = |t|^2 = 1 - |r|^2$$

The scattering (Landauer) formalism



$$b_L = r a_L + t a_R$$

$$\hat{I} \propto \int v(E)\rho(E)dE(\, a_L^+ a_L - b_L^+ b_L)$$

The quantum current operator

For ω=0, *r* and *t* energy independent, with p = |t|^2 = 1-|r|^2 :

$$\hat{I} = \frac{e}{h}\int dE\,|t|^2\,(a_L^+ a_L - a_R^+ a_R) - (r^* t a_L^+ a_R + r t^* a_R^+ a_L)$$

Dc current:

$$\left\langle \hat{I} \right\rangle = \frac{e}{h}\int dE\ p(\left\langle a_L^+ a_L \right\rangle - \left\langle a_R^+ a_R \right\rangle)$$

$$= p\frac{e}{h}\int dE(f_L(E) - f_R(E)) = p\frac{e^2}{h}V$$

Büttiker-Landauer formula

Zero frequency noise

$$\left\langle \Delta I^2 \right\rangle = \left\langle \left(I - \left\langle \hat{I} \right\rangle \right)^2 \right\rangle = \left\langle \hat{I}^2 \right\rangle - \left\langle \hat{I} \right\rangle^2$$

$$= \left(\frac{e}{h}\right)^2 p^2 \int dE\left(f_L(E)(1 - f_L(E)) + f_R(E)(1 - f_R(E)\right)$$

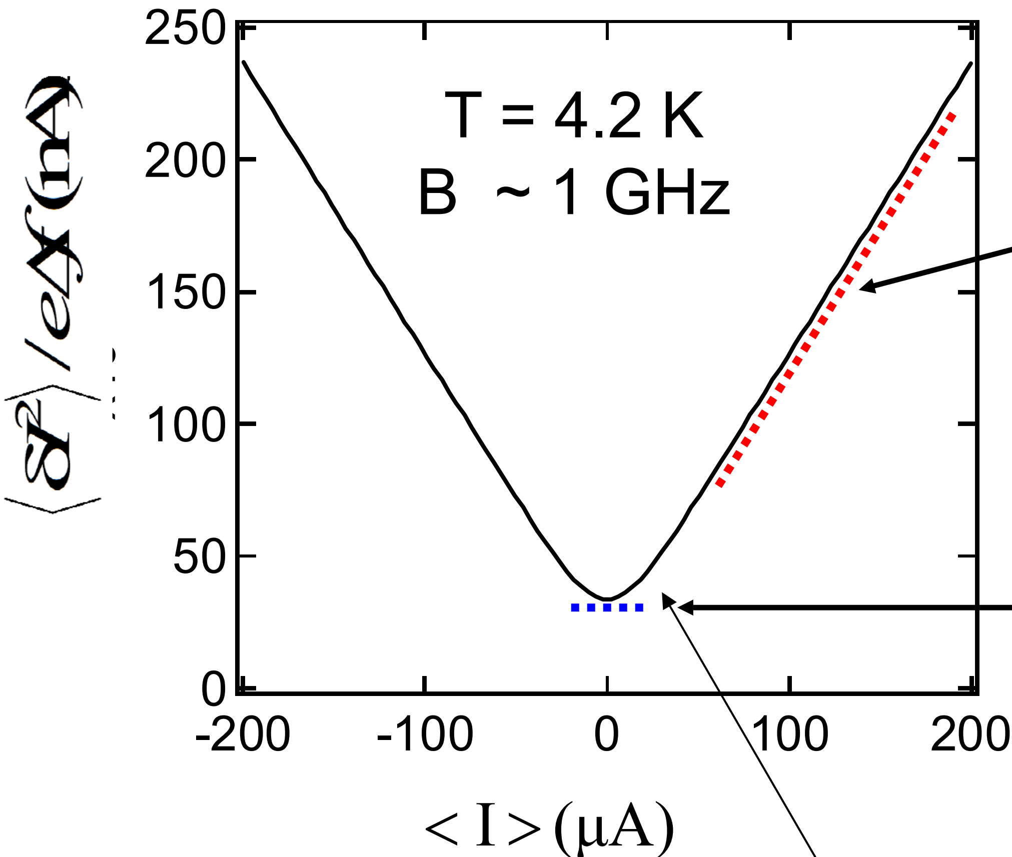
$$+ \left(\frac{e}{h}\right)^2 p(1-p) \int dE\left(f_L(E)(1 - f_R(E)) + f_R(E)(1 - f_L(E)\right)$$

First term: proportionnal to p^2 x TEMPERATURE

Second term: proportionnal to p(1-p) x VOLTAGE for eV>>k_BT

For a review: Blanter & Büttiker, Phys. Rep. 336 (00)

Example: tunnel junction



$$p << 1$$

Full shot noise

$$S_2 = eI$$

Equilibrium noise (fluctuation-dissipation theorem)

$$S_2 = 2k_B TG$$

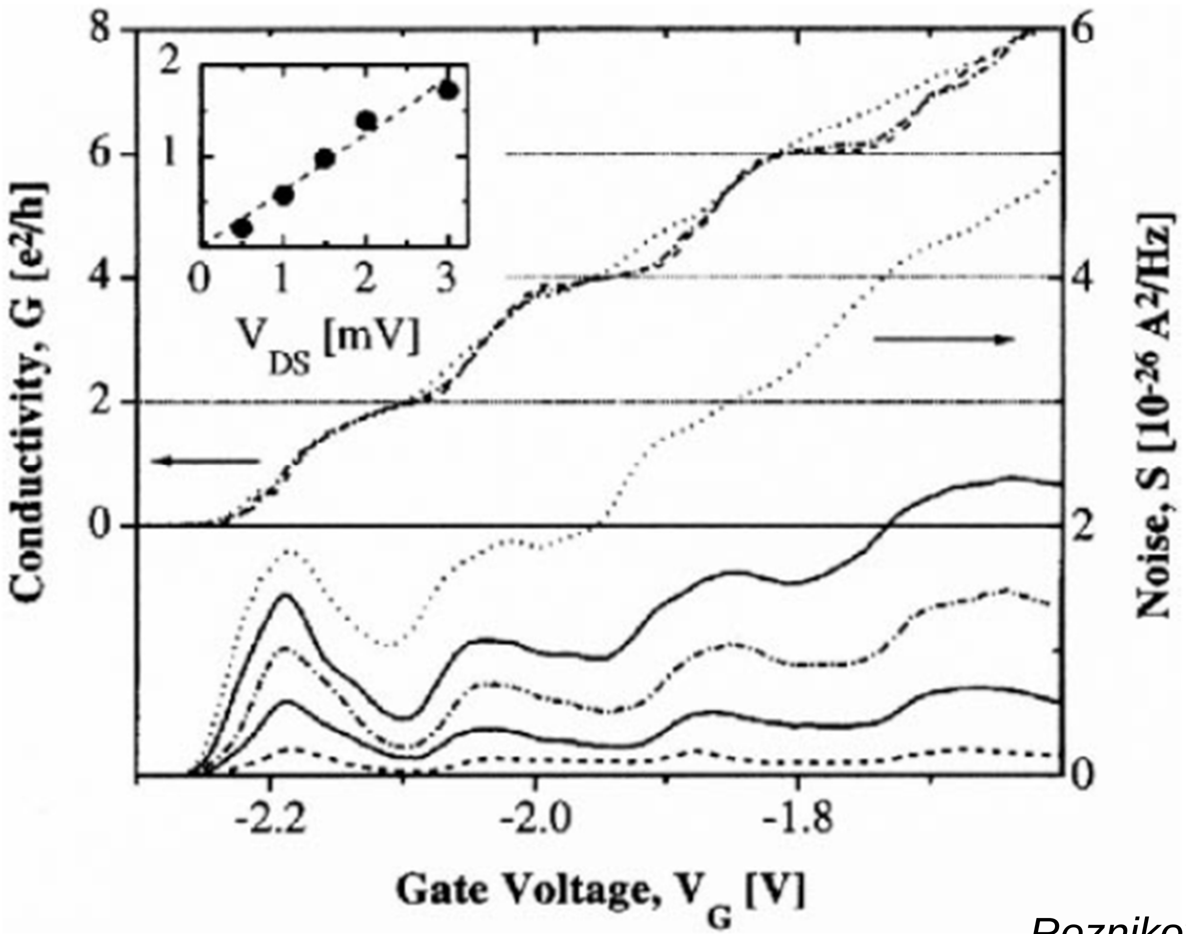
crossover

$$eV = k_B T$$

Example: QPC, p tunable

For eV>>kT, S_2=FeI, with F the Fano factor

$$F = \frac{\sum p_i(1-p_i)}{\sum p_i}$$

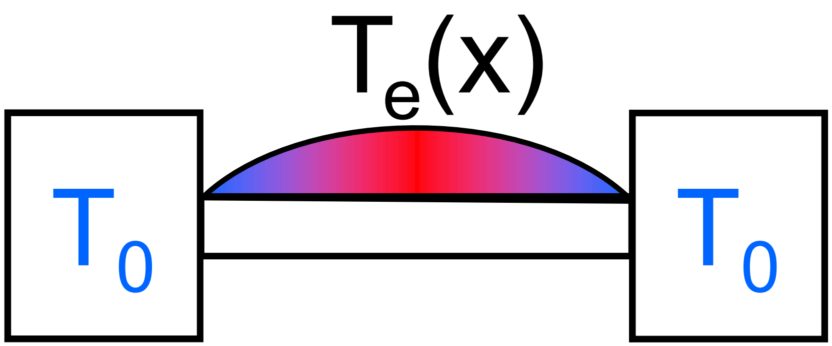


Reznikov et al., PRL (95)

Example: diffusive wire, p distributed

$$F = 1 - \frac{\langle p^2 \rangle}{\langle p \rangle} = \frac{1}{3}$$

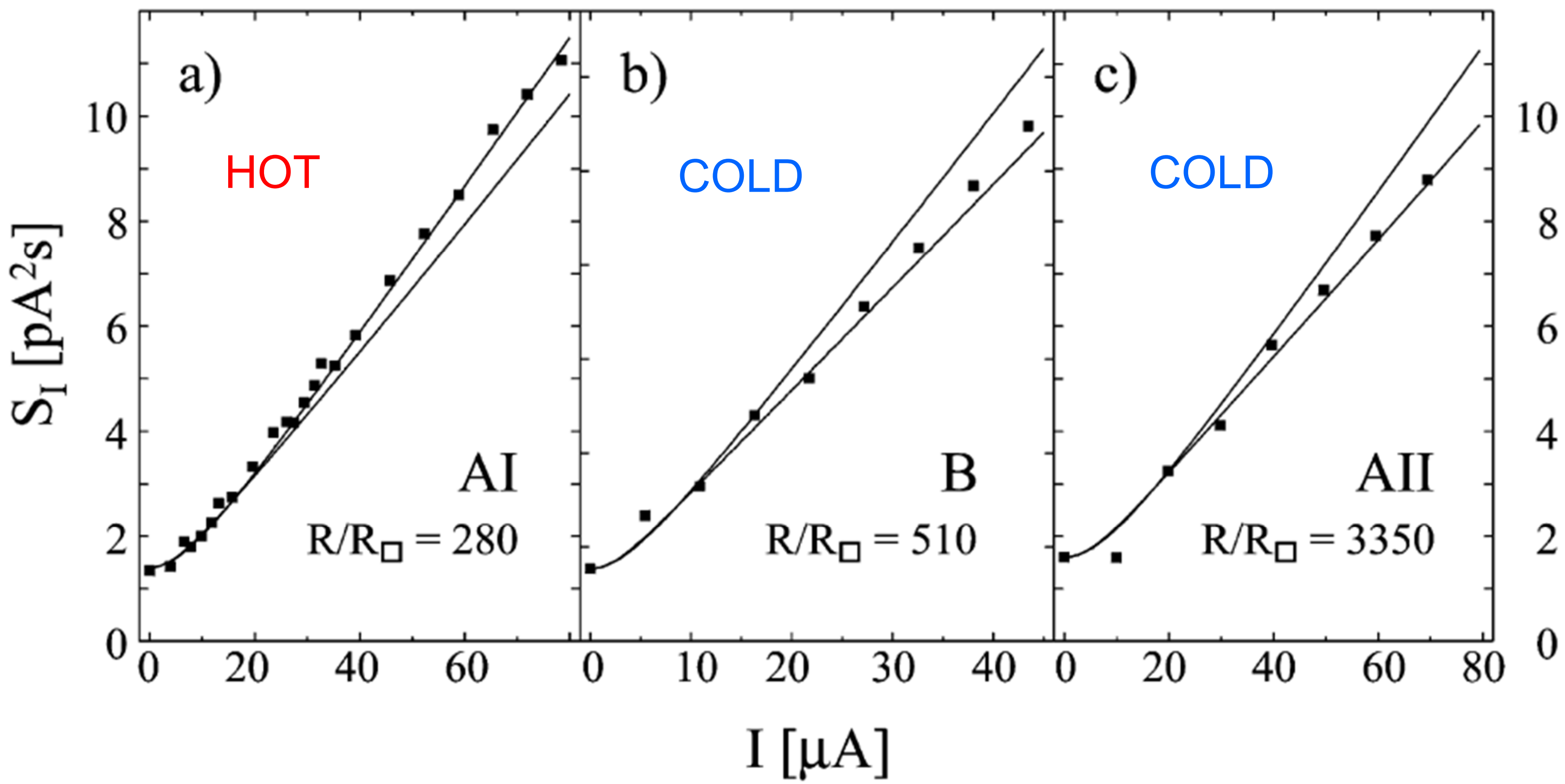
But: what if electrons interact (hot electrons regime) ?



