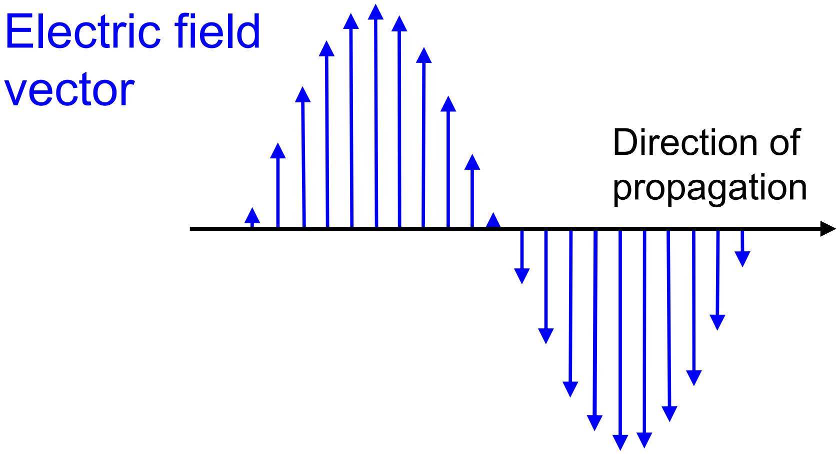
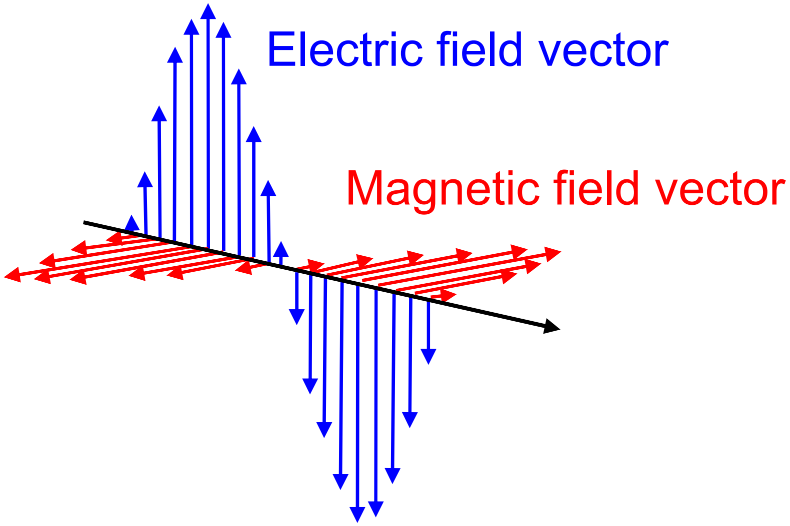
Negative refraction
Why light bends the wrong way

Aloyse Degiron
Institut d’Electronique Fondamentale
Université Paris-Sud and CNRS
Orsay, France
aloyse.degiron@u-psud.fr

European Summer Campus June 28, 2010

Propagation of light in media





The propagation of an electromagnetic wave is governed by Maxwell's equations:

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

Quantity	Units
E	Electric Field (V/m)
H	Magnetic Intensity (A/m)
D	Electric Flux (C/m^2)
B	Magnetic Flux (Wb/m^2)

To solve Maxwell's equations, we must have additional information on how E, D, B and H are related. This is done generally through the constitutive relations, which, for most common materials, take the form:

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$\vec{B} = \mu_0 \vec{H} + \vec{M}$$

Quantity	Units
ε_0	$\varepsilon_0 = 8.854 \times 10^{-12}$ Farad/m
μ_0	$4\pi \times 10^{-7}$ Henries/m
P	Polarization
M	Magnetization

The response of the material enters through these parameters. The constitutive relations can also be written as:

$$\vec{D} = \varepsilon \vec{E} \qquad \vec{B} = \mu \vec{H}$$

where $\varepsilon = \varepsilon_0 + P/E$ is the dielectric permittivity of the material

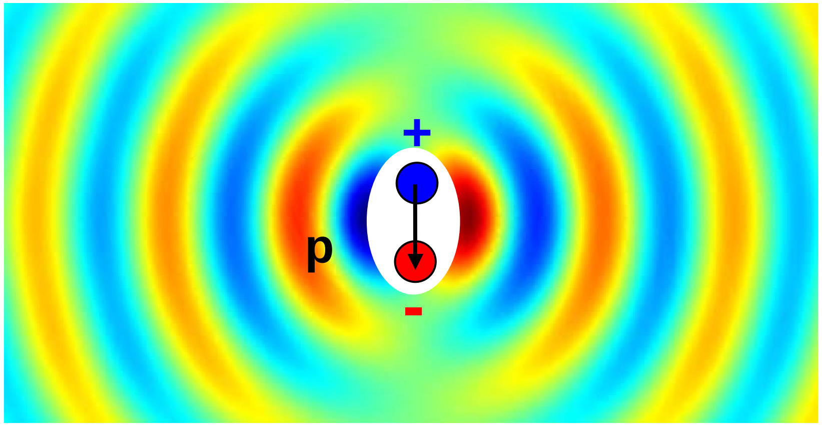
and $\mu = \mu_0 + M/H$ is the magnetic permeability of the material

A simplified description of the permittivity ε

The electromagnetic properties of materials result from the displacement of charges in the presence of electric and magnetic fields.

Under an applied electric field, the molecules and atoms forming a material become polarized, resulting in complex distribution of charges.

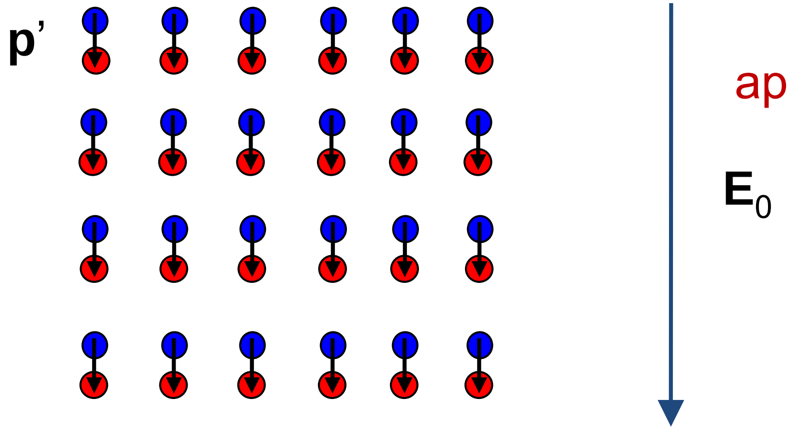
The most important contributions of the material response is the electric dipole:



The electric dipole moment is a measure of the linear charge separation

We can model most materials as being composed of a collection of polarizable dipoles, each element having a polarizability α.

$$\mathbf{p}' = \alpha \mathbf{E}_{loc}$$

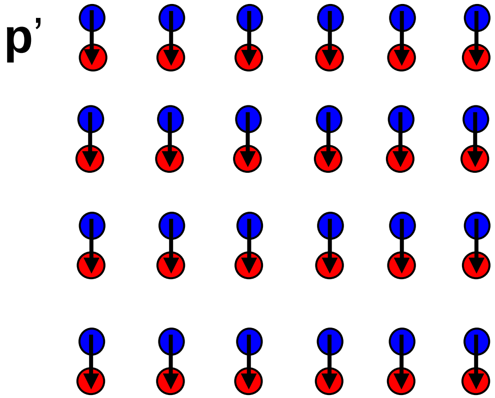


The collection of dipoles interact through their fields and therefore he dipole moment of each entity is **p**' rather than **p**.

That is, the dipole is induced according to the local field that acts at the location of the dipole, E_{loc}.

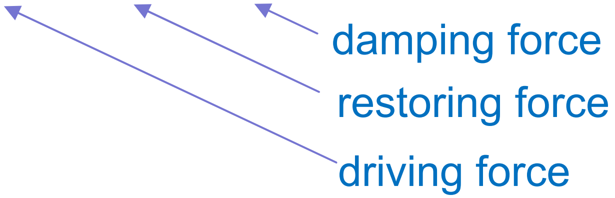
Physical Basis of polarizability

$$\mathbf{p}' = \alpha \mathbf{E}_{loc}$$



If x is the displacement of the charge, then we can model the response as that of a harmonic oscillator:

$$m\ddot{x} = qE - kx - \gamma'\dot{x}$$



$$\ddot{x} = \frac{q}{m}E - \frac{k}{m}x - \frac{\gamma'}{m}\dot{x}$$

$$\ddot{x} + \omega_0^2 x + \Gamma\dot{x} = \frac{q}{m}E$$

If x is the displacement of the charge, then we can model the response as that of a harmonic oscillator:

$$\ddot{x}+\omega_0^2 x+\Gamma\dot{x}=\frac{q}{m}E$$

Assuming the charges are driven with an oscillating field,

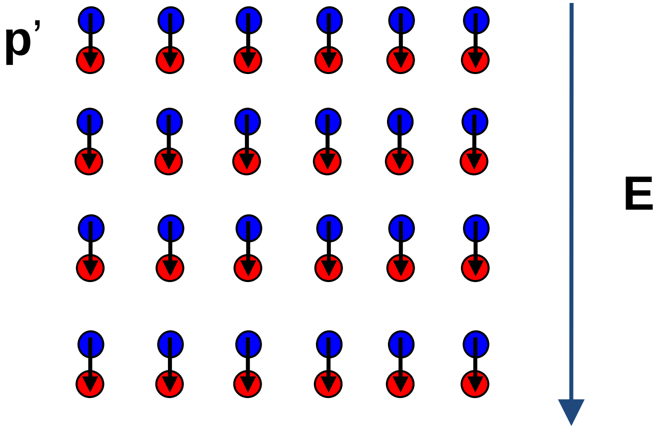
$$E=E_0 e^{-i\omega t} \qquad x=x_0 e^{-i\omega t}$$

$$-\omega^2 x_0+\omega_0^2 x_0-i\Gamma\omega x_0=\frac{q}{m}E_0$$

$$p=qx_0=\frac{q^2/m}{-\omega^2+\omega_0^2-i\Gamma\omega}E_0$$

The Polarization Field

Because the density of dipoles in a material can usually be taken to be large, and if we assume the polarizability does not change much over small distances, we can define a new field called the polarization density:



$$\mathbf{P} = n\mathbf{p}' = n\alpha\mathbf{E}_{loc} \equiv \varepsilon_0 \chi_e \mathbf{E}_{loc}$$

susceptibility

$$n = \frac{\#dipoles}{volume}$$

$$P = Np = \frac{Nq^2/m}{-\omega^2 + \omega_0^2 - i\Gamma\omega} E_0 = \frac{\dfrac{Nq^2}{m\varepsilon_0}}{-\omega^2 + \omega_0^2 - i\Gamma\omega} \varepsilon_0 E_{loc}$$

P is a continuous field even though the collection of dipoles is inhomogeneous!

$$P = \frac{\omega_p^2}{-\omega^2 + \omega_0^2 - i\Gamma\omega}\varepsilon_0 E_{loc} \qquad \omega_p^2 = \frac{Nq^2}{m\varepsilon_0}$$

Here we have a relation between P and the local field.

To obtain an expression for the permittivity ε of the medium, we must relate E_{loc} to E_0, the driving field.

In other words the collective interaction of all dipoles must be taken into account using **effective medium theory**.

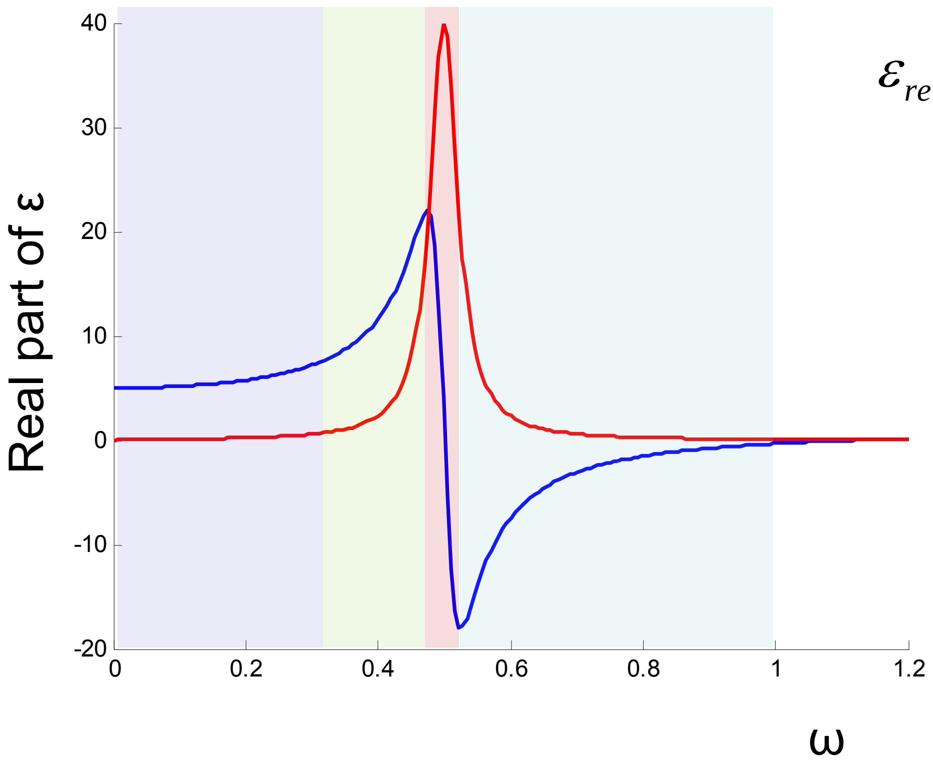
Example - Clausius-Mossoti equations:

$$P = n\alpha E_{loc} = \boxed{\frac{n\alpha}{1 - \frac{n\alpha}{3\varepsilon_0}}} E_0$$

Homogeneous susceptibilty χ_E

The Drude-Lorentz form

Suppose that the collective interactions can be neglected. Then we have the well known Drude-Lorentz form:



$$\varepsilon_{rel} = \frac{\varepsilon}{\varepsilon_0} = 1 + \frac{\omega_p^2}{\left(-\omega^2 + \omega_0^2\right) - i\Gamma\omega}$$

The Drude-Lorentz form provides a good qualitative description of most material responses.

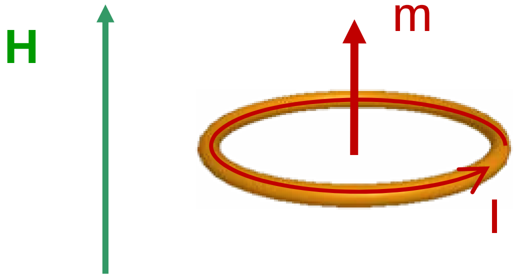
This form implies that the dipolar response of most materials exhibits a **resonant behavior** in a small frequency range.

Response to the magnetic field

We have presented a model for the response of a medium under an applied electric field. To be complete, we need a model for the magnetic response of a medium.

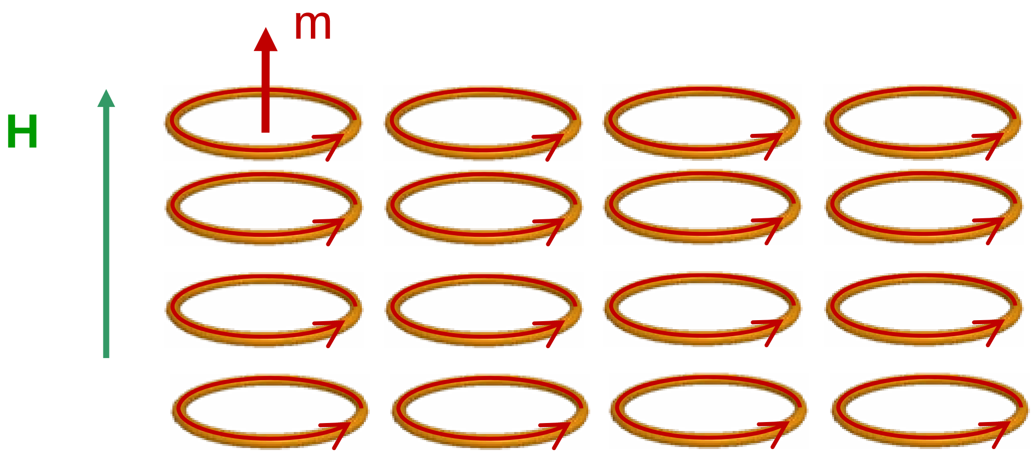
The essence of magnetism is due to the spin of the electrons.

Classically, the spin can be thought as a current that flows in a closed circuit.



$$\mathbf{m} = \frac{1}{2}\int \mathbf{r} \times \mathbf{J} dV = IA\hat{z}$$

Response to the magnetic field



Similar to the electric response, where the atoms forming the material create a continuous polarization field, we can define the magnetization as:

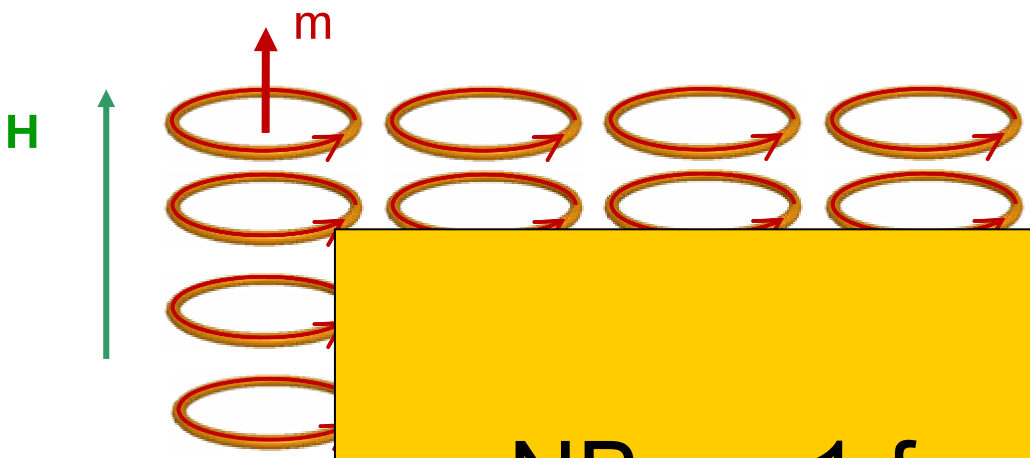
$$\vec{M} = n\vec{m} = n\alpha_m\vec{H} = \chi_m\vec{H}$$

moment density magnetic polarizability magnetic susceptibility

The permeability μ is given by:

$$\mu = \frac{B}{H} = \frac{\mu_0 H + M}{H} = \mu_0(1 + \chi_m) \equiv \mu_0\mu_r$$

Response to the magnetic field



Similar to the erial create

a continuous s:

NB: μ~1 for most Materials

(no magnetic response)

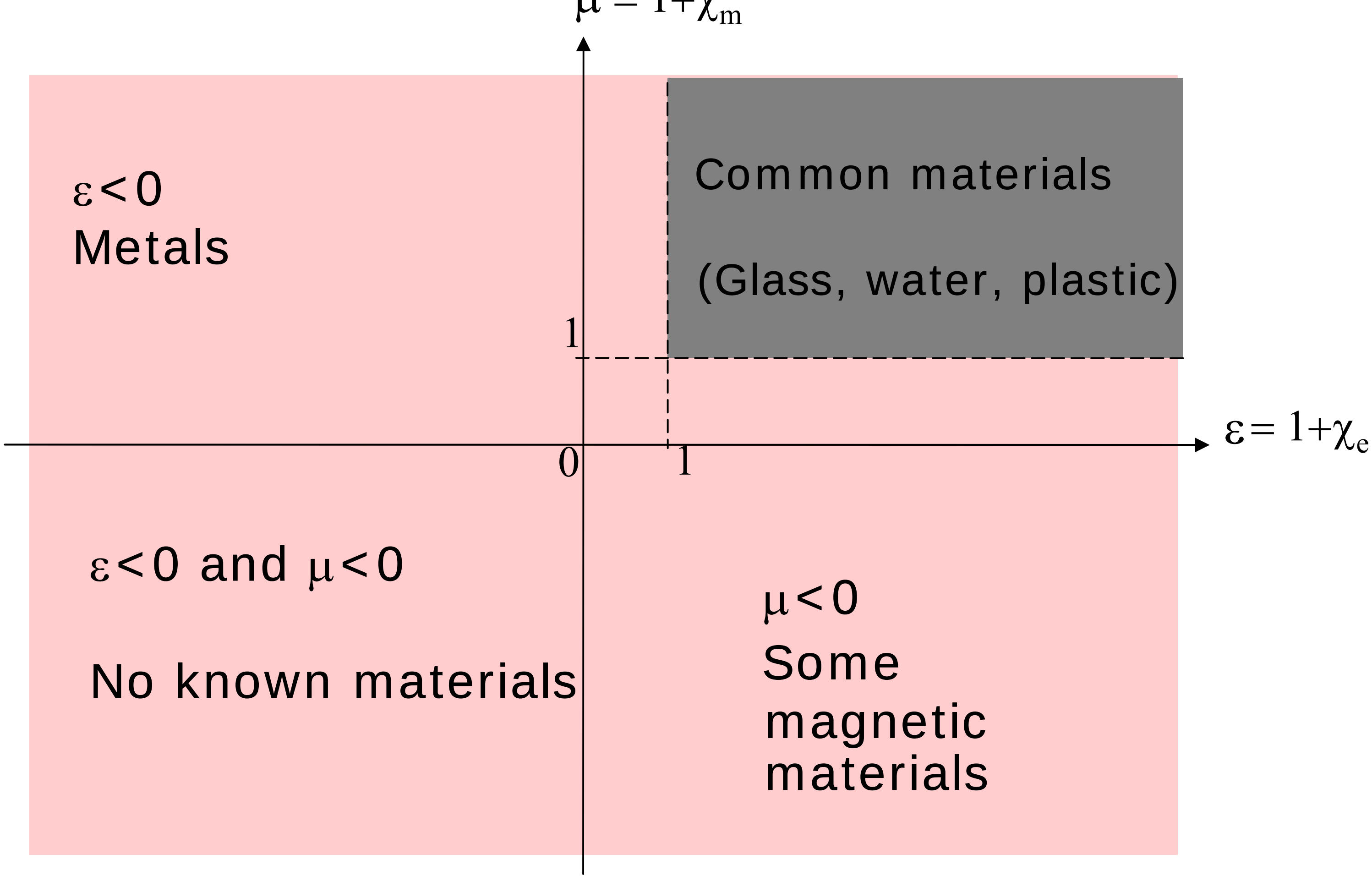
moment tibility

The permeability μ is given by:

$$\mu = \frac{B}{H} = \frac{\mu_0 H + M}{H} = \mu_0 (1 + \chi_m) \equiv \mu_0 \mu_r$$

Classification of materials in terms of ε and μ

We have seen that the material response in Maxwell's equations is given by ε and μ. What are the values of these parameters for common materials ?