$$S_2 = 4k_B R \int_0^L T_e(x, V) \frac{dx}{L}$$

$$F = \sqrt{3}/_4 \approx 0.43$$

Elastic transport vs. hot electrons: effect of length & T



Henny et al. PRB (99)

From meso- to macroscopic: electron-phonon interaction

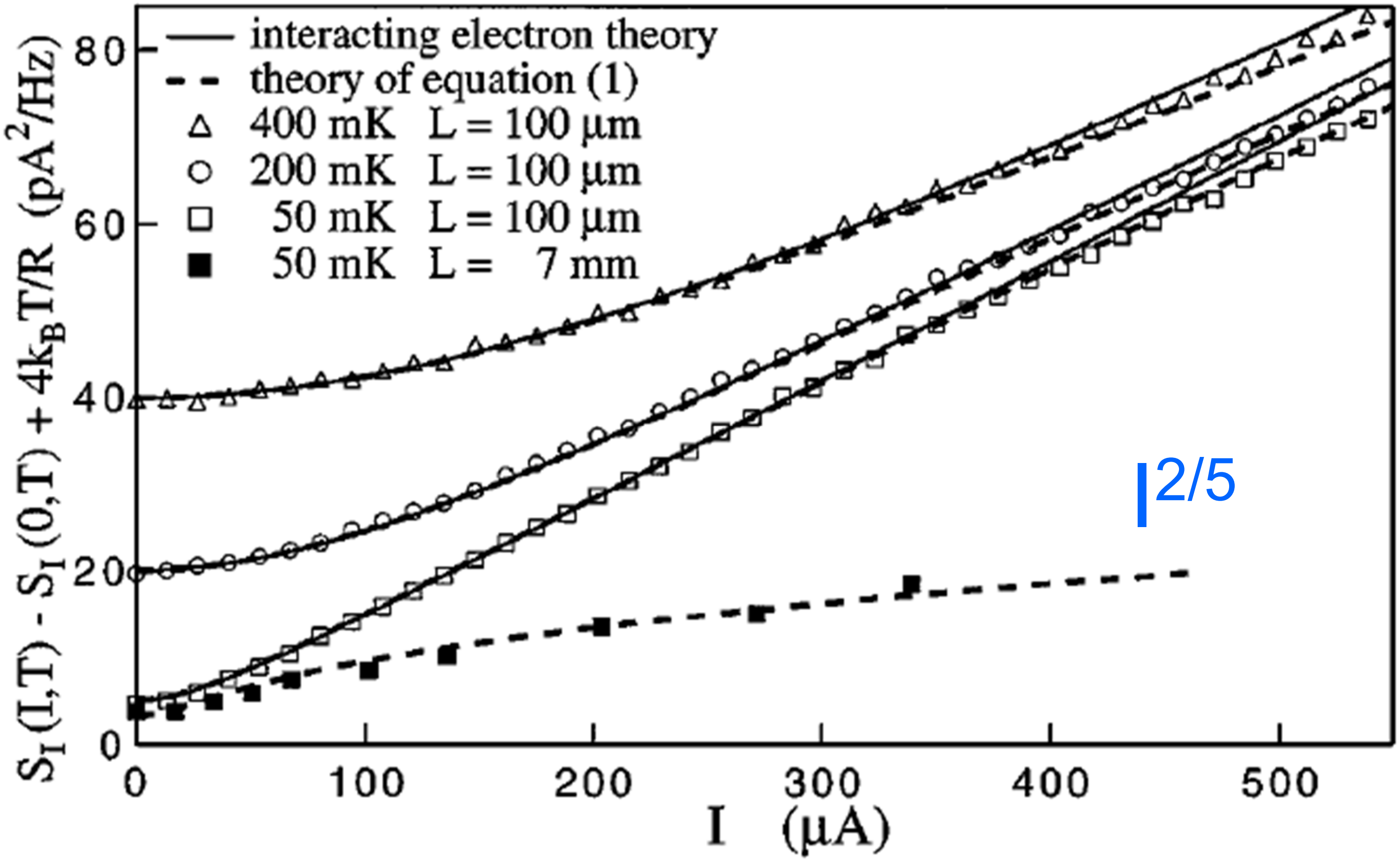
$$S_2 = 2k_B TG$$

Electrons are heated up by Joule effect and cooled down by emission of phonons: $T \neq T_{ph}$

$$P_{Joule} = RI^2 = P_{e-ph} = \Sigma V\left(T^5 - T_{ph}^5\right)$$

$$\Rightarrow T \propto I^{2/5}$$

Noise is suppressed by inelastic scattering (e-phonon)



Steinbach et al., PRL (96)

Shot noise measure the charge of the carriers

$$S_2 = FqI$$

Examples:

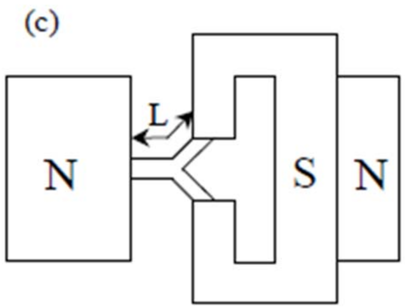
- Normal metal / Superconductor interface: q=2e
- Fractionnal Quantum Hall Effect: q=e/3

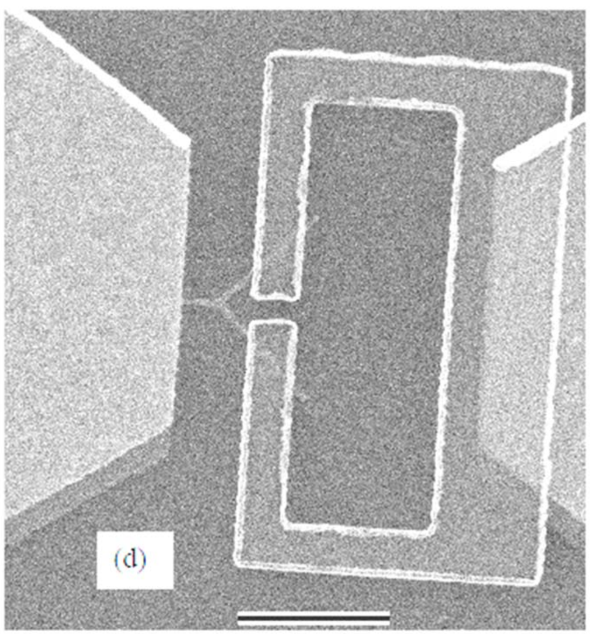
Effects of quantum coherence (i.e., of the phase of the wavefunctions) ?

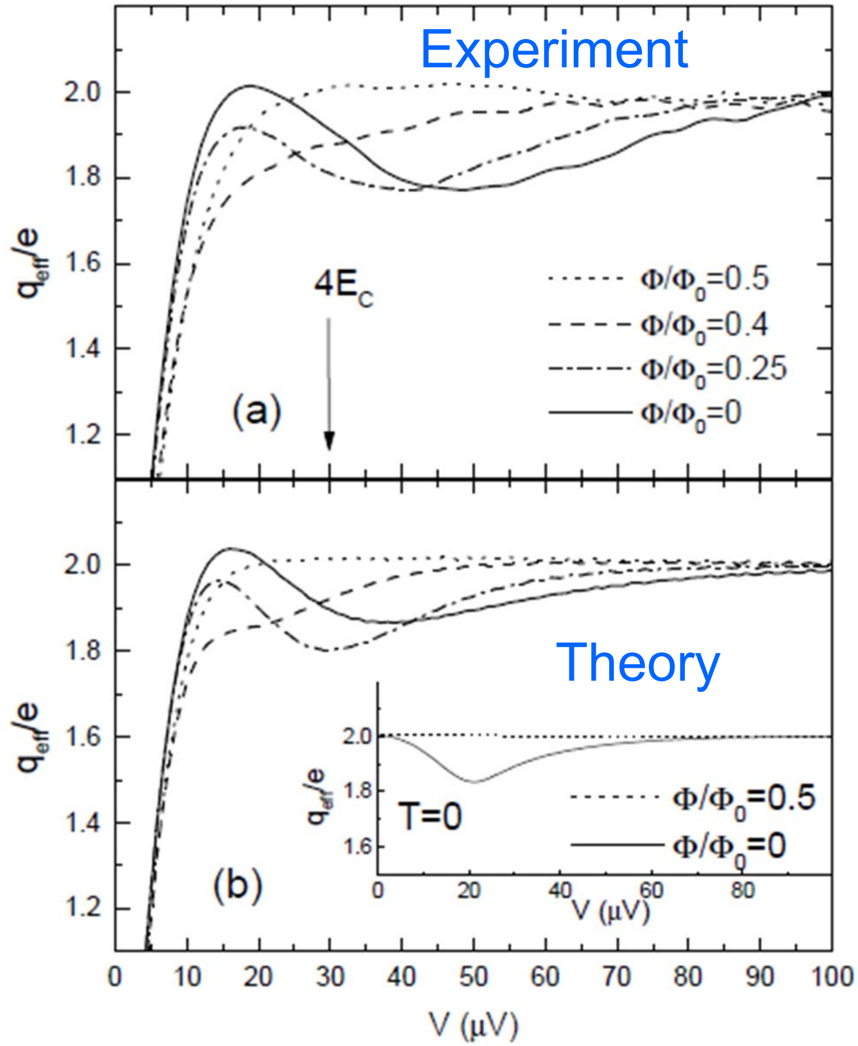
The phase of the wavefunctions can be modified with the help of a small magnetic field or flux (in a ring).

- Quantum corrections to S_2: small (weak localization, Aharonov-Bohm effect), never measured
- Andreev interferometer: effective charge depends on magnetic flux
- Two-particles Aharonov-Bohm effect: noise depends on magnetic flux whereas conductance does not

Example: Andreev Interferometer

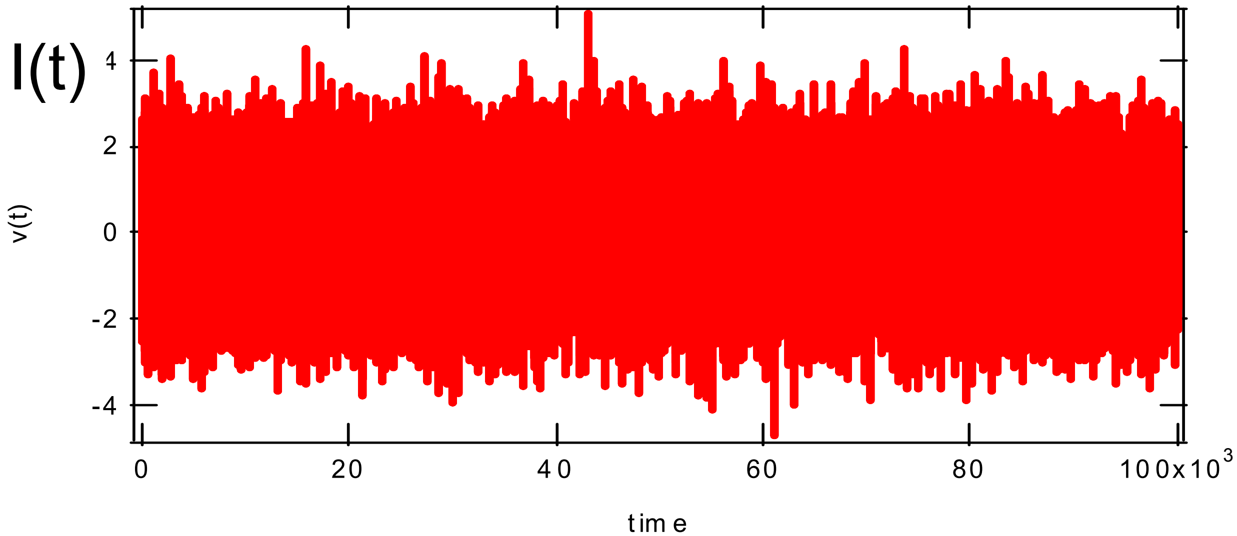






BR et al., PRL90 (03)

Noise at finite frequency ?



Time average

Correlation function: $$C_2(V,\tau) = \langle \delta I(t+\tau)\delta I(t) \rangle$$

Noise spectral density: $$S_2(V,\omega) = \langle \delta I(\omega)\delta I(-\omega) \rangle$$

$$\begin{aligned} I(\omega) = \frac{h}{e}\int dE \quad & [(1 - r^*(E)r(E+\hbar\omega))a_L^+(E)a_L(E+\hbar\omega) - r^*(E)t(E+\hbar\omega)a_L^+(E)a_R(E+\hbar\omega) \\ & -t^*(E)r(E+\hbar\omega)a_R^+(E)a_L(E+\hbar\omega) - t^*(E)t(E+\hbar\omega)a_R^+(E)a_R(E+\hbar\omega)] \end{aligned}$$

Quantum mechanics: ordering of operators?

Average current:

$$I_{DC} = \langle \hat{I} \rangle$$

Noise S_2:

$$S_2(\omega) = \int dt e^{i\omega t} \begin{cases} \langle \hat{I}(0)\hat{I}(t) \rangle \\ \langle \hat{I}(t)\hat{I}(0) \rangle \\ \frac{1}{2}\left(\langle \hat{I}(0)\hat{I}(t) \rangle + \langle \hat{I}(t)\hat{I}(0) \rangle \right) \end{cases}$$

Absorption

Emission

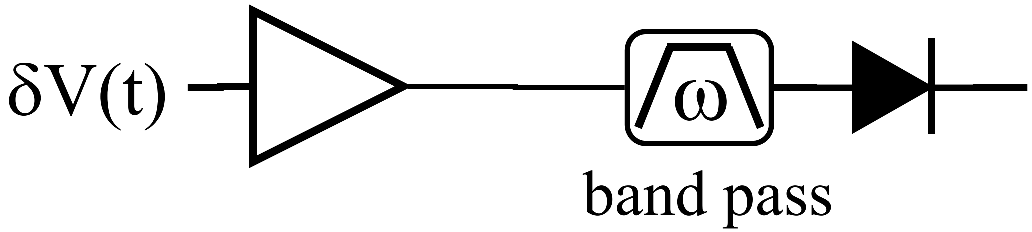
Classical

Zero point fluctuations

$$S_2^{abs}(\omega) = S_2^{em}(-\omega)$$

$$S_2^{sym}(\omega) = S_2^{em}(\omega) + \frac{1}{2} G\hbar\omega$$

S_2 in the quantum regime $\hbar\omega > k_BT, eV$



J. Gabelli & BR, (10)
First observation:
R. Schoelkopf et al. PRL78 (97)

Tunnel junction R=50Ω

$T_{phonons}$= 22 mK
$T_{electrons}$= 27 mK

f = 5.5 - 6.5 GHz
hf/k_B = 290 mK
Ghf/e = 0.50μA

It is not possible to separate the noise of the amplifier from the ZPF !

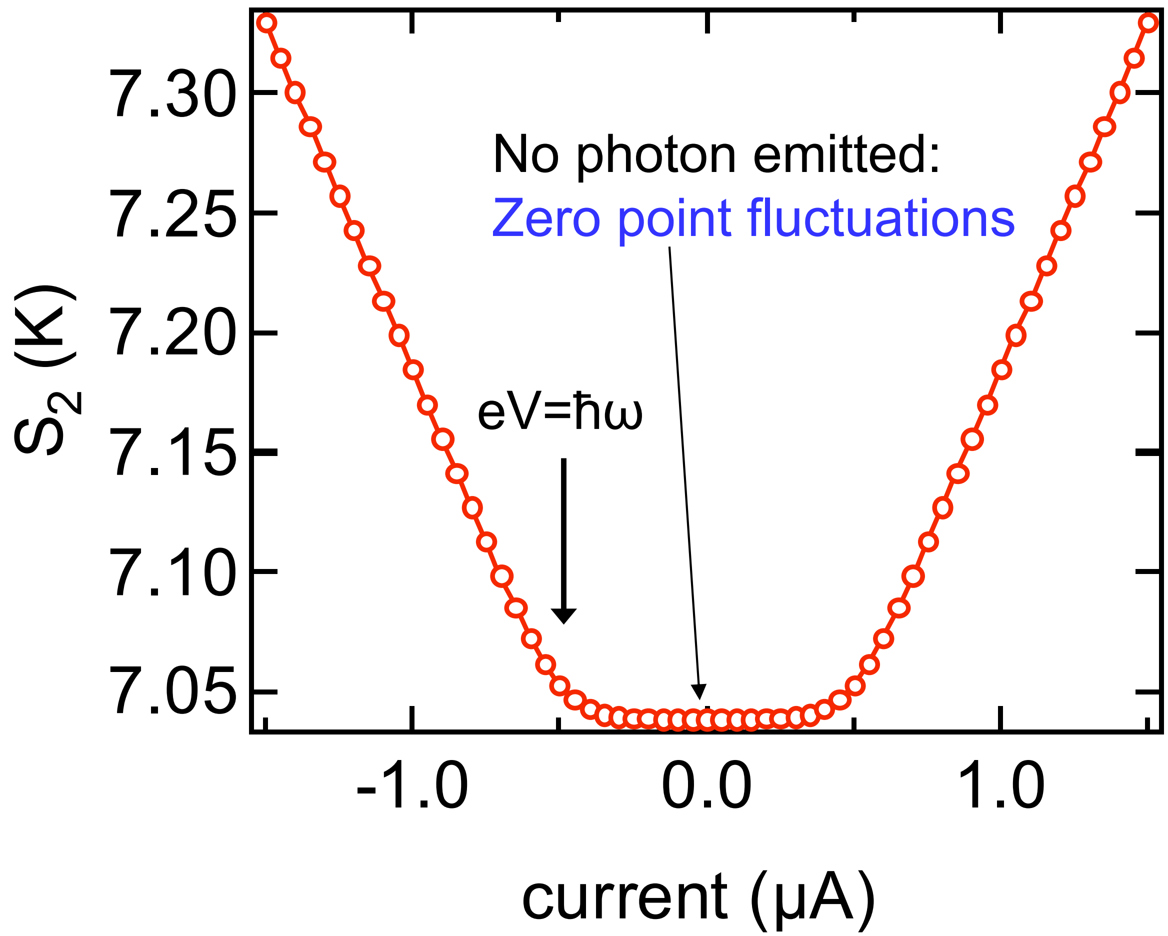


Photo-assisted noise: $S_2(\omega)$ in the presence of AC excitation at freq. ω_0

$$V(t) = V_{dc} + V_{ac}\cos\omega_0 t$$

$$a_L(E,t) = a_L \exp\left(-\frac{iEt}{\hbar}\right)$$

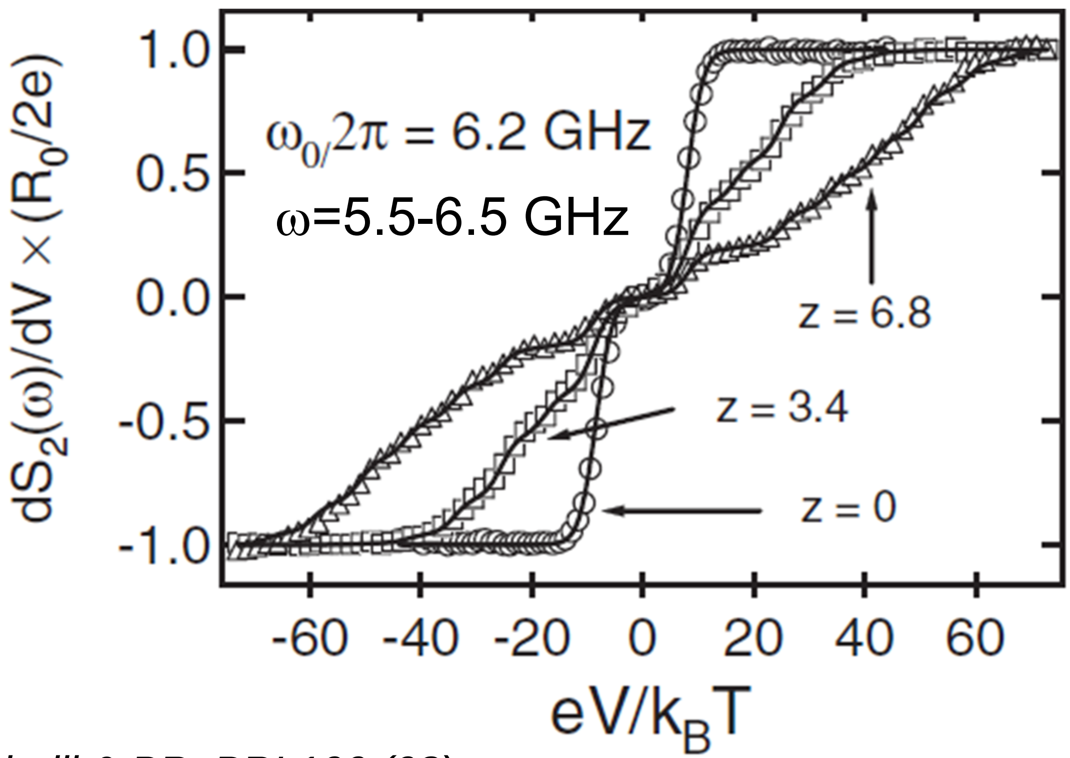
$$\downarrow$$

$$a_L(E,t) = a_L \exp - \frac{i}{\hbar}\left(Et + \int_0^t eV(t')dt'\right)$$

The energy levels of the reservoir follow adiabatically the voltage (Tien-Gordon)

For a dc voltage, this is equivalent to a shift of the Fermi level

Photo-assisted noise: $S_2(\omega)$ in the presence of AC excitation at freq. ω_0



$$z = \frac{eV_{ac}}{\hbar\omega_0}$$

« Kinks » for

$$eV_{dc} = \pm\hbar\omega + n\hbar\omega_0$$

give access to the effective charge (no calibration necessary)

J. Gabelli & BR, PRL100 (08): $\omega \sim \omega_0$
First observation:
R. Schoelkopf et al. PRL80 (98) $\omega << \omega_0$

For a diffusive wire, importance of the diffusion time

Summary

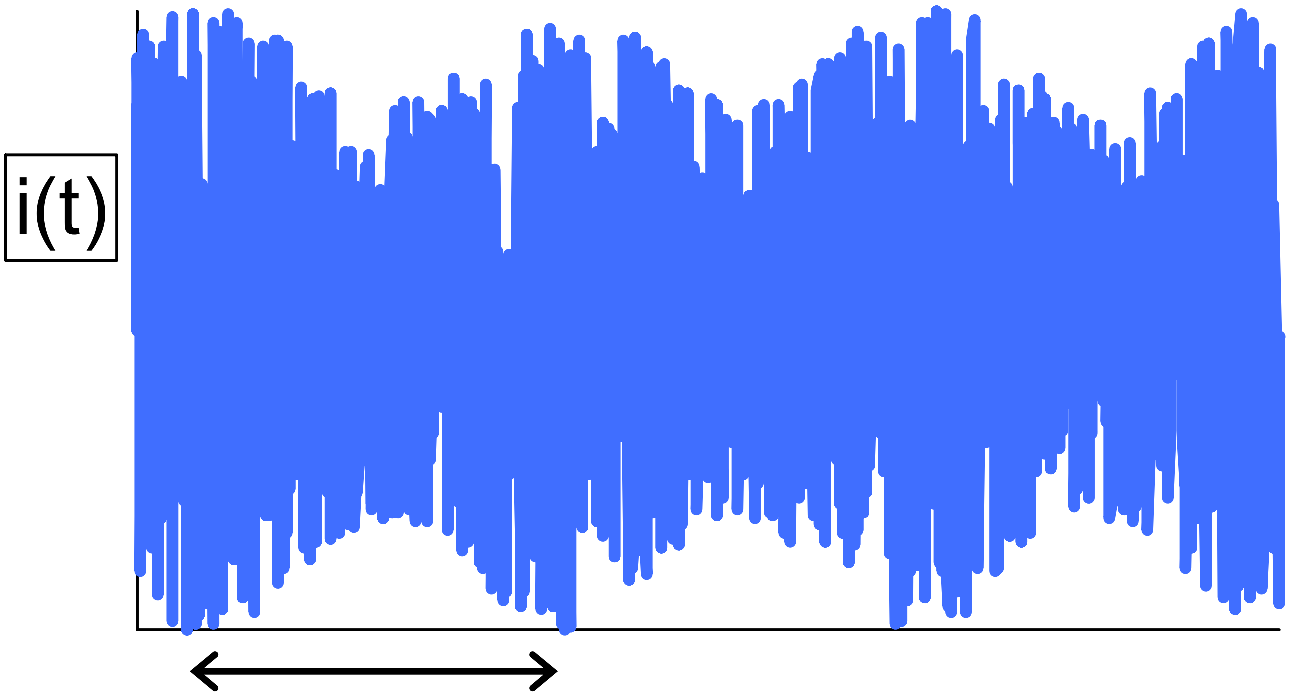
	$\langle \bullet \rangle$	$\langle \bullet \bullet \rangle$
	$I(V)$	$\langle \delta I(\omega)\delta I(-\omega) \rangle$
$\frac{\partial \bullet}{\partial V_{\omega_0}}$	$\frac{\partial I_{\omega_0}}{\partial V_{\omega_0}}$	?
$\frac{\partial^2 \bullet}{\partial V_{\omega_0} \partial V_{\omega_1}}$	$\frac{\partial^2 I_{\omega_0 \pm \omega_1}}{\partial V_{\omega_0} \partial V_{\omega_1}}$	Photo-assisted noise