Influence of ε and μ on the propagation

$$\vec{\nabla} \cdot \overrightarrow{E} = 0 \qquad \vec{\nabla} \times \overrightarrow{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \overrightarrow{H} = \frac{\partial \vec{D}}{\partial t}$$

Using the curl equations and assuming all quantities have a time dependence of exp(-iωt), a wave equation can be found

$$\nabla \times \mathbf{E} = i\omega \mathbf{B} \qquad \nabla \times \mathbf{H} = -i\omega \mathbf{D}$$

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \mathbf{E} \right) = i\omega \nabla \times \mathbf{H} = \omega^2 \varepsilon \mathbf{E}$$

$$\nabla^2 E + \varepsilon_0 \varepsilon_r \mu_0 \mu_r \frac{\omega^2}{c^2} E = 0$$

$$\nabla^2 E + n^2 \frac{\omega^2}{c^2} E = 0 \qquad \text{with} \quad c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \quad \text{and} \quad \mu_r \varepsilon_r = n^2$$

$$\nabla^2 E + n^2 \frac{\omega^2}{c^2} E = 0 \quad (1) \qquad \text{with} \quad c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \quad \text{and} \quad \mu_r \varepsilon_r = n^2$$

c is the speed of light in vacuum and n is the index of refraction.

Equation (1) admits propagative solutions only if $n^2>0$:

$$\vec{E} = \vec{E}_0 \exp\left[i(\vec{k}\vec{r} - \omega t) \right] \qquad k = \frac{\omega}{c} n$$

Once we have an expression for the E field, we can calculate H as follows:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t} \qquad \begin{bmatrix} ik_y E_z - ik_z E_y \\ ik_z E_x - ik_x E_z \\ ik_x E_y - ik_y E_x \end{bmatrix} = i\omega\mu\vec{H}$$

$$\vec{k} \times \vec{E} = \omega\mu\vec{H}$$

Structure of the wave

We have seen that the wave equation, $\nabla^2 E + n^2 \frac{\omega^2}{c^2} E = 0$ admits propagative solutions only if $n^2>0$.

However there is not restriction on the sign of n: $n = \pm\sqrt{\varepsilon\mu}$

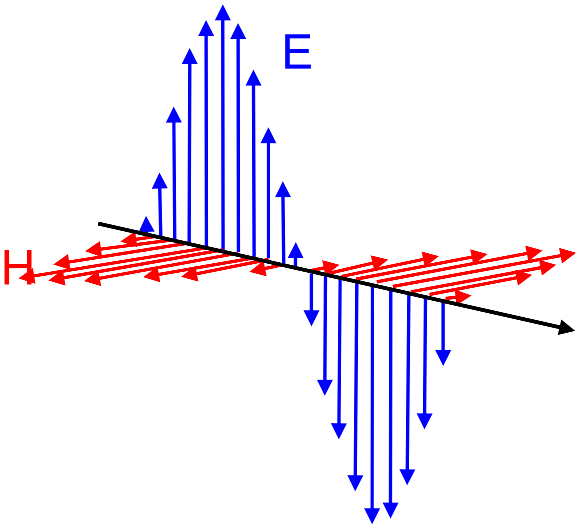
The condition $n^2>0$ implies two sets of solutions:

***First solution*: ε>0 and μ >0:**

Direction of energy flow: $\mathbf{S} = \mathbf{E} \times \mathbf{H}$

Direction of phase velocity: $\mathbf{k} \times \mathbf{E} = \omega \mathbf{B}$

$$\mathbf{k} \parallel \mathbf{E} \times \mathbf{B} = \mu \mathbf{E} \times \mathbf{H} = \mu \mathbf{S}$$



Structure of the wave

We have seen that the wave equation, $\nabla^2 E + n^2 \frac{\omega^2}{c^2} E = 0$ admits propagative solutions only if $n^2>0$.

However there is not restriction on the sign of n: $n = \pm\sqrt{\varepsilon\mu}$

The condition $n^2>0$ implies two sets of solutions:

***First solution*: ε>0 and μ >0:**

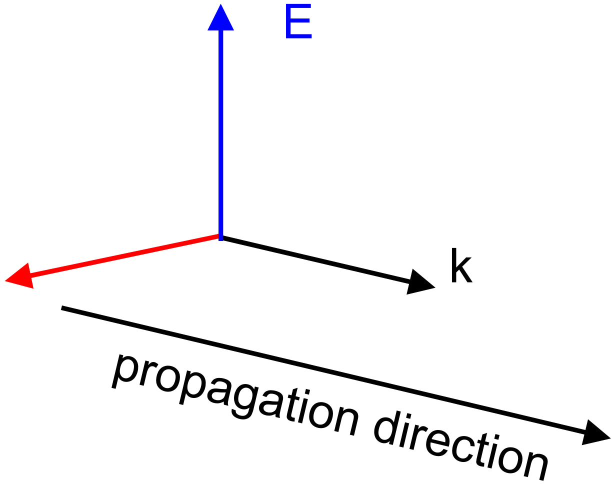
Direction of energy flow: $\mathbf{S} = \mathbf{E} \times \mathbf{H}$

Direction of phase velocity: $\mathbf{k} \times \mathbf{E} = \omega \mathbf{B}$

$$\mathbf{k} \parallel \mathbf{E} \times \mathbf{B} = \mu \mathbf{E} \times \mathbf{H} = \mu \mathbf{S}$$

-> E, H and k form a right-handed triplet.

->Phase and energy velocities *parallel*



Structure of the wave

We have seen that the wave equation, $\nabla^2 E + n^2 \frac{\omega^2}{c^2} E = 0$ admits propagative solutions only if $n^2>0$.

However there is not restriction on the sign of n: $n = \pm\sqrt{\varepsilon\mu}$

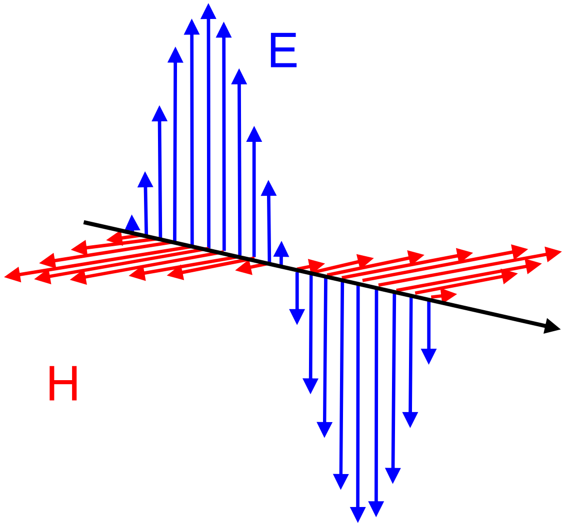
The condition $n^2>0$ implies two sets of solutions:

Second solution: ε<0 and μ <0:

Direction of energy flow: $\mathbf{S} = \mathbf{E}\times\mathbf{H}$

Direction of phase velocity: $\mathbf{k}\times\mathbf{E} = \omega\mathbf{B}$

$$\mathbf{k} \parallel \mathbf{E}\times\mathbf{B} = \mu\mathbf{E}\times\mathbf{H} = \mu\mathbf{S}$$



Structure of the wave

We have seen that the wave equation, $\nabla^2 E + n^2 \frac{\omega^2}{c^2} E = 0$ admits propagative solutions only if $n^2>0$.

However there is not restriction on the sign of n: $n = \pm\sqrt{\varepsilon\mu}$

The condition $n^2>0$ implies two sets of solutions:

***Second solution*: ε<0 and μ <0:**

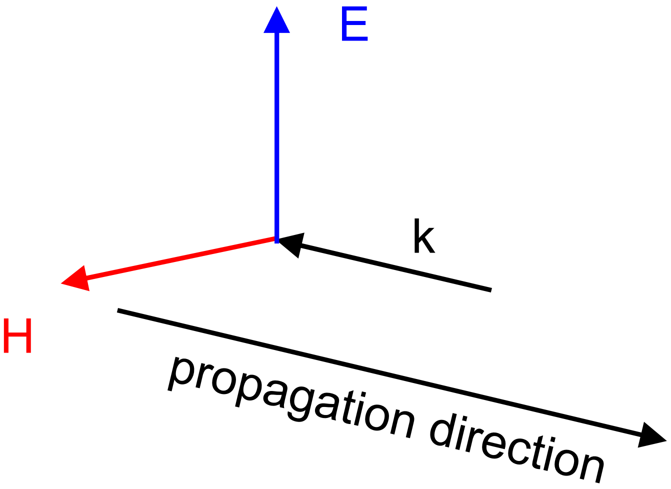
Direction of energy flow: $\mathbf{S} = \mathbf{E} \times \mathbf{H}$

Direction of phase velocity: $\mathbf{k} \times \mathbf{E} = \omega \mathbf{B}$

$$\mathbf{k} \,\|\, \mathbf{E} \times \mathbf{B} = \mu \mathbf{E} \times \mathbf{H} = \mu \mathbf{S}$$

-> E, H and k form a left-handed triplet.