NOISE DYNAMICS

Noise susceptibility – How fast can one modulate noise ?

$$\frac{\partial S_2(\omega)}{\partial V_{\omega_0}}$$

$$V(t) = V_{dc} + \delta V \cos \omega_0 t$$



ω_0^{-1}

$$\omega_0 << \omega$$

Noise susceptibility – the case of a macroscopic conductor

Fluctuation-dissipation theorem, at equilibrium and low frequency $\hbar\omega \ll k_B T$:

$$S_2 = 4k_B TG$$

NOISE = electron THERMOMETER

$$\delta V(t) \Rightarrow \delta P_{Joule}(t) = I\delta V(t) \Rightarrow \delta T(t) \Rightarrow \delta S_2(t)$$

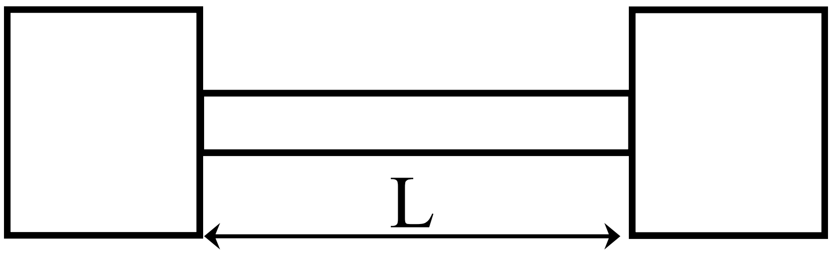
$$\boxed{\chi_{\omega_0}(\omega) = \frac{\partial S_2(\omega)}{\partial V_{\omega_0}} \propto Z_{thermal}^{-1}(\omega_0)}$$

$$\omega >> \omega_0$$

Its frequency dependence gives ENERGY RELAXATION (i.e. INELASTIC) time

Noise susceptibility – from macro- to mesoscopic conductor

diffusive metallic wire: length L >> mean free path



$$L^2 = D\tau_D$$

* long wire or SNS: phonon cooling

The noise suceptibility gives the ELECTRON-PHONON time

* intermediate wire: diffusion cooling

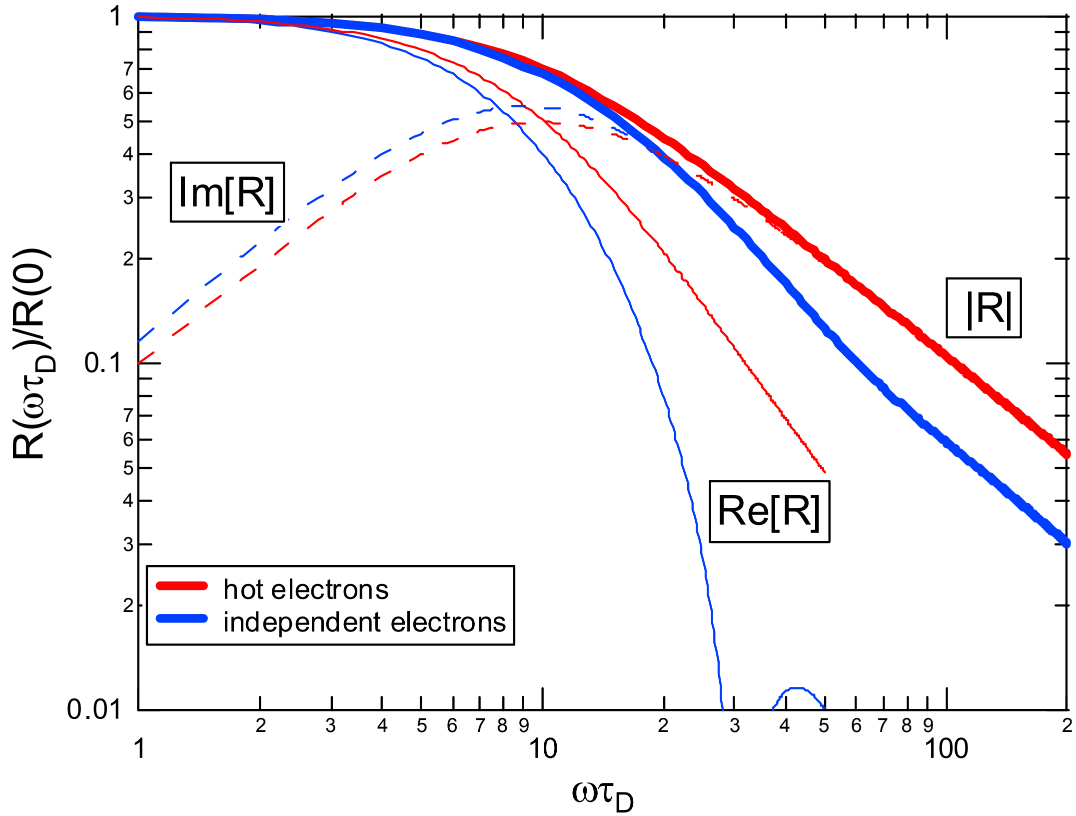
The noise suceptibility gives the DIFFUSION time

* short wire: elastic transport (independent electrons)

The noise suceptibility gives the ELECTRON-ELECTRON time

* ballistic wire (nanotube), quasi-crystals (sub-diffusive), … ??

The noise susceptibility of a diffusive wire



No calibration needed !

Measures the thermalization (inelastic) time.

Reminder: $S_2(\omega)$ indep. of ω for a diffusive wire

BR & D. Prober, PRL95 (05)

Noise susceptibility – beyond the classical regime: theory

What if $\omega_0 > \omega$?
What if $\hbar\omega > k_B T$?

$$\boxed{\chi_{\omega_0}(\omega) \propto \left\langle \delta I(\omega)\, \delta I(\omega_0 - \omega) \right\rangle}$$

Calculation:
* Landauer-Büttiker formalism
* SYMMETRIZATION of the operators and $\omega \to -\omega$

The symmetrization rule depends on the experimental setup !

In particular: $\omega \sim \omega_0$

$$\boxed{\chi_{\omega}(\omega) \propto \left\langle \delta I(\omega)\, \delta I(0) \right\rangle}$$

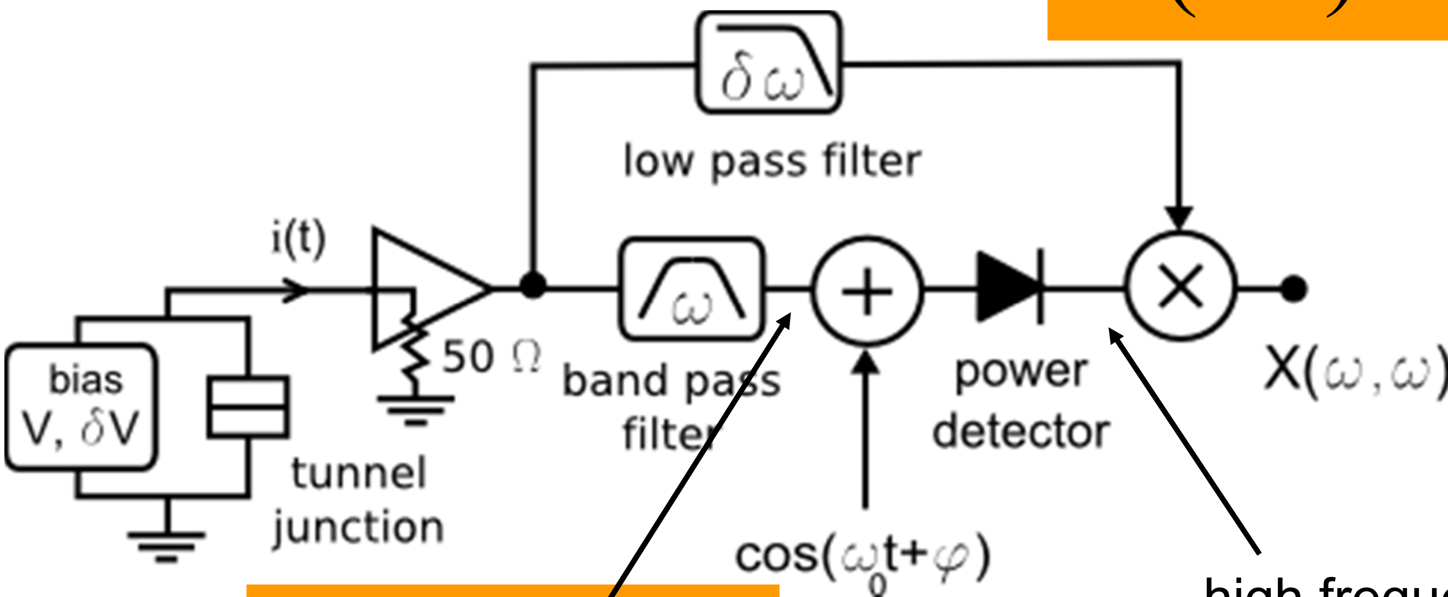
Measures correlation between currents at different frequencies.
This correlation is induced by the excitation.

Noise susceptibility – the quantum regime: experiment

$\omega_0 \sim \omega \sim 6$ GHz
$\hbar\omega/k_B T \sim 8.5$
$\delta\omega \sim 100$ MHz

low frequency current

$$\delta I(\pm\varepsilon)e^{\pm i\varepsilon t}$$

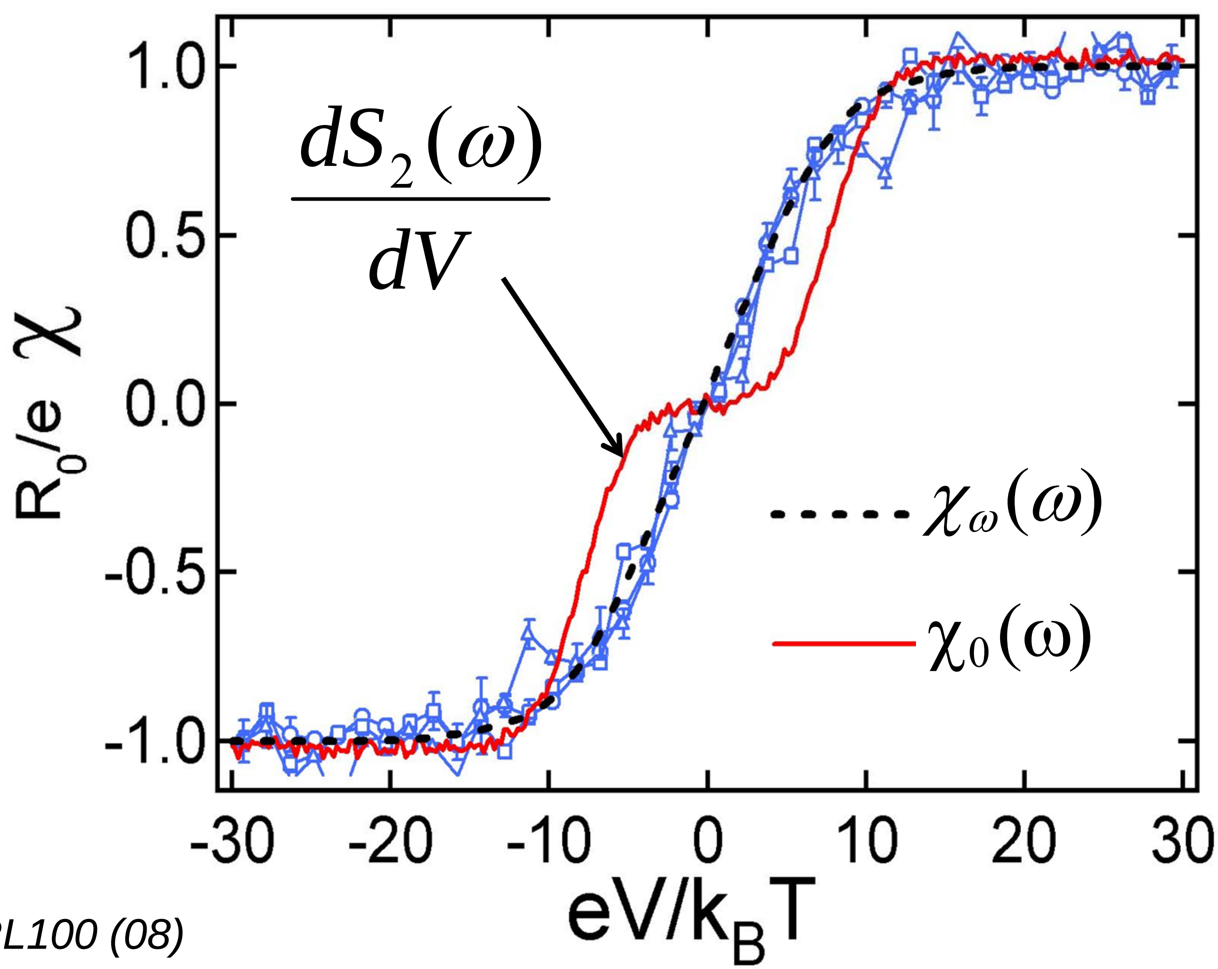


$$\delta I(\pm\omega)e^{\pm i\omega t}$$

high frequency current shifted to low freq.