->Phase and energy velocities *antiparallel*



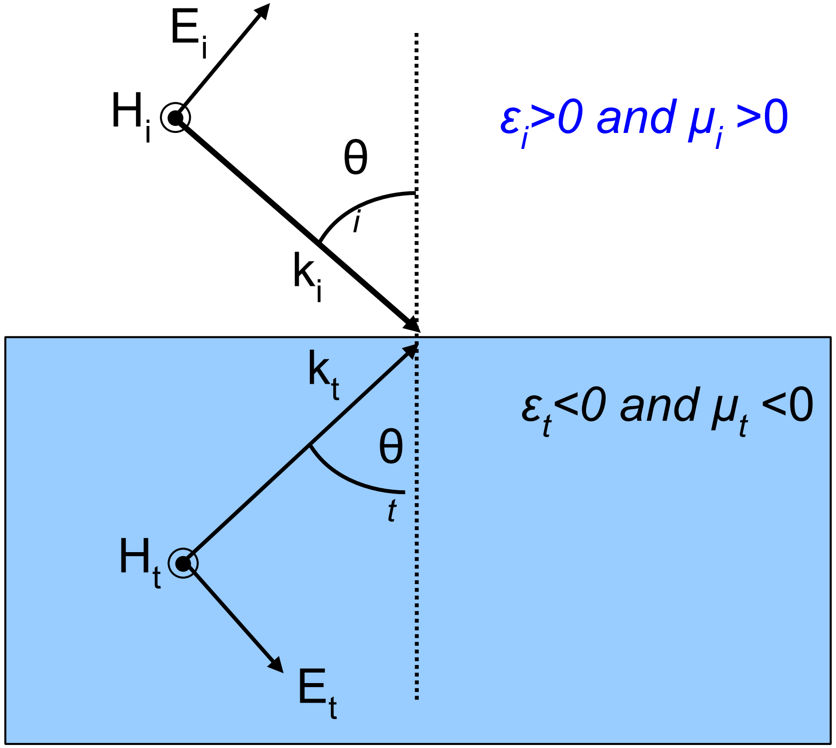
Left-handed media = negative index media

Boundary conditions at an interface between two media: →

$E_{t1}=E_{t2}$;
$\varepsilon_1 E_{n1}=\varepsilon_2 E_{n2}$

$H_{t1}=H_{t2}$
$\mu_1 H_{n1}=\mu_2 H_{n2}$

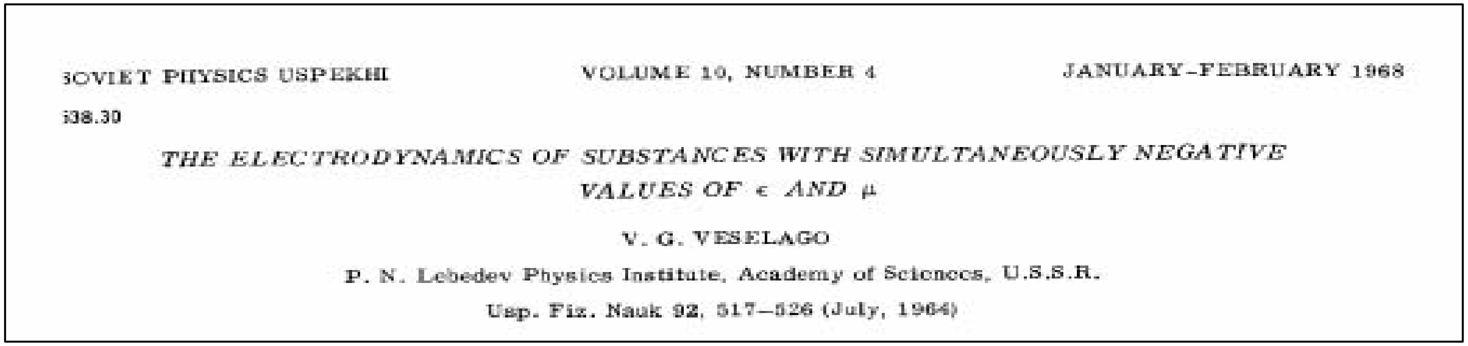
For two media with different rightnesses, the following situation follows:



Snell's law: $n_i \sin\theta_i = n_t \sin\theta_t$

$\varepsilon_t<0$ and $\mu_t<0 \rightarrow n_t<0$

Negative index and Veselago



SOVIET PHYSICS USPEKHI VOLUME 10, NUMBER 4 JANUARY-FEBRUARY 1968

538.30

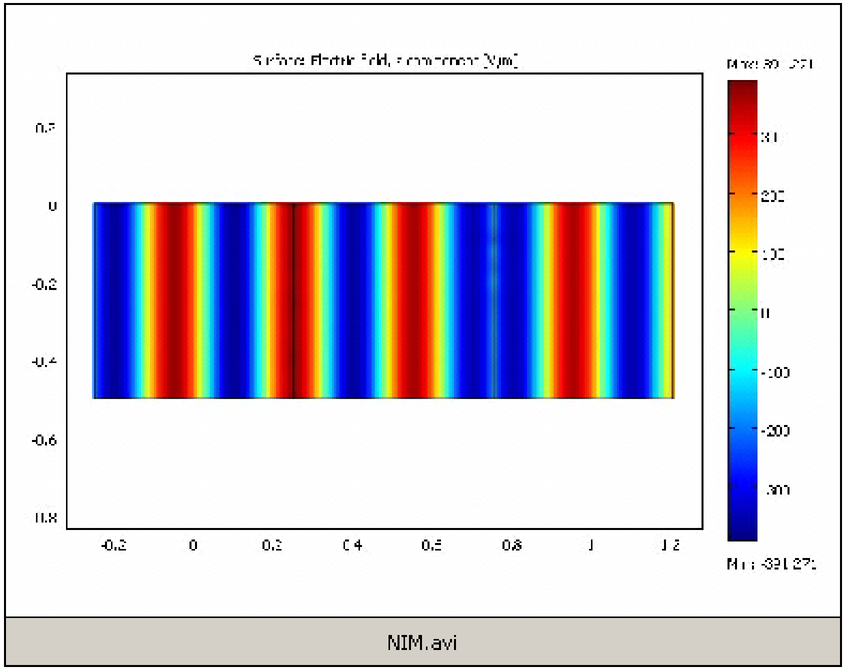
THE ELECTRODYNAMICS OF SUBSTANCES WITH SIMULTANEOUSLY NEGATIVE VALUES OF ϵ AND μ

V. G. VESELAGO

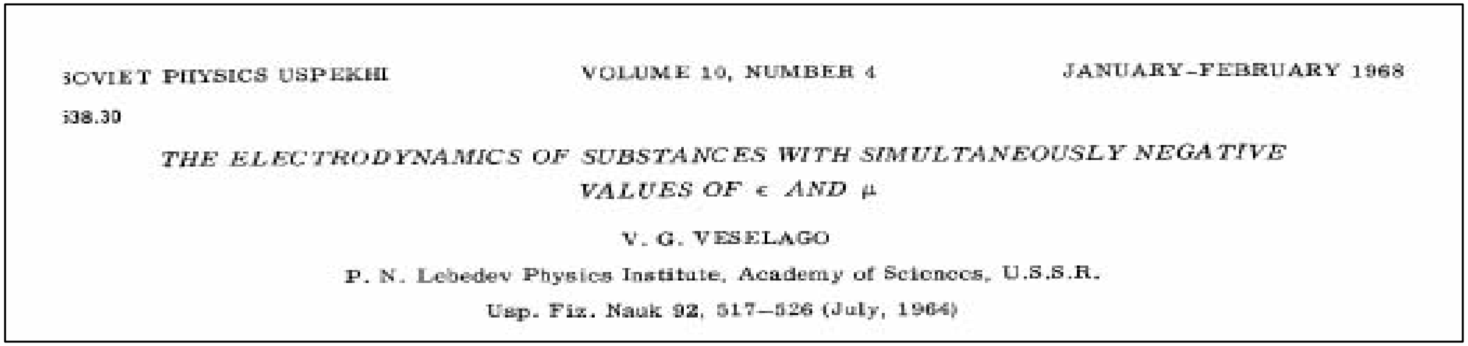
P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Usp. Fiz. Nauk 92, 517–526 (July, 1964)

Phase and energy velocities are *antiparallel*:



Negative index and Veselago



SOVIET PHYSICS USPEKHI VOLUME 10, NUMBER 4 JANUARY-FEBRUARY 1968

538.30

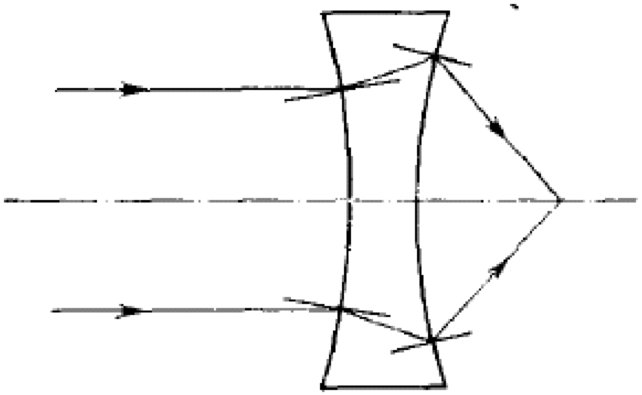
THE ELECTRODYNAMICS OF SUBSTANCES WITH SIMULTANEOUSLY NEGATIVE VALUES OF ϵ AND μ

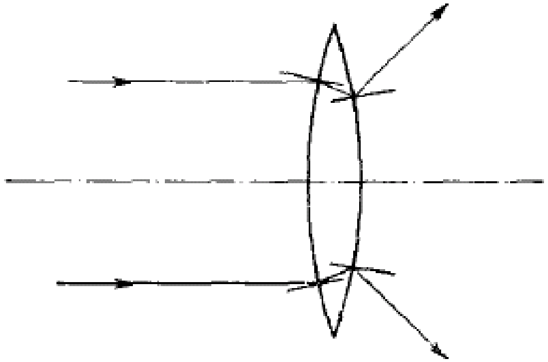
V. G. VESELAGO

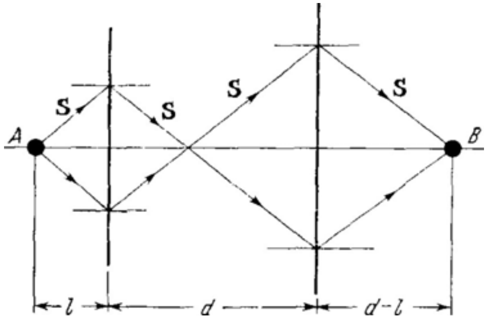
P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Usp. Fiz. Nauk 92, 517–526 (July, 1964)

Diverging lenses are convex, converging lenses are concave (or planar):

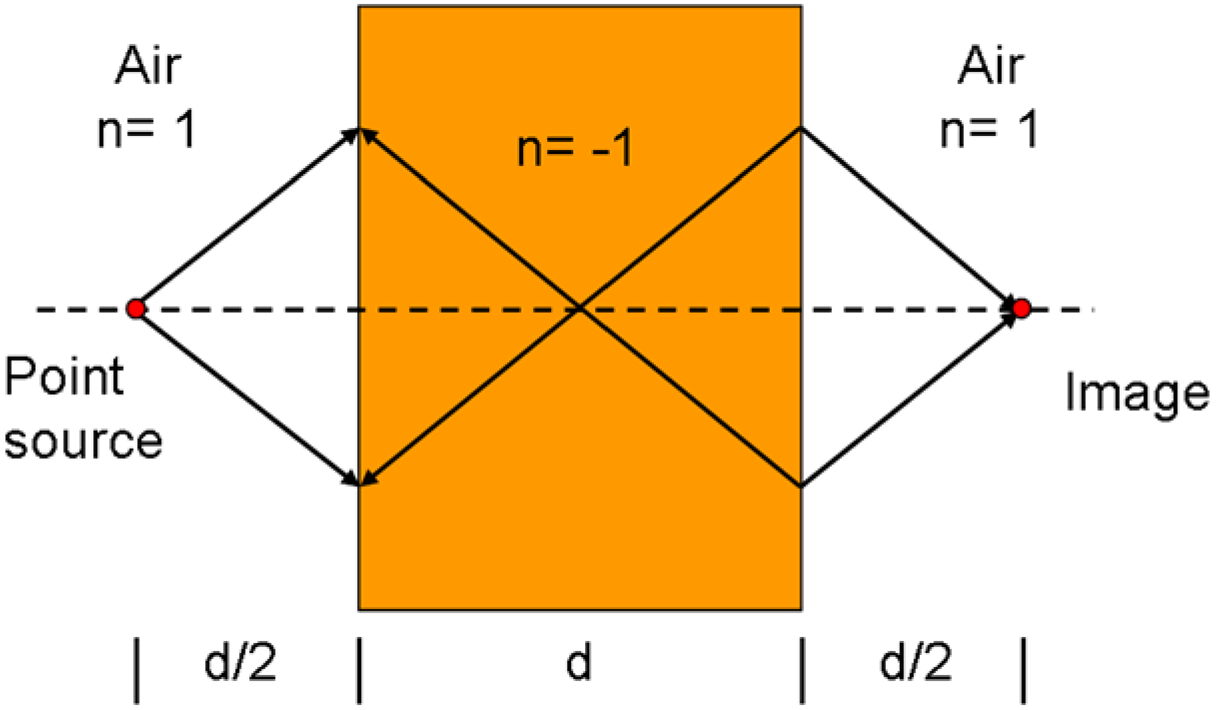






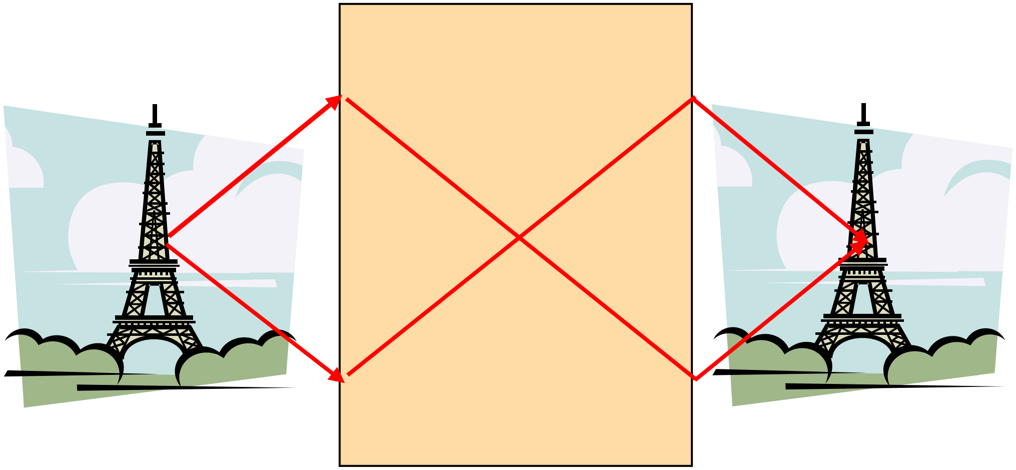
Negative index planar lens

- A slab of negative index medium (NIM) is capable of forming an image of an object placed at some finite distance from the slab [Veselago, *Sov. Phys. Uspekhi* **10,** 509 (1968)].



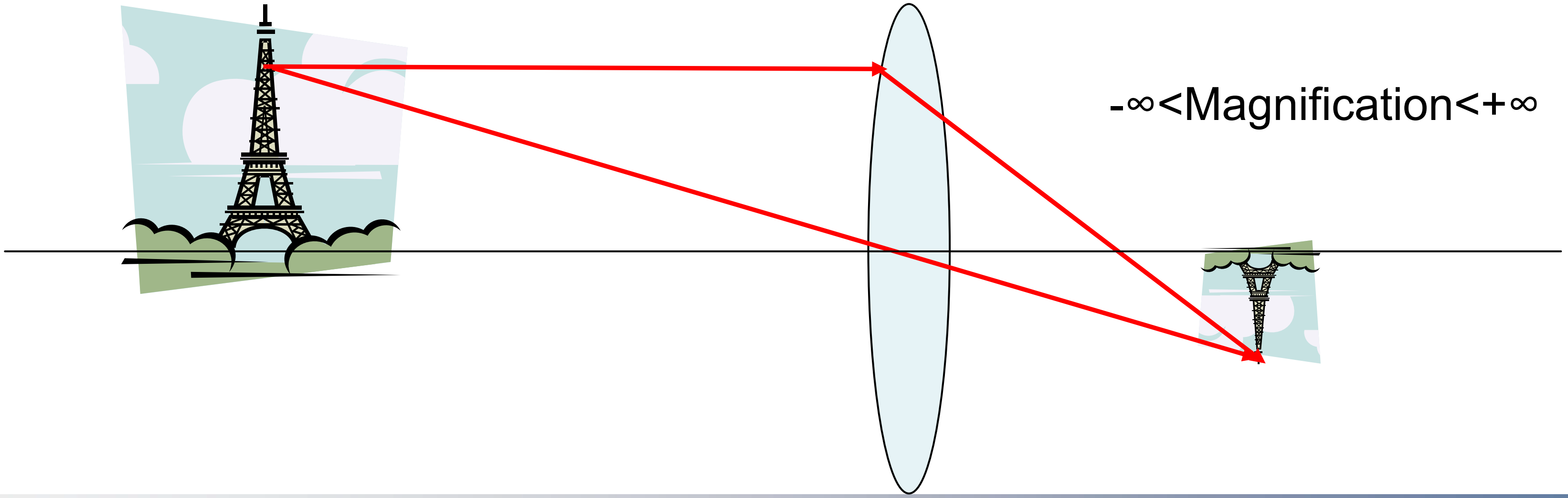
A NIM lens is not a conventional lens!

NIM lens

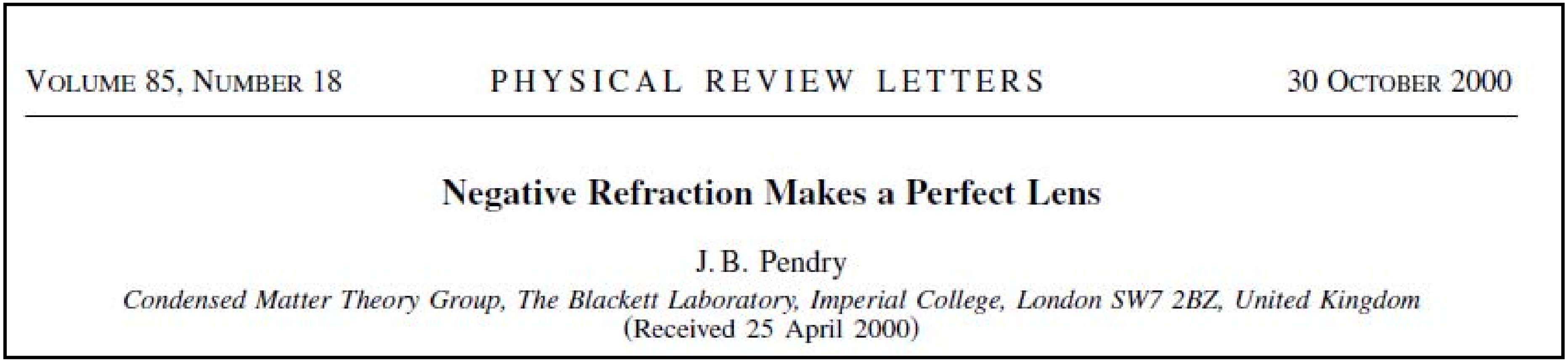


Magnification=+1
(does not focus
parallel waves)

Conventional converging lens



-∞<Magnification<+∞

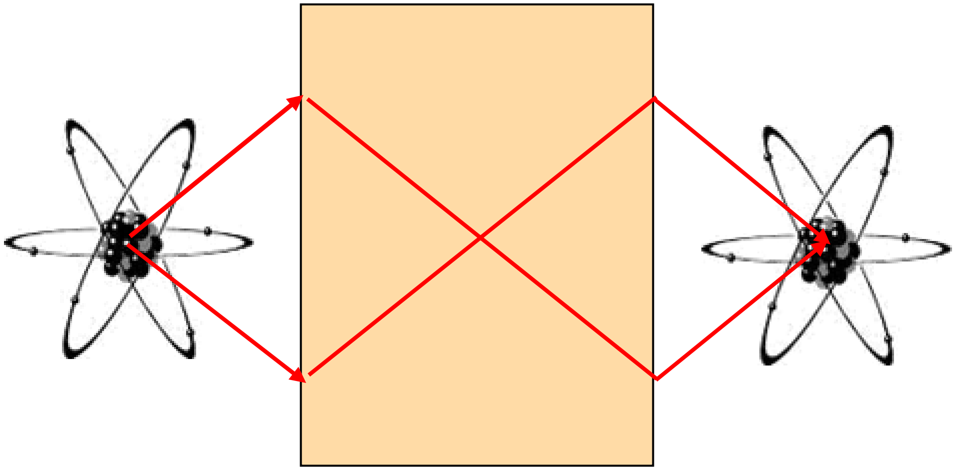


Volume 85, Number 18 PHYSICAL REVIEW LETTERS 30 October 2000

Negative Refraction Makes a Perfect Lens

J. B. Pendry
Condensed Matter Theory Group, The Blackett Laboratory, Imperial College, London SW7 2BZ, United Kingdom

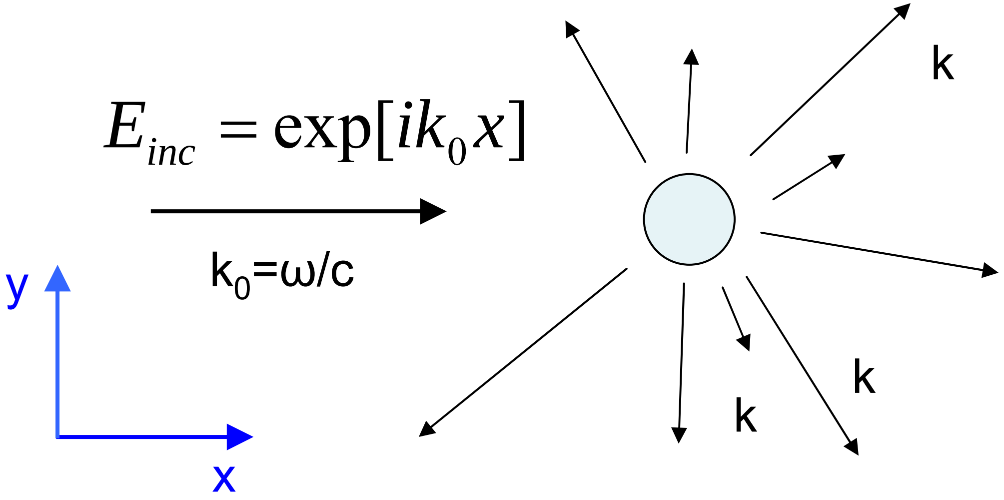
(Received 25 April 2000)



A lossless NIM slab creates **a perfect image** of the source.

To understand what is a perfect image, we first need to define what is an “object”.