Noise susceptibility – the quantum regime: experiment

ω_0~ ω~ 6 GHz
T=35 mK
$\hbar\omega/k_B T$~8.5



J. Gabelli & BR, PRL100 (08)

What else ?

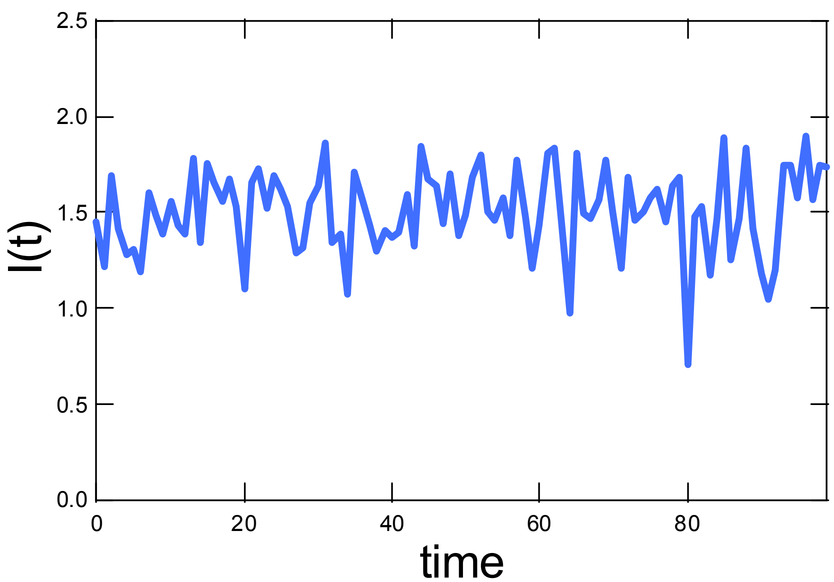
	$\langle \bullet \rangle$	$\langle \bullet\bullet \rangle$	$\langle \bullet\bullet\bullet \rangle$
	$I(V)$	$\langle \delta I(\omega)\delta I(-\omega)\rangle$	$\langle \delta I(\omega)\delta I(\omega')\delta I(-\omega-\omega')\rangle$
$\dfrac{\partial \bullet}{\partial V_{\omega_0}}$	$\dfrac{\partial I_{\omega_0}}{\partial V_{\omega_0}}$	$\dfrac{\partial S_2(\omega)}{\partial V_{\omega_0}}$	
$\dfrac{\partial^2 \bullet}{\partial V_{\omega_0}\partial V_{\omega_1}}$	$\dfrac{\partial^2 I_{\omega_0\pm\omega_1}}{\partial V_{\omega_0}\partial V_{\omega_1}}$		

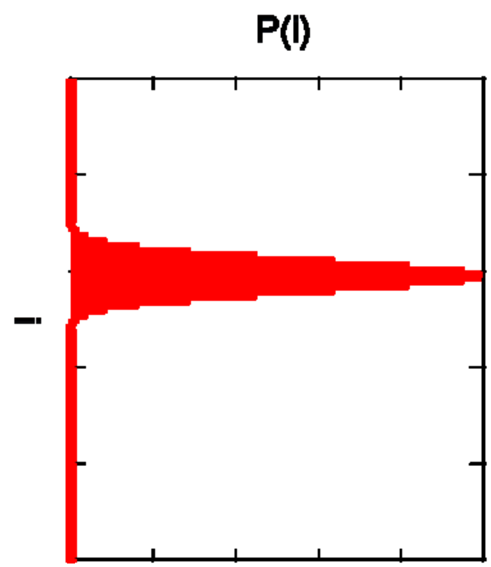
THIRD CUMULANT

The third cumulant of noise

$$S_3(\omega,\omega') = \left\langle \delta I(\omega)\delta I(\omega')\delta I(-\omega-\omega')\right\rangle$$

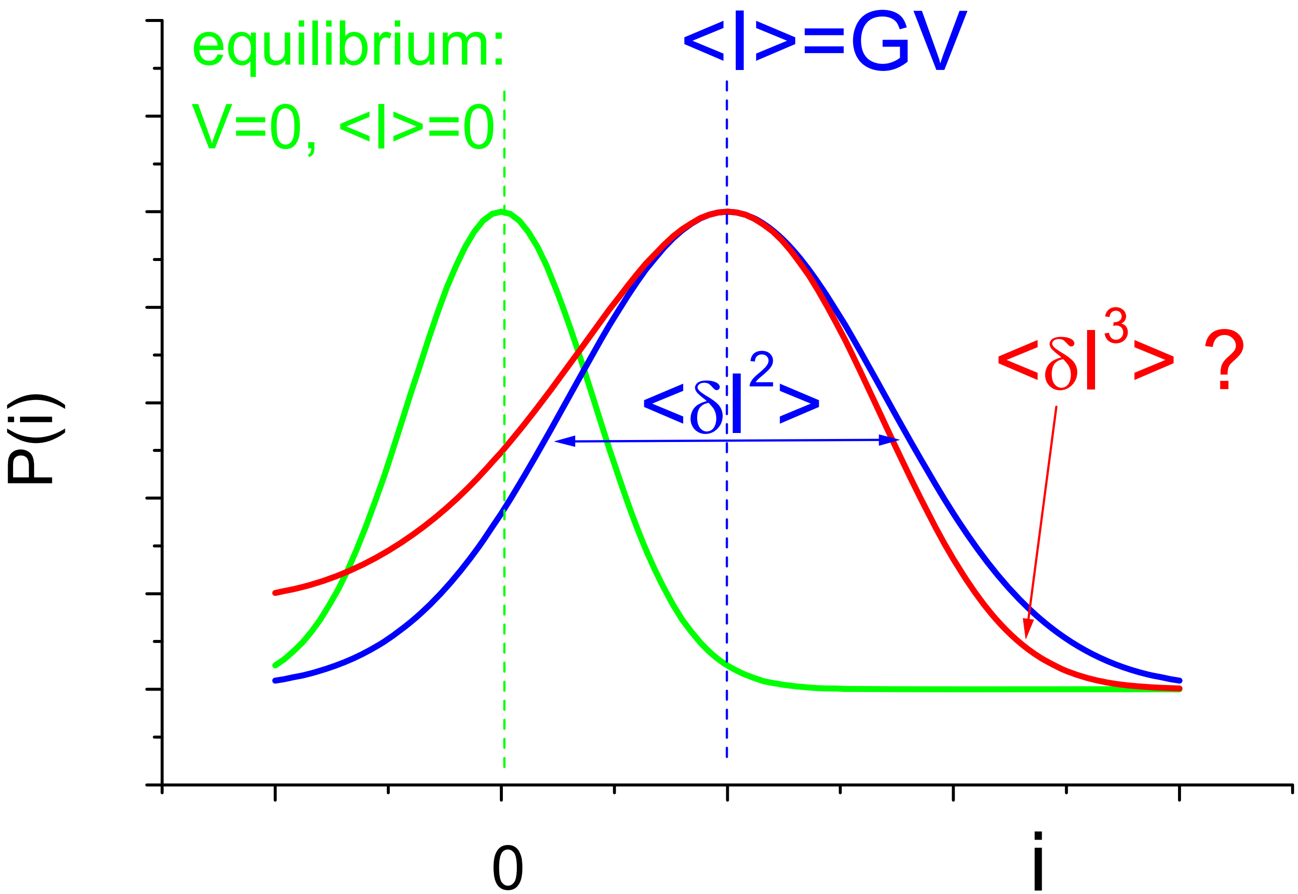
At low frequency: S_3 = SKEWNESS of the probability distribution of current fluctuations P(I): zero for gaussian noise

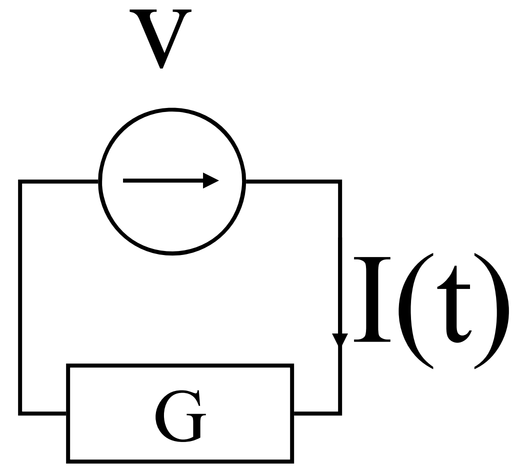




$$S_3(0,0) \approx \left\langle \delta I^3 \right\rangle$$

Statistics of the current

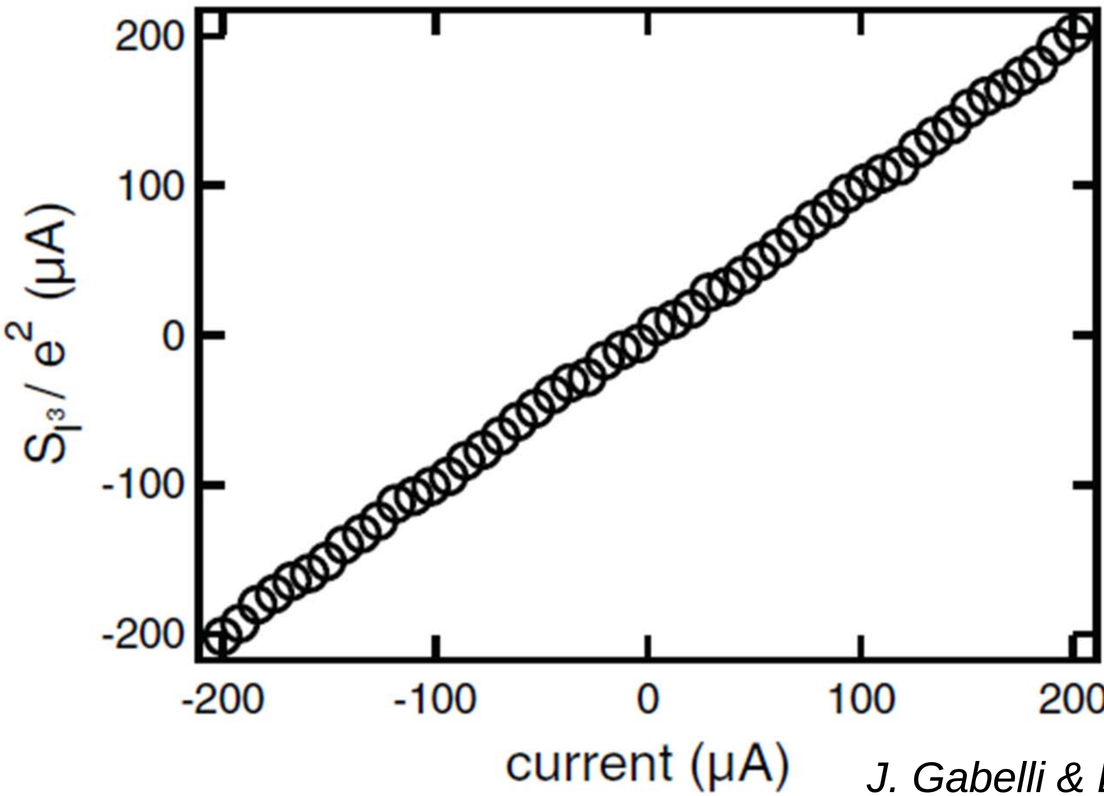




$$I(t) = \langle I \rangle + \delta I(t)$$

At equilibrium: Johnson noise: $<\delta I^2>=4k_B TGB$, (B=bandwidth)

Classical result: $S_3(0,0)$ for a tunnel junction

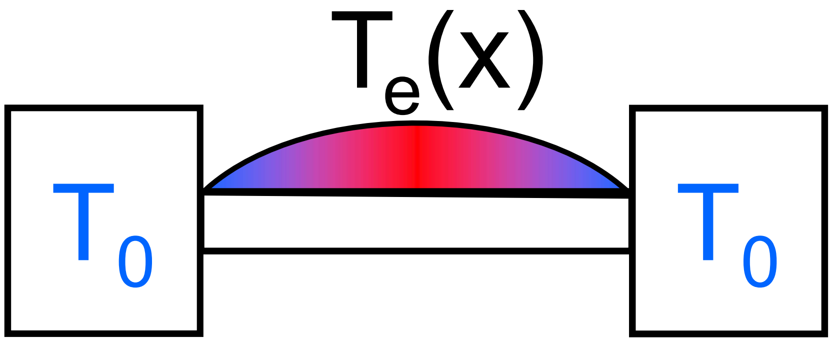


J. Gabelli & BR, (10)
First observation:
BR et al. PRL91 (03)

$$S_3(0,0) = e^2 I$$

Independent of temperature ! (between RT and 20 mK)

S_3 in other systems : a diffusive wire in the hot electrons regime



$$S_2 = 4k_B R \int_0^L T_e(x,V)\frac{dx}{L}$$

A cascade mechanism (thermal feedback) is responsible for S_3:

$$\delta I(t) \Rightarrow \delta P_{Joule}(t) \Rightarrow \delta T(t) \Rightarrow \delta S_3 = \langle \delta I \delta T \rangle$$

K. Nagaev, PRB66 (02)

S_3 at finite frequency ?

$$\delta I(t) \Rightarrow \delta P_{Joule}(t) \Rightarrow \delta T(t+\tau) = \int_{-\infty}^{t+\tau} Z(t+\tau-t')\delta I(t')$$

$$S_3(\tau) = \langle \delta I(t)\delta T(t+\tau) \rangle = \begin{cases} 0 \text{ for } \tau < 0 \text{ (causality)} \\ \neq 0 \text{ for } 0 \leq \tau \leq \tau_{diff} \\ 0 \text{ for } \tau >> \tau_{diff} \end{cases}$$

S_3 shows a frequency dependence at the scale of the inverse diffusion time, whereas S_2 does not because of screening.

S. Pilgram et al., PRB70 (04)

Quantum regime: what is measured vs. what is calculated

Real instrument: number $a(t) \rightarrow \langle a \rangle_{time}, \langle a^2 \rangle_{time}, \langle a^n \rangle_{time}$

Quantum mechanics: Operator $\hat{A} \rightarrow \langle \hat{A} \rangle_{stat}, \langle \hat{A}^2 \rangle_{stat}, \langle \hat{A}^n \rangle_{stat}$

What about correlations at different times ?

$$a(t) \rightarrow \langle a(t_1)a(t_2) \rangle, \langle a(t_1)a(t_2)a(t_3) \rangle, \ldots$$

$$\langle a(t_1)a(t_2) \rangle \neq \langle \hat{A}(t_1)\hat{A}(t_2) \rangle \text{ or } \langle \hat{A}(t_2)\hat{A}(t_1) \rangle \text{ or whatever !!}$$

In principle, one cannot measure current fluctuations with an ampmeter !

The third cumulant S_3 at finite frequency ?

$$S_3(\omega_1, \omega_2) = \left\langle I(\omega_1) I(\omega_2 - \omega_1) I(-\omega_2) \right\rangle$$

Measures phase correlations at 3 different frequencies !

* Classical result: in a Dirac peak, all the Fourier components are IN PHASE

* Quantum regime: correlations involving zero point fluctuations ?

We have measured: $\boxed{S_3(0, \omega) = \left\langle I(0) \underbrace{I(\omega) I(-\omega)}_{\text{ZPF}} \right\rangle}$

low freq. current fluctuations

S_3 and Q mechanics: ordering ???

$$S_3(\omega,\omega') = \int dt dt' e^{i(\omega t+\omega' t')} \left\langle \hat{I}(0,t,t'?) \hat{I}(0,t,t'?) \hat{I}(0,t,t'?) \right\rangle$$

The result depends on ORDERING:

$$S_3(0,0) = \frac{e^2}{h} V \cdot \begin{cases} p(1-p)(1-2p) & \text{Keldysh ordering} \\ p^2(1-p) & \text{Fully symmetrized} \end{cases}$$

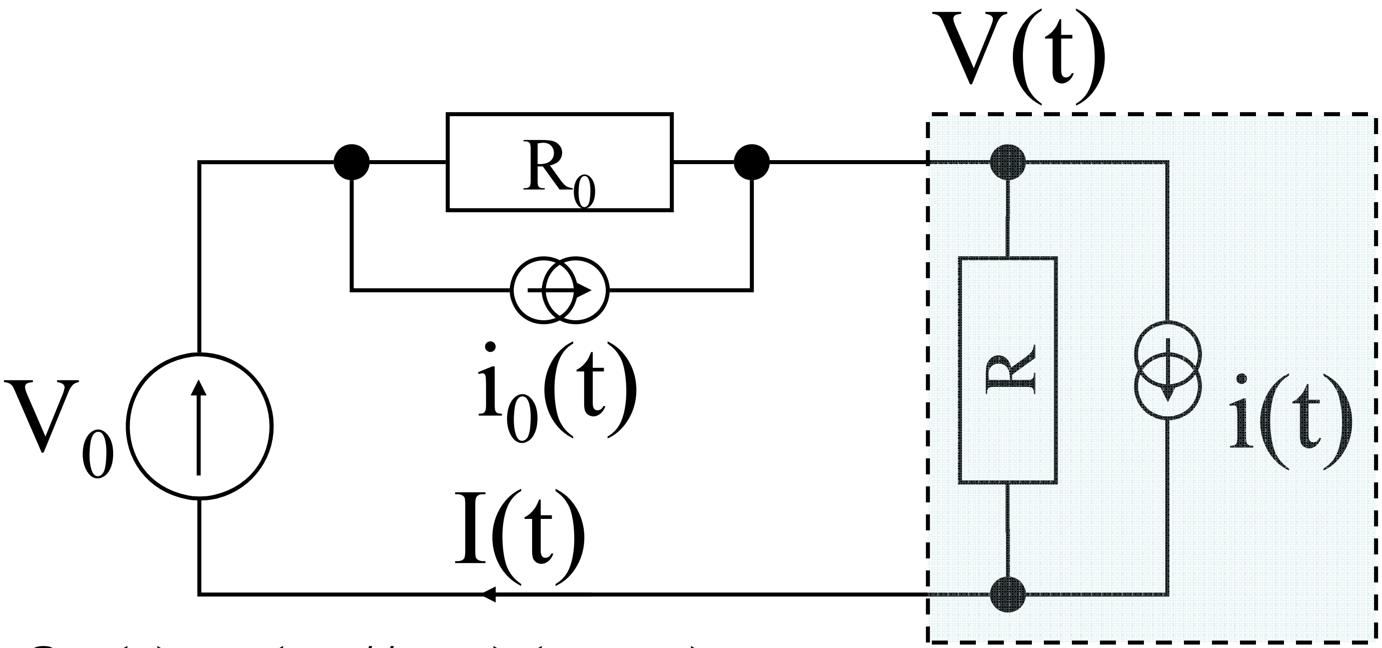
At finite frequency, Keldysh ordering, for a tunnel junction:

$$S_3(\omega_1,\omega_2) = e^2 I$$

Independent of frequency !!

Galaktionov, Golubev & Zaikin, PRB68 (03)
Salo, Hekking & Pekola, PRB74 (06)

Environmental effects



$$\delta V(t) = (R // R_0)(i_0 - i)$$

$$\langle \delta V^2 \rangle = (R // R_0)^2 \left(\langle i_0^2 \rangle + \langle i^2 \rangle - 2 \langle i_0 i \rangle \right)$$

$$\langle \delta V^3 \rangle = (R // R_0)^3 \left(\langle i_0^3 \rangle - \langle i^3 \rangle + 3 \langle i_0 i^2 \rangle - 3 \langle i_0^2 i \rangle \right)$$

The probability distribution P(i) depends on V(t)

Feedback and noise of the environment

Beenakker
Kindermann
Nazarov
PRL90 (03)

* The noise of the sample is modulated by external voltage fluctuations:

$$\left\langle i_0 i^2 \right\rangle = \left\langle i_0 S_2(V(t)) \right\rangle \cong \left\langle i_0 \frac{dS_2}{dV} \delta V(t) \right\rangle = \left\langle i_0^2 \right\rangle (R // R_0) \frac{dS_2}{dV}$$

Noise of the environment: T_{env}

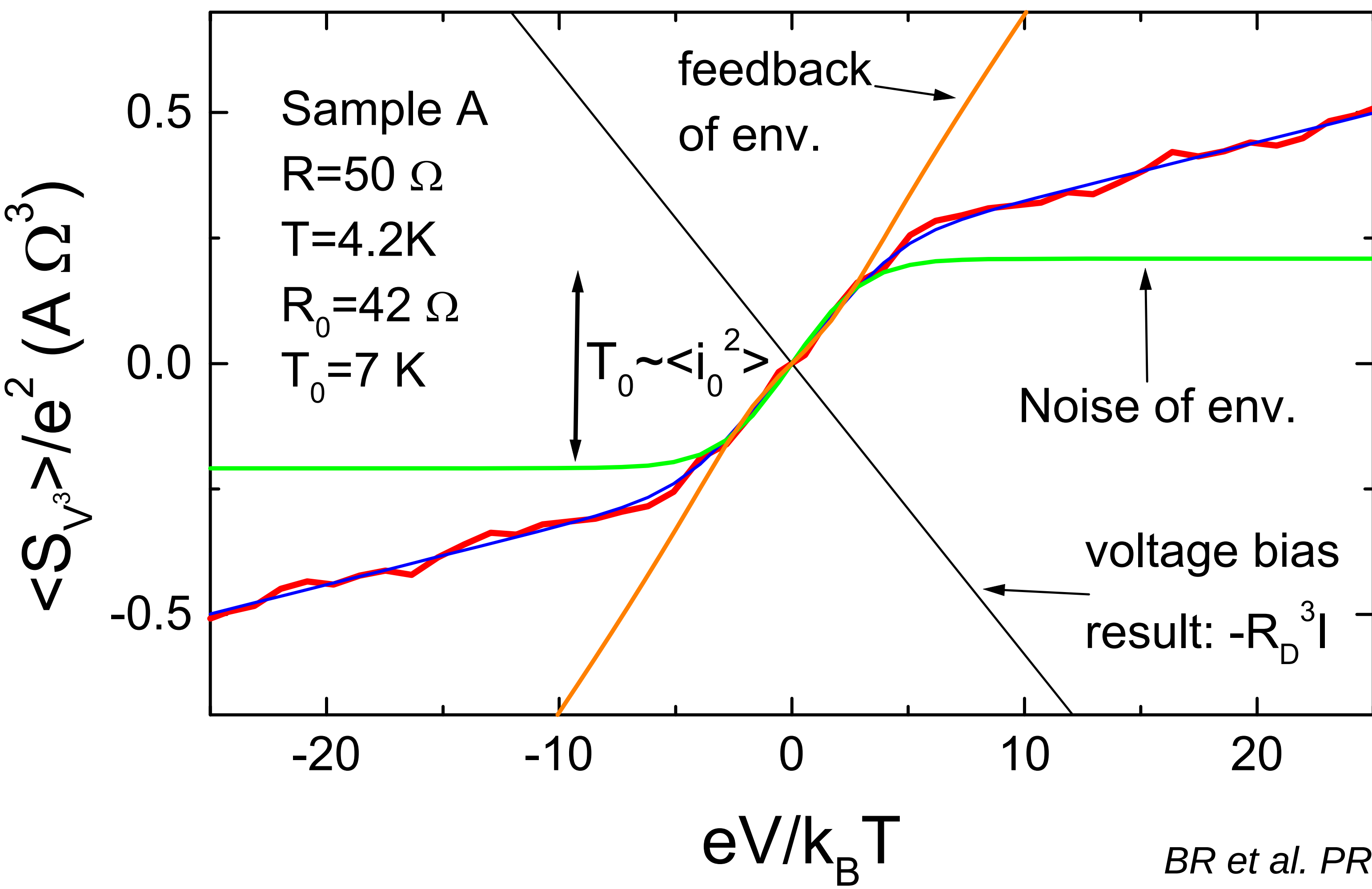
Noise susceptibility

* The noise of the sample is modulated by its own current fluctuations through the external impedance:

$$\left\langle i^3 \right\rangle = \left\langle i i^2 \right\rangle = \left\langle i^3 \right\rangle_V + 3\left\langle i \ S_2(V(t)) \right\rangle \cong \left\langle i^3 \right\rangle_V - 3\left\langle i^2 \right\rangle (R // R_0) \frac{dS_2}{dV}$$

Feedback (even for T_{env}=0)