Example of a sub-wavelength sphere:



$$E_{inc} = \exp[ik_0 x]$$

$k_0 = \omega/c$

$$E(x,y) = \sum_{k_y=-\infty}^{+\infty} \exp\left[i(k_x x + k_y y)\right]$$

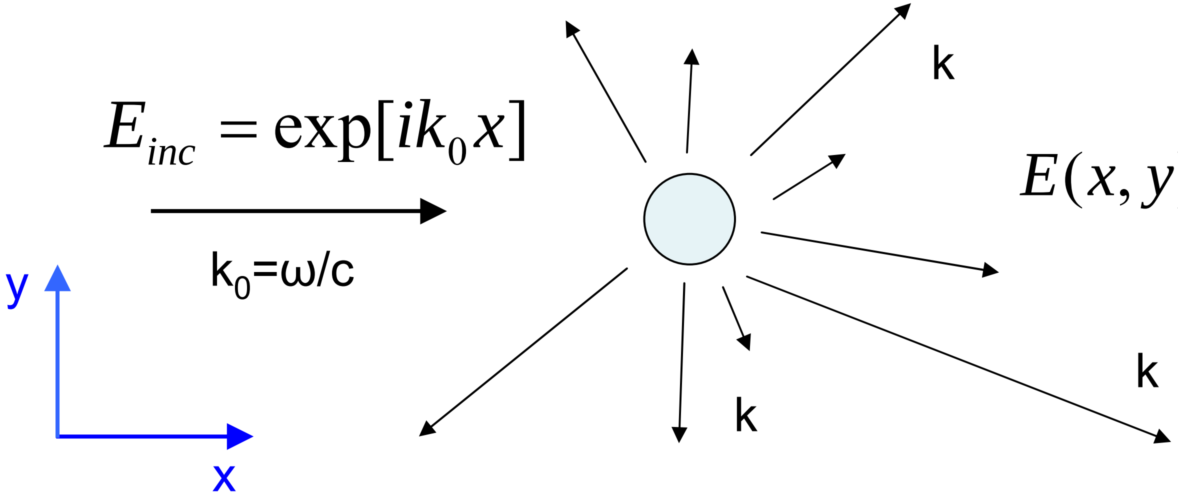
All the information about the sphere is encoded in the A_{ky} coefficients and the different values of k_y

Remember that k= ω/c. Thus $k_x = \sqrt{\frac{\omega^2}{c^2} - k_y^2}$

$\text{if } |k_y| \le \frac{\omega}{c} \equiv \frac{2\pi}{\lambda},$ then k_x is real (propagating wave)

$\text{if } |k_y| > \frac{\omega}{c} \equiv \frac{2\pi}{\lambda},$ then k_x is imaginary (evanescent, non-propagating wave)

Example of a sub-wavelength sphere:



$$E_{inc} = \exp[ik_0 x]$$

$k_0=\omega/c$

$$E(x,y) = \sum_k A_k \exp\left[i(k_x x + k_y y)\right]$$

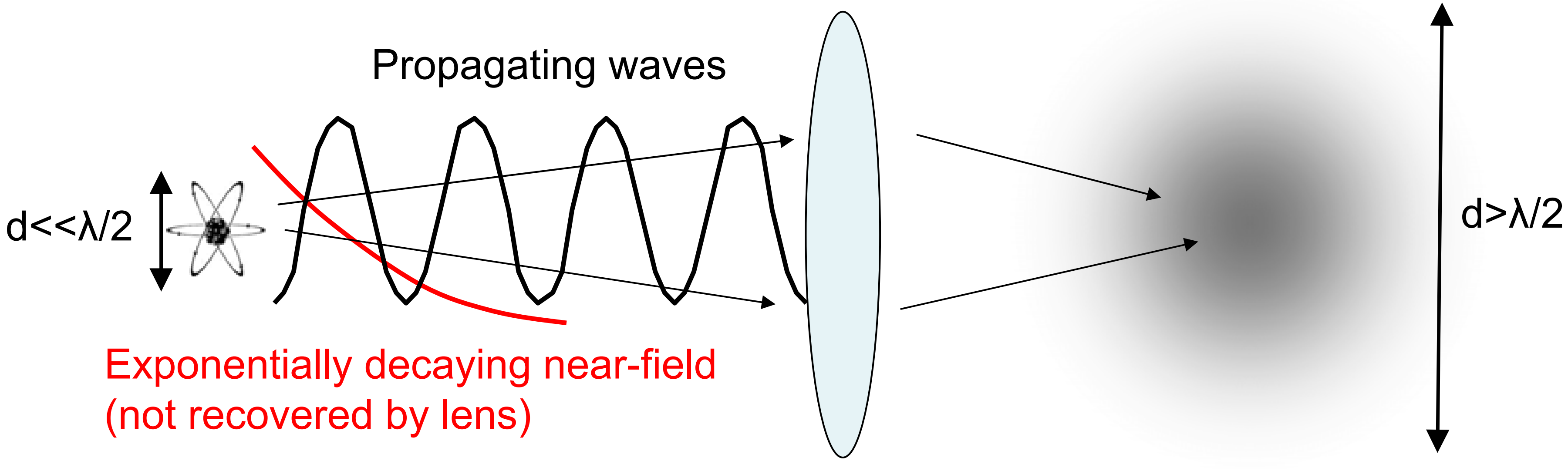
Hence, the electromagnetic field scattered by the sphere consists of two sets of solutions:

1) **The far-field distribution**, i.e. propagating waves that can be collected by a lens or a detector

2) **The near-field distribution**, i.e. evanescent waves that exponentially decay away the object.

Only the far-field can be recovered by conventional optics, so information is lost in the process

Conventional imaging setups are limited by diffraction



The planar lens: theoretical predictions

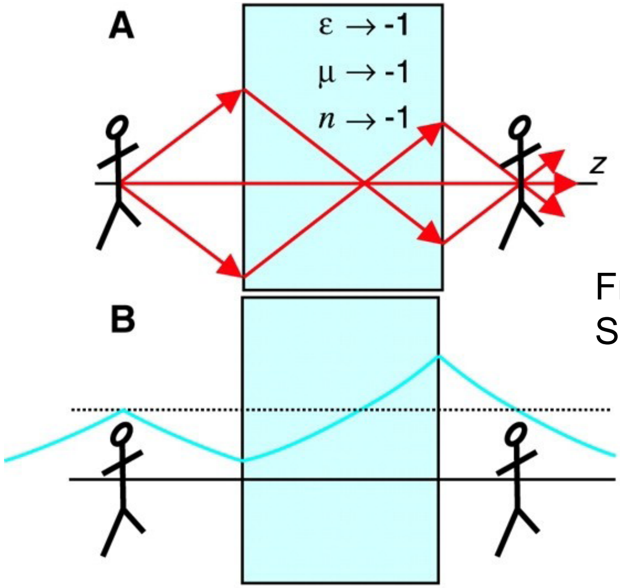
- In 2000 Pendry showed that the planar negative index lens can recover the far-field as well as the near-field of an object.

- In other words, the evanescent fields that exponentially decay are amplified by the lens

- This amplification does not violate energy conservation because evanescent waves do not carry energy

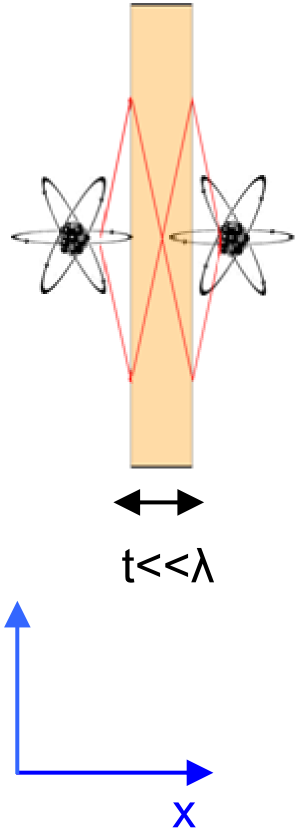
Propagating waves:

Evanescent waves:



From Smith et al., Science 305 (2004)

The case of a planar lens with subwavelength thickness



In the case t<<λ, we are in the quasistatic limit—waves can be treated as uncoupled electrostatic and magnetostatic fields.

If the E field is parallel to the x,y plane, then the H field does not play a role in the propagation.

Hence, dependence on μ is eliminated. In other words, perfect images do not require n=-1 but only ε=-1.

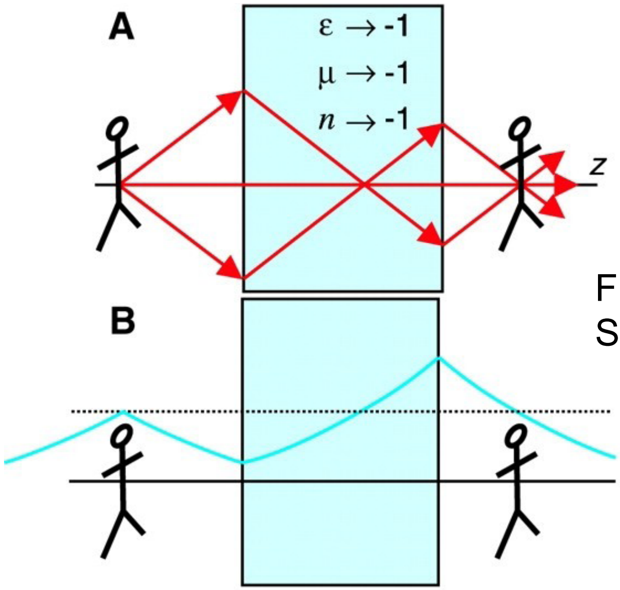
(conversely, if the H field is parallel to the x,y plane, then perfect images require μ<-1)

Pendry PRL 2000

The issue of losses

Propagating waves:

Evanescent waves:



From Smith et al., Science 305 (2004)

To obtain a perfect image, the evanescent waves forming the near-field distribution of the object must be amplified by the NIM slab.

If the slab suffer from resistive losses, then these evanescent waves are not completely restaured, thus degrading the image.

If losses are small, then sub-diffraction (but not perfect) imaging is still possible

If losses are too high, then the image becomes diffraction-limited.