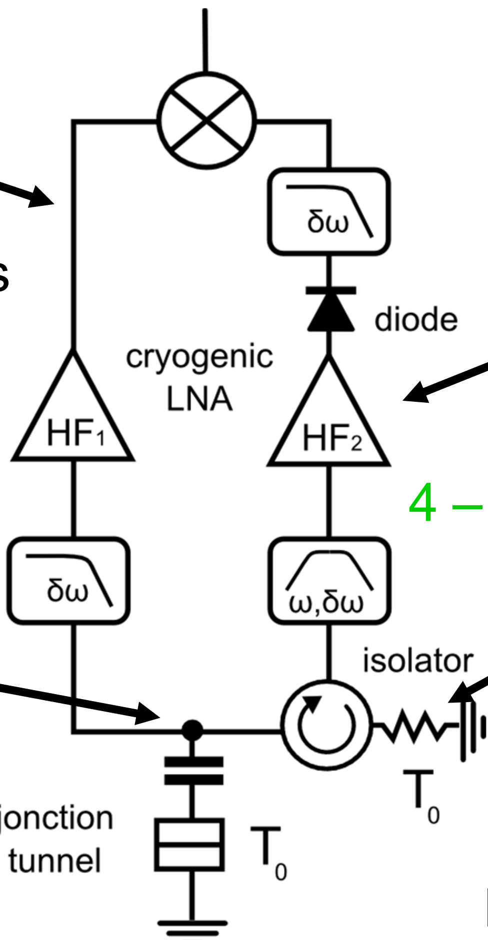
Environmental contributions at zero frequency



BR et al. PRL91 (03)

Experimental setup for $S_3(0,\omega)$

Low frequency current fluctuations



0 – 1 GHz

4 – 8 GHz

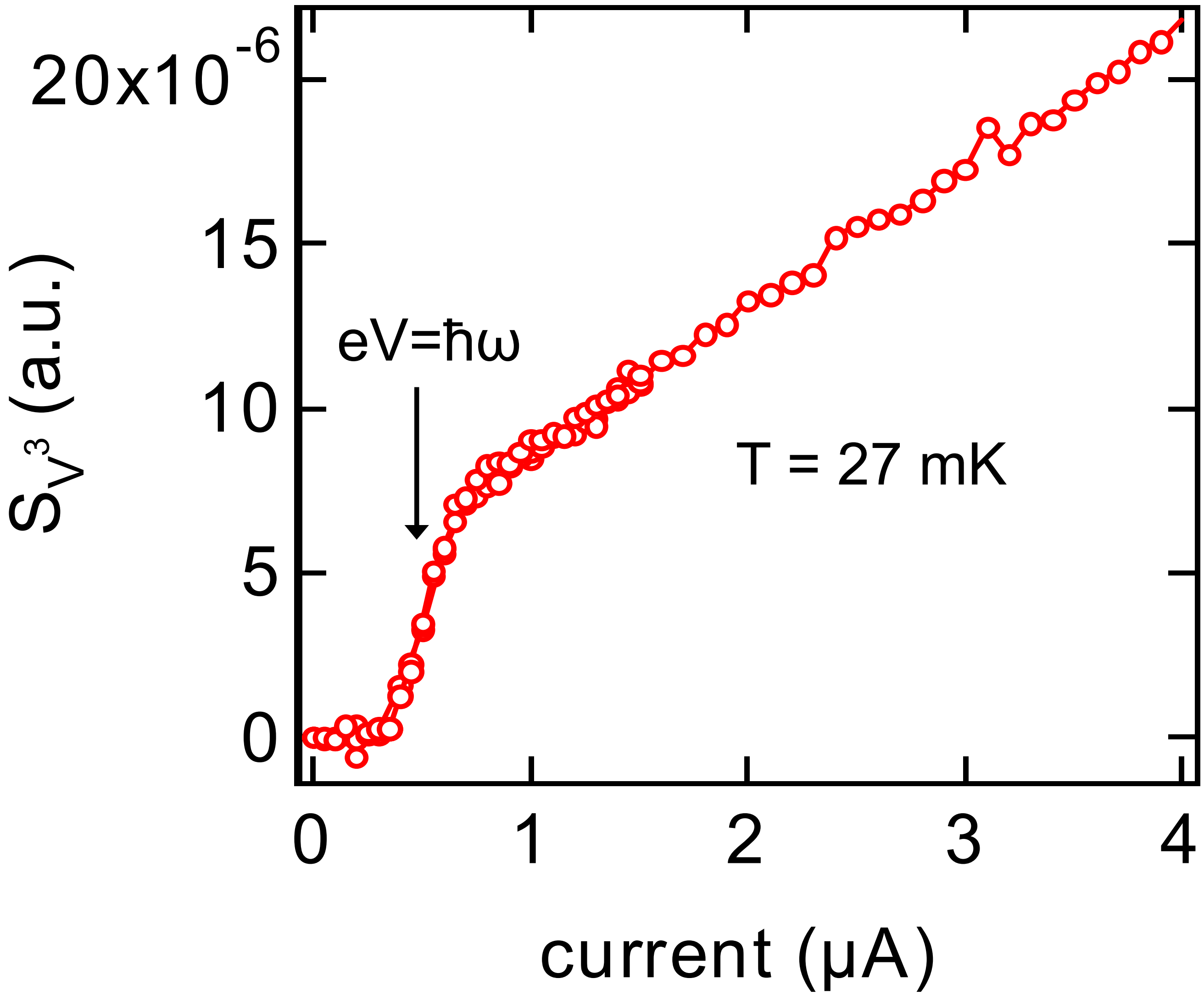
Diplexer, not splitter

Zero point fluctuations of the sample amplified

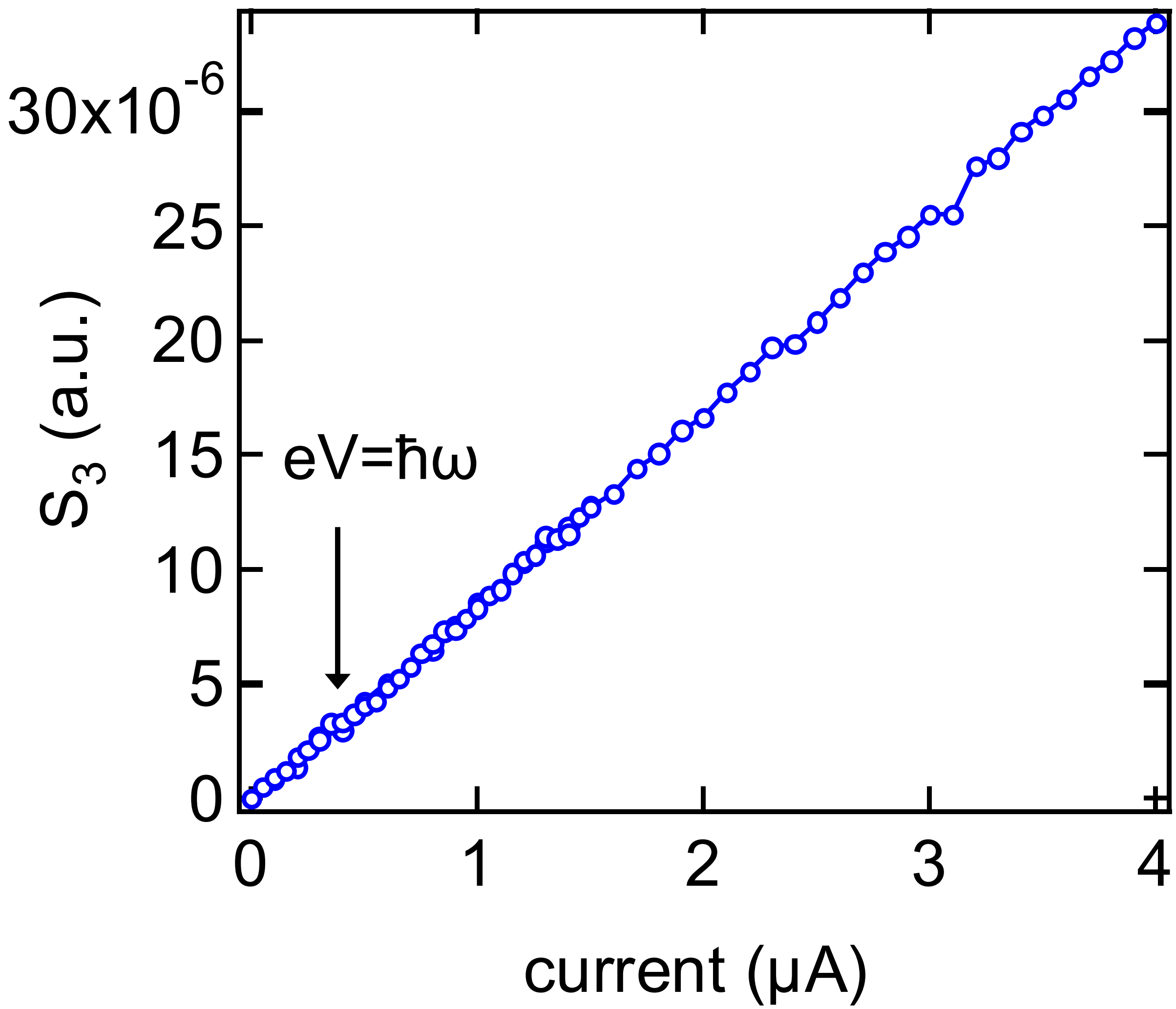
High frequency environment: 50 Ω at the sample's temperature

Environmental effects: only the low freq. environmental noise temp. is not well known.

Third cumulant of VOLTAGE



INTRINSIC 3rd cumulant of CURRENT

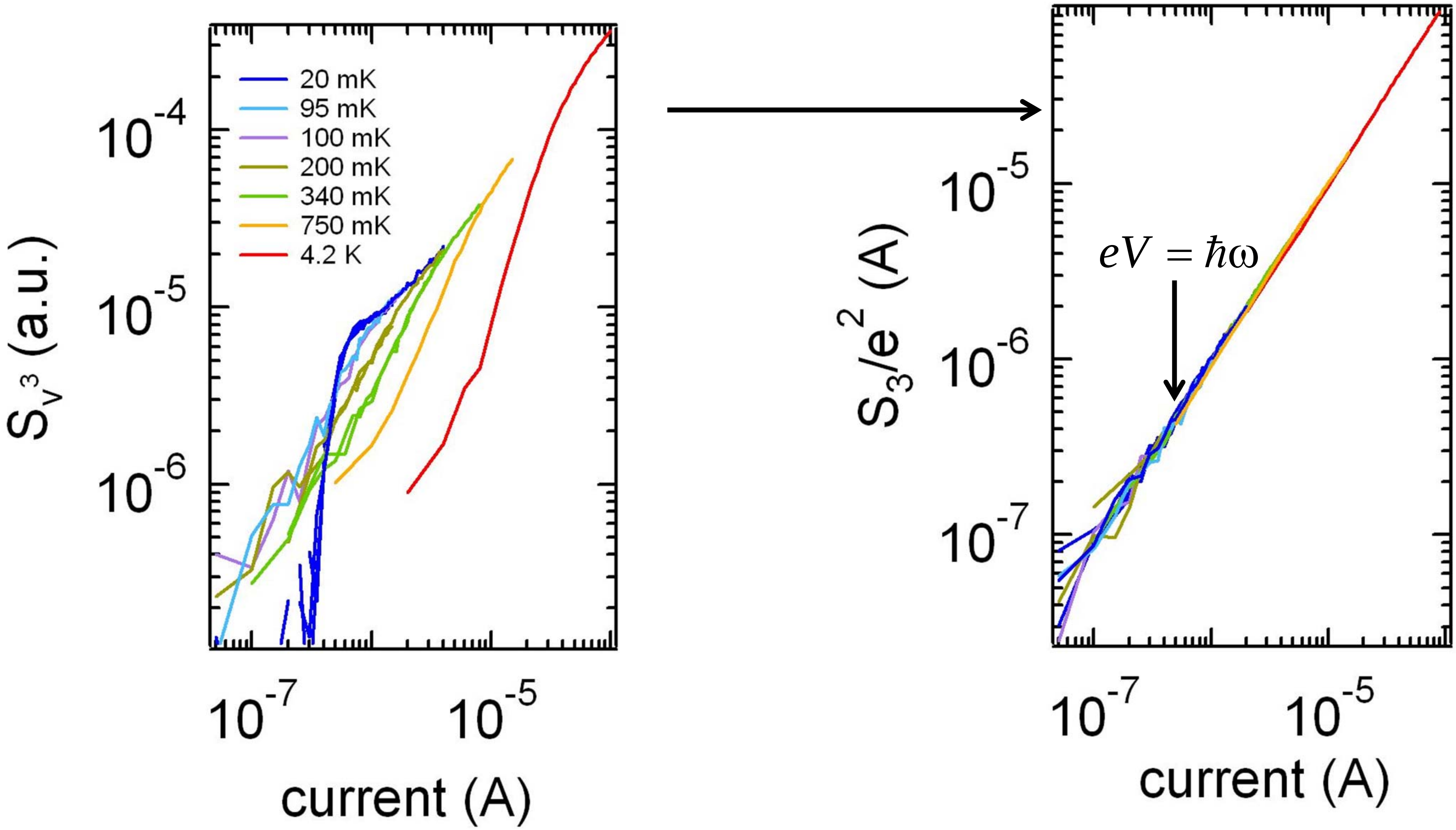


$$e^2 I$$

Correlation between ZPF and low freq. I(t)

J. Gabelli & BR, unpublished

Environmental contributions vs. T



One unknown parameter: the effective noise temperature of the LF amplifier

Another way to measure $S_3(0,f)$?

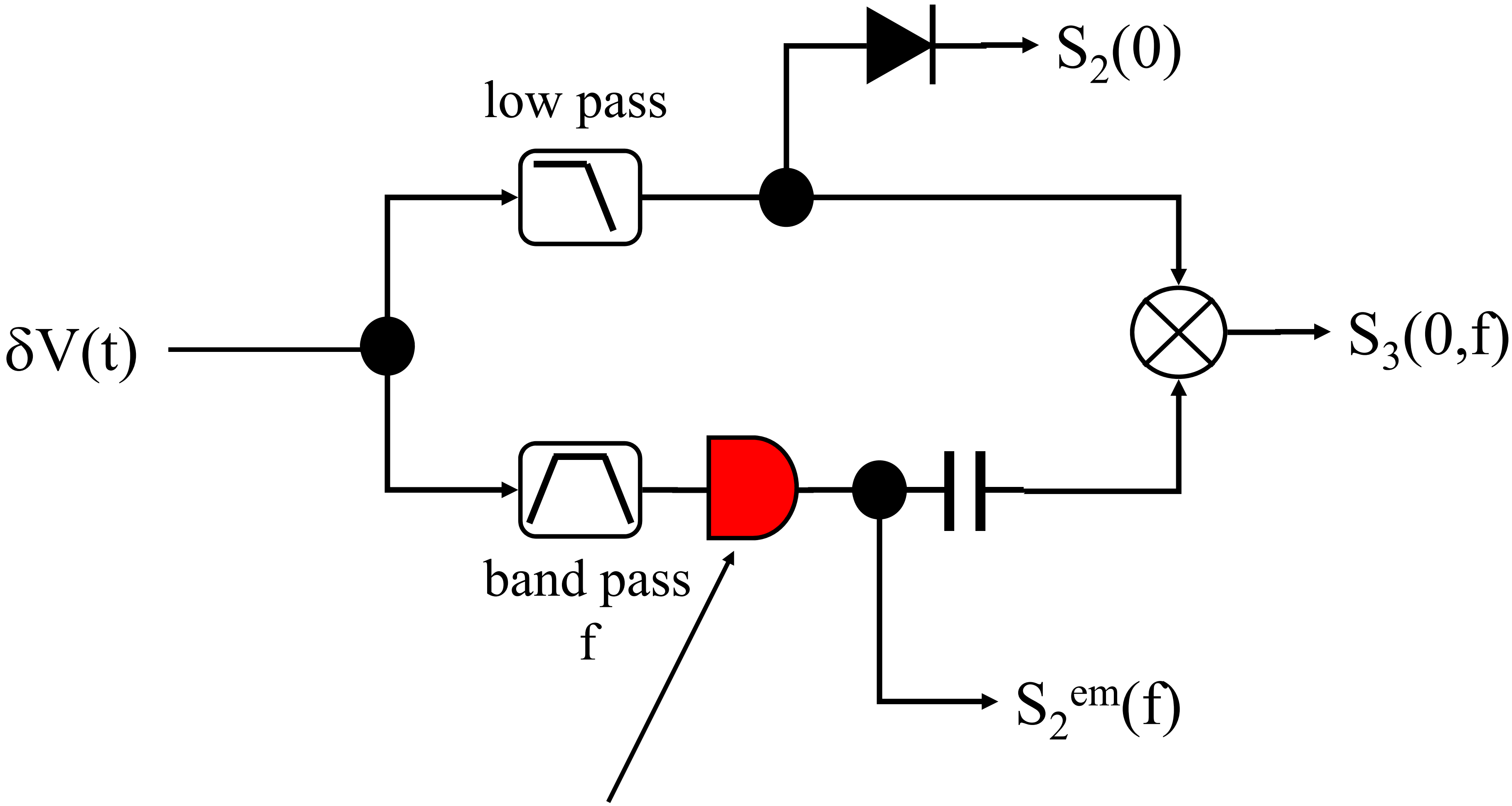
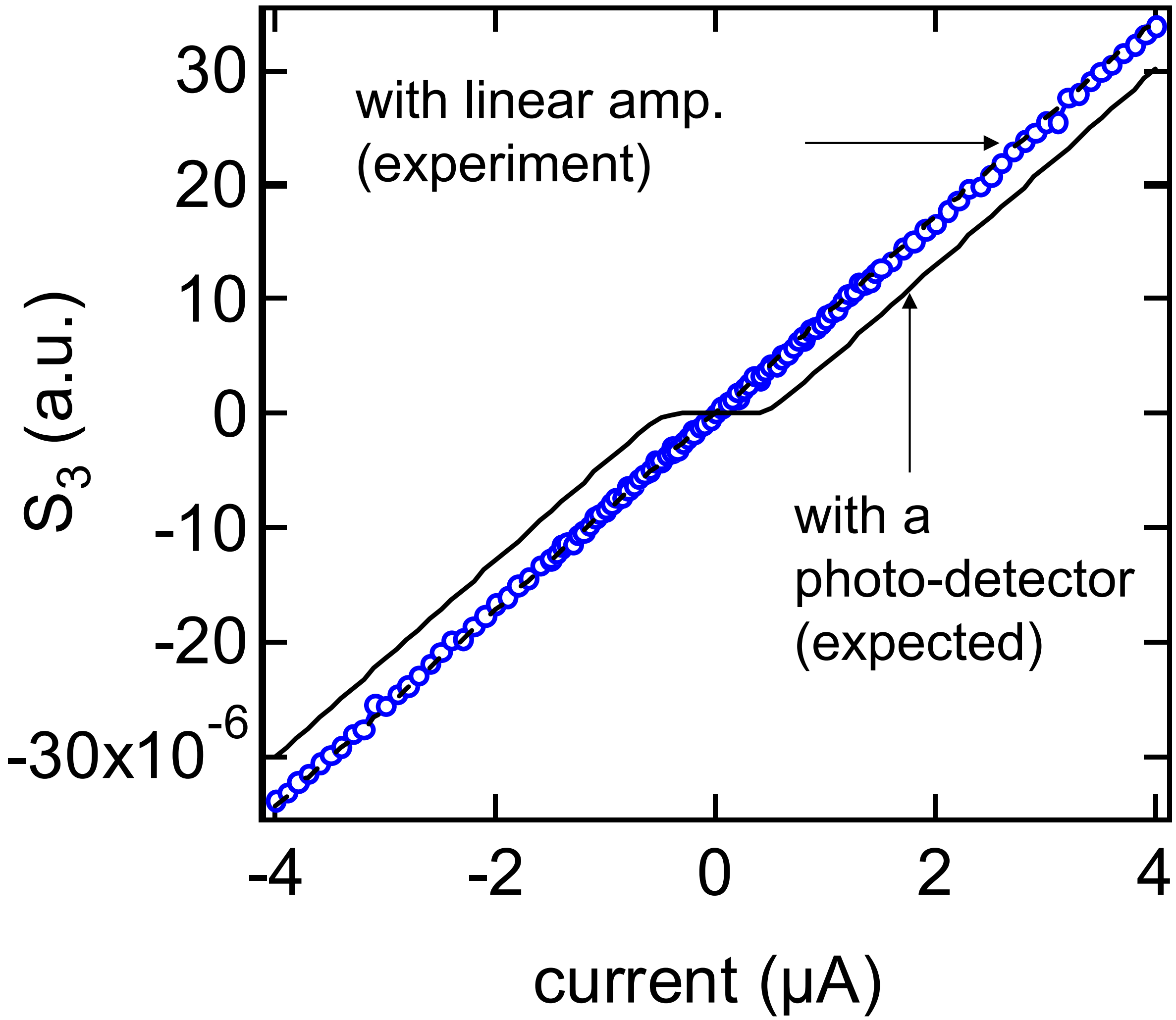


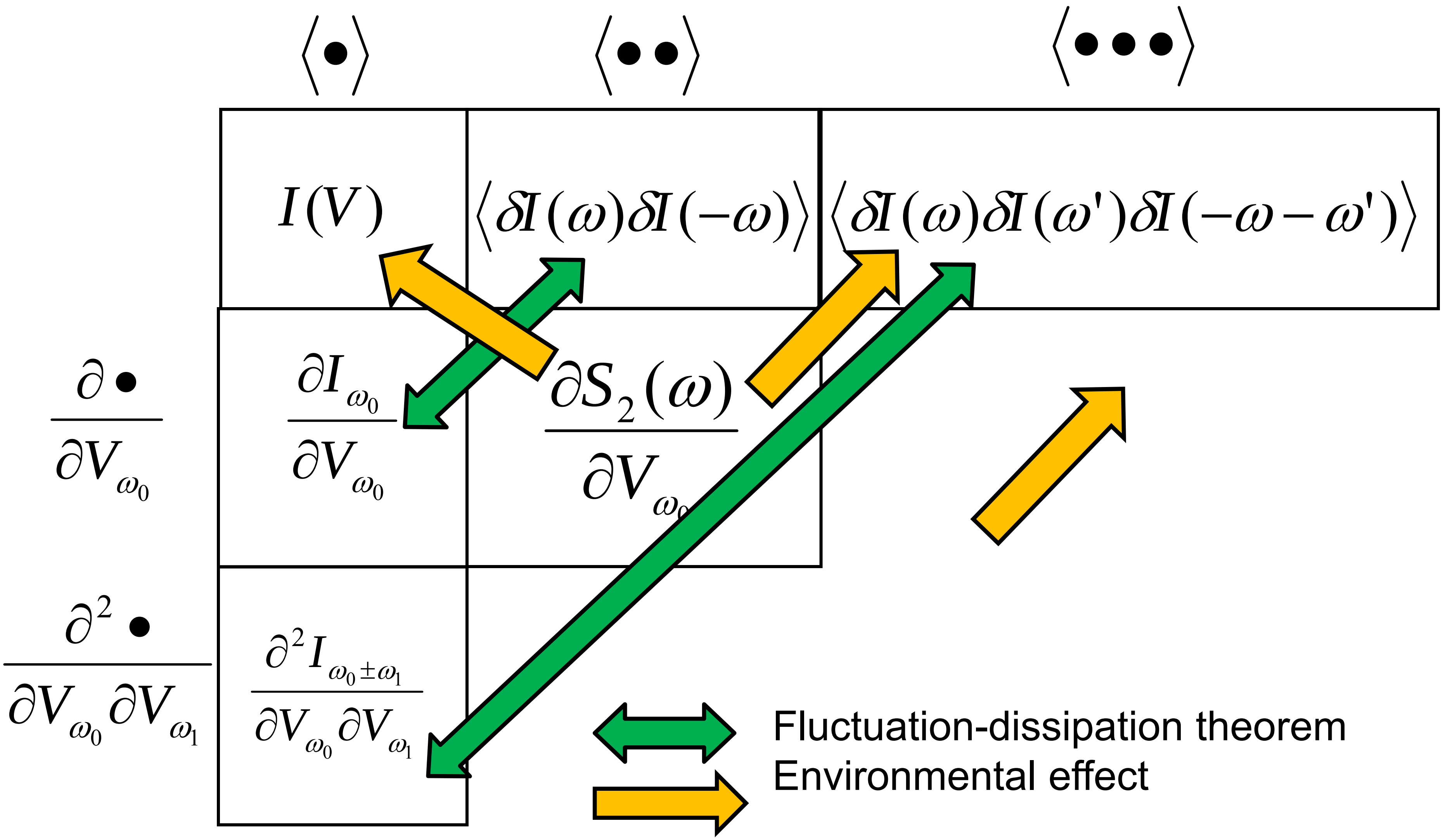
Photo-multiplier: absorbs photons

Gives zero for $eV<hf$: another ordering of the operators ?

Third cumulant of CURRENT



Summary (final)



Anything else ?

	$\langle \bullet \rangle$	$\langle \bullet\bullet \rangle$	$\langle \bullet\bullet\bullet \rangle$	$\langle \bullet\bullet\bullet\bullet \rangle$
	DC I(V)	Noise $S_2(\omega)$	3rd moment $S_3(\omega, \omega')$	4th cumulant $C_4(\omega, \omega', \omega'')$
$\frac{\partial \bullet}{\partial V_{\omega_0}}$	Conductanc e G(ω)	Noise suceptibility	?	?
$\frac{\partial^2 \bullet}{\partial V_{\omega_0} \partial V_{\omega_1}}$	Photo-voltaic effect, mixing	Photo-assisted noise	0	Photo-assisted C_4: see poster J.C. Forgues
$\vdots$	?	?	Photo-assisted S_3	?

Conclusion

There is plenty of work to do for interested students !!