Smith et al., APL 82, 1506 (2003)

Speaking of losses…

Negative refraction is not equivalent to negative index of refraction

The condition for negative refraction is: $n = -\sqrt{\varepsilon\mu}$

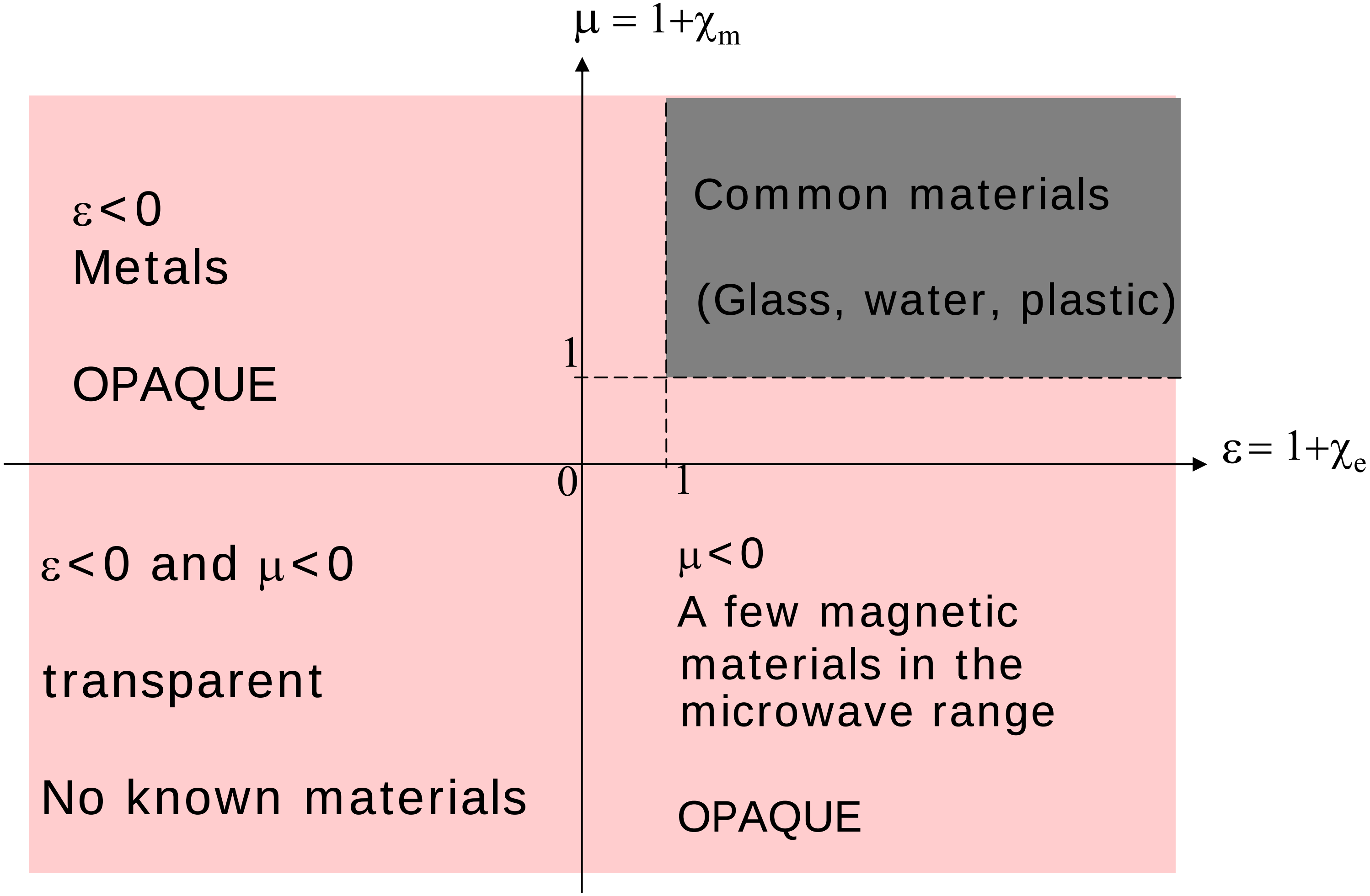
What if ε and μ are complex quantitities? $\varepsilon = \varepsilon' + i\varepsilon''$

$$\mu = \mu' + i\mu''$$

ε′ and μ’ account for the losses in the material.

The condition n<0 is verified if ε’< 0 and μ>0
and ε’|μ |+μ’|ε|<0.

SUMMARY



$\mu = 1+\chi_m$

$\varepsilon = 1+\chi_e$

$\varepsilon<0$
Metals

OPAQUE

Common materials

(Glass, water, plastic)

$\varepsilon<0$ and $\mu<0$

transparent

No known materials

$\mu<0$
A few magnetic materials in the microwave range

OPAQUE

In the first part of this lecture, we have seen that media with negative permittivity and permeability have non-intuitive interesting properties.

However there is no such thing as negative index materials in nature!

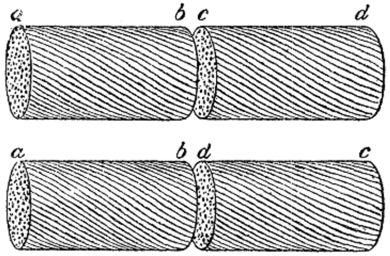
Can somehow artificial structures behave as effective negative index media?

Definition: artificial composite medium = metamaterial

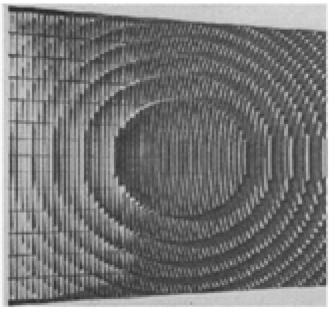
Artificial composites in electromagnetism

The concept of artificial electromagnetic composites is almost as old as Maxwell’s equations

1898: Twisted jute elements acting as chiral molecules (J.C. Bose, Proceeding of Royal Soc. London)

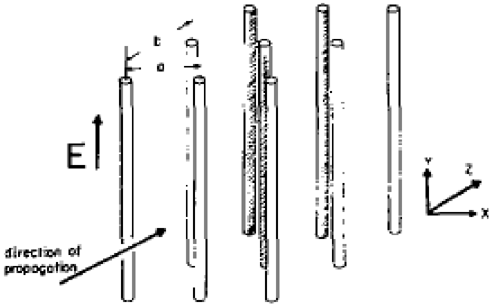


1922: “Artificial magnetic medium” (A. Goldhammer, physikalische Zeitschrift).

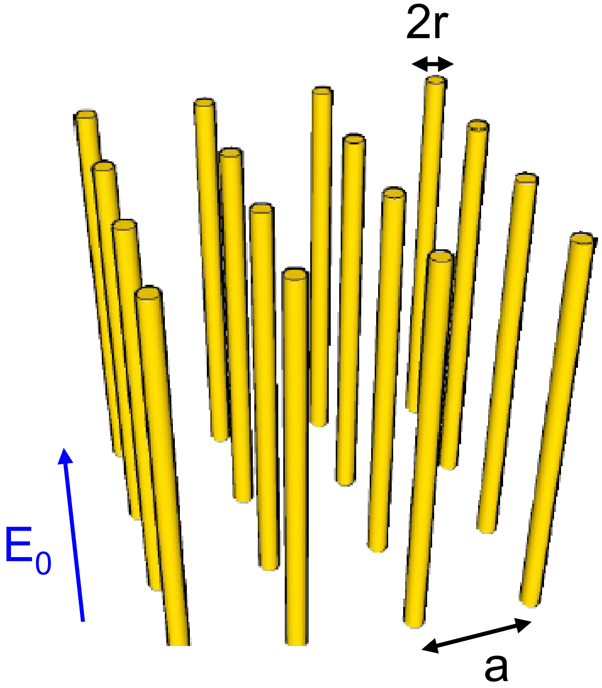


1946: Artificial “metal lens antennas” (W.E Kock, Proc. of the IRE and Waves and Electrons)

1962: “Plasma simulation by artificial dielectrics” (W. Rotman, IRE Trans. Antennas Propag.)



An array of perfectly conducing wires

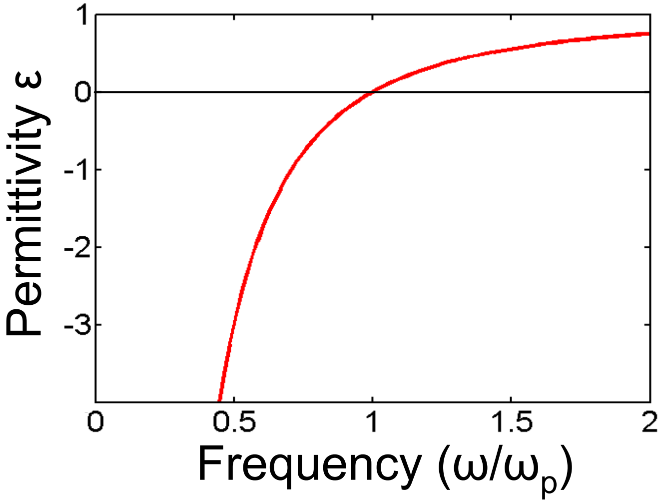


Pendry et al., PRL 76 (1996)

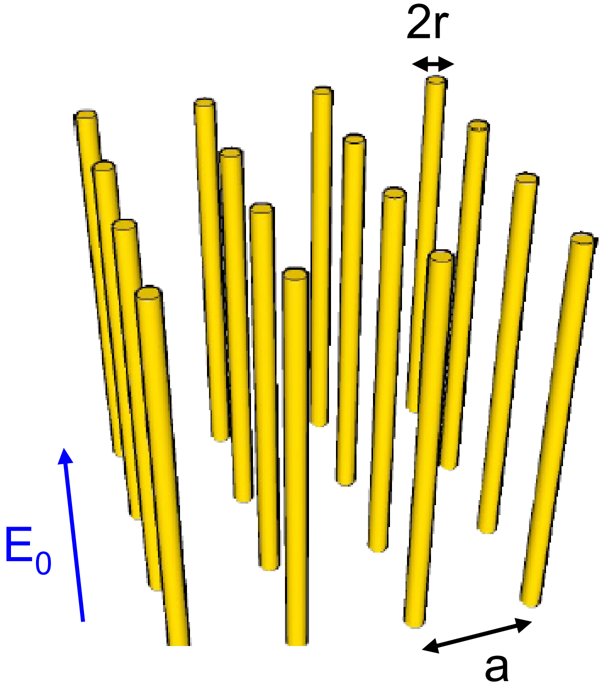
In 1996 Pendry et al. rediscovered the wire medium and showed that its properties are those of a homogeneous metal at optical frquencies.

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

with $$\omega_p^2 = \frac{2\pi\, c_0^2}{a^2 \ln(a/r)}$$



An array of perfectly conducing wires



Pendry et al., PRL 76 (1996)

In 1996 Pendry et al. rediscovered the wire medium and showed that its properties are those of a homogeneous metal at optical frquencies.

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

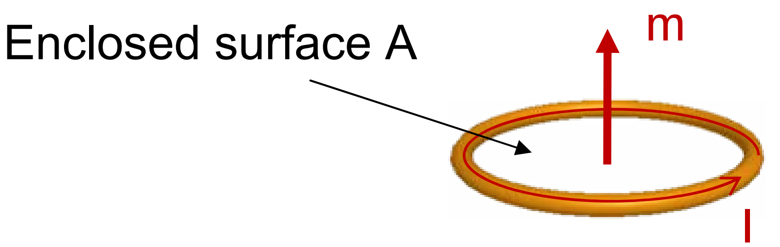
with $$\omega_p^2 = \frac{2\pi\, c_0^2}{a^2 \ln(a/r)}$$

Note that ω_p depends only on the geometrical parameters of the system: the wire radius and the lattice constant!

Magnetic Response

We have discussed the means of engineering electric response in materials. How do we now obtain a magnetic response?

Recall that a magnetic dipole moment occurs when current flows in a closed circuit.

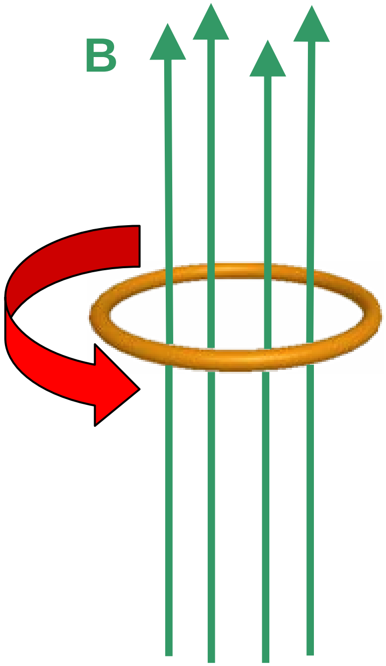


$$\mathbf{m} = \frac{1}{2}\int \mathbf{r} \times \mathbf{J} dV = IA\hat{z}$$

While the dominant contribution to magnetism in materials is that of spin—a quantum mechanical phenomenon—all we need is to cause currents in a ring to flow.

Magnetic Response

When an alternating magnetic field is directed along the axis of a ring, an electromotive force is induced that can cause currents to flow.



We can calculate the electromotive force (emf) induced by the time-varying magnetic field as follows:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = i\omega\mu_0 \mathbf{H}_0$$

$$EMF = \oint \mathbf{E} \cdot d\mathbf{l} = i\omega\mu_0 \pi a^2 H_0$$

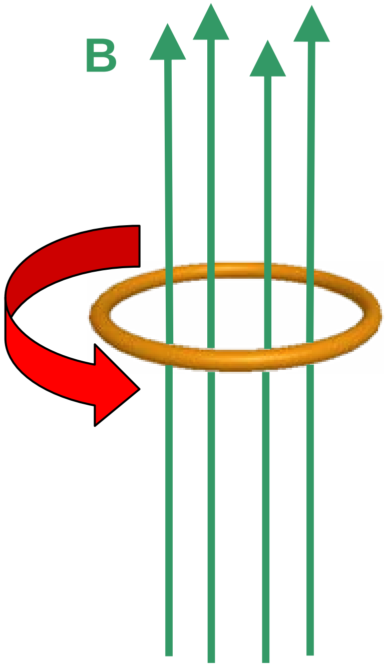
Faraday's induction law
a is the radius of the loop

NB: the emf is not a force, but a voltage!

$$I = \frac{EMF}{Z_0} = \frac{i\omega\mu_0 \pi a^2 H_0}{Z_0}$$

Magnetic Response

When an alternating magnetic field is directed along the axis of a ring, an electromotive force is induced that can cause currents to flow.



$$I = \frac{EMF}{Z_0} = \frac{i\omega\mu_0\pi a^2 H_0}{Z_0}$$

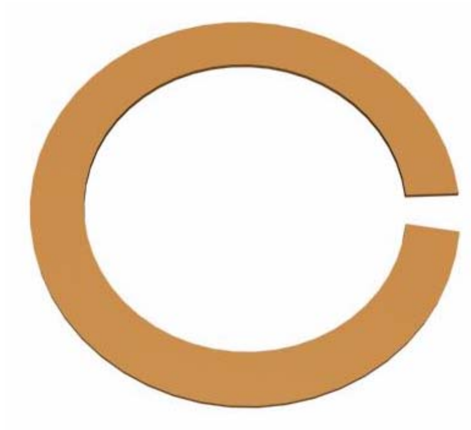
$$m = IA = \pi a^2 I = \frac{i\omega\mu_0\pi^2 a^4 H_0}{Z_0} = \alpha_m H_0$$

$$\chi_m = N\alpha_m = \frac{i\omega\mu_0\pi^2 \frac{a^4}{d^3}}{Z_0}$$

$$\frac{\mu}{\mu_0} = 1 + \chi_m = 1 + \frac{i\omega\mu_0\pi^2 \frac{a^4}{d^3}}{Z_0}$$

The Split Ring Resonator

When an alternating magnetic field is directed along the axis of a ring, an electromotive force is induced that can cause currents to flow.



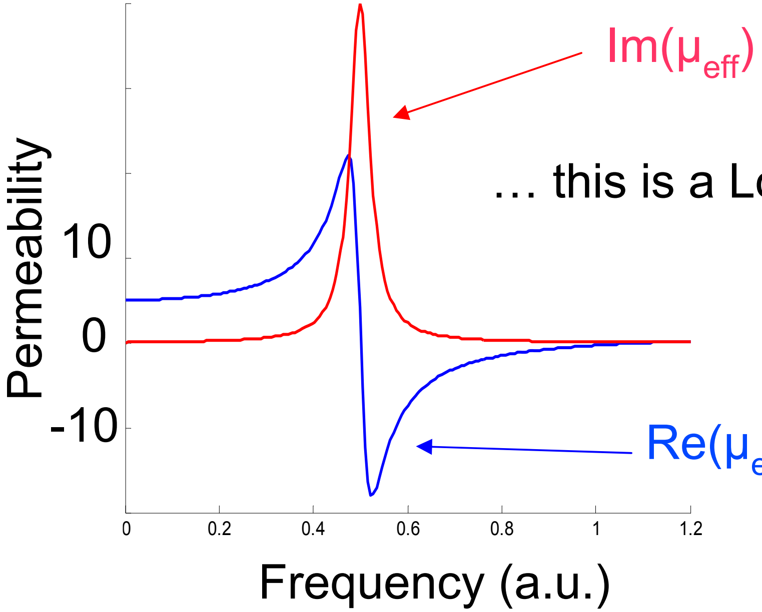
$$Z_0 = R - i\omega L - \frac{1}{\omega C}$$

$$\frac{\mu}{\mu_0} = 1 + \chi_m = 1 - \frac{\omega^2 \mu_0 \pi^2 \frac{a^4}{d^3 L}}{\omega^2 - \frac{1}{LC} + i\omega \frac{R}{L}}$$

$$\frac{\mu}{\mu_0} = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2 + i\omega\Gamma}$$

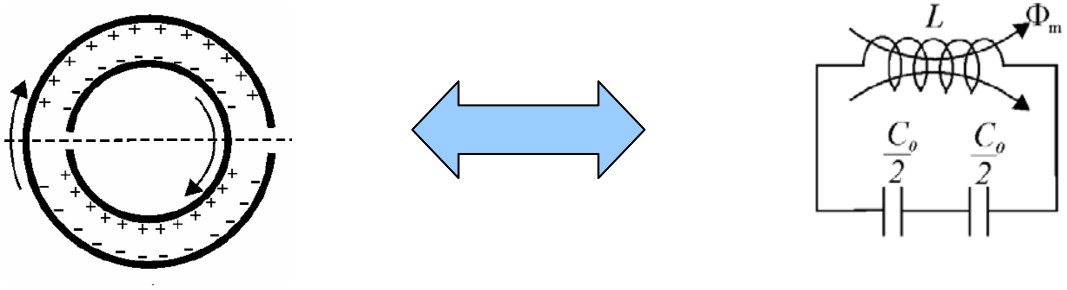
- The effective permeability can be analytically expressed as:

$$\mu_{eff} = 1 - \frac{\dfrac{\pi r^2}{a^2}}{1 + \dfrac{2\sigma i}{\omega r \mu_0} - \dfrac{3dc_0^2}{\pi^2 \omega^2 r^3}}$$

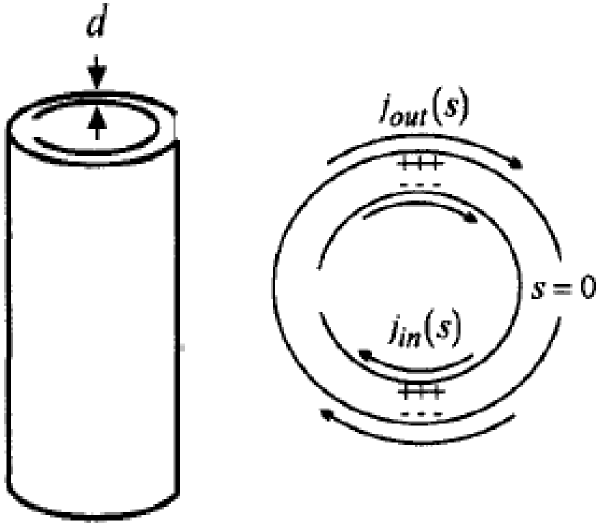


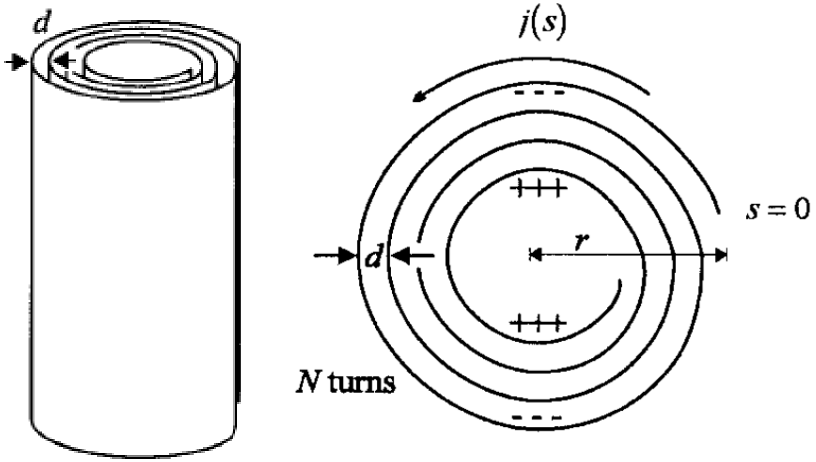
… this is a Lorentzian.

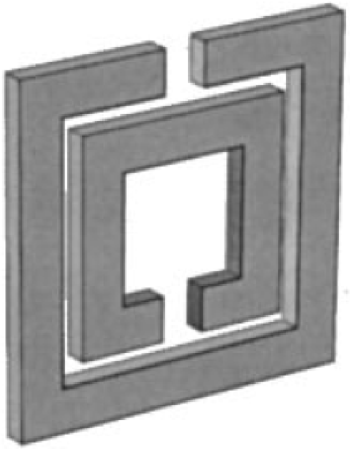
- Useful circuit analogy: a split ring behaves as a RLC resonator.

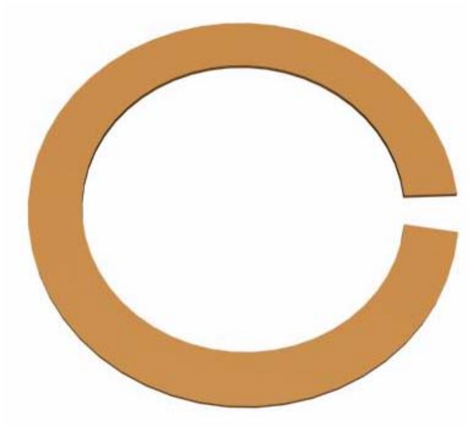


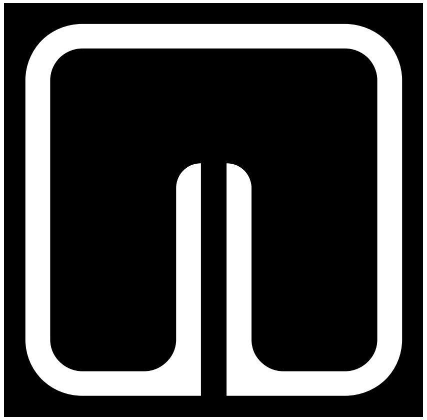
Split ring resonators (SRRs): variants.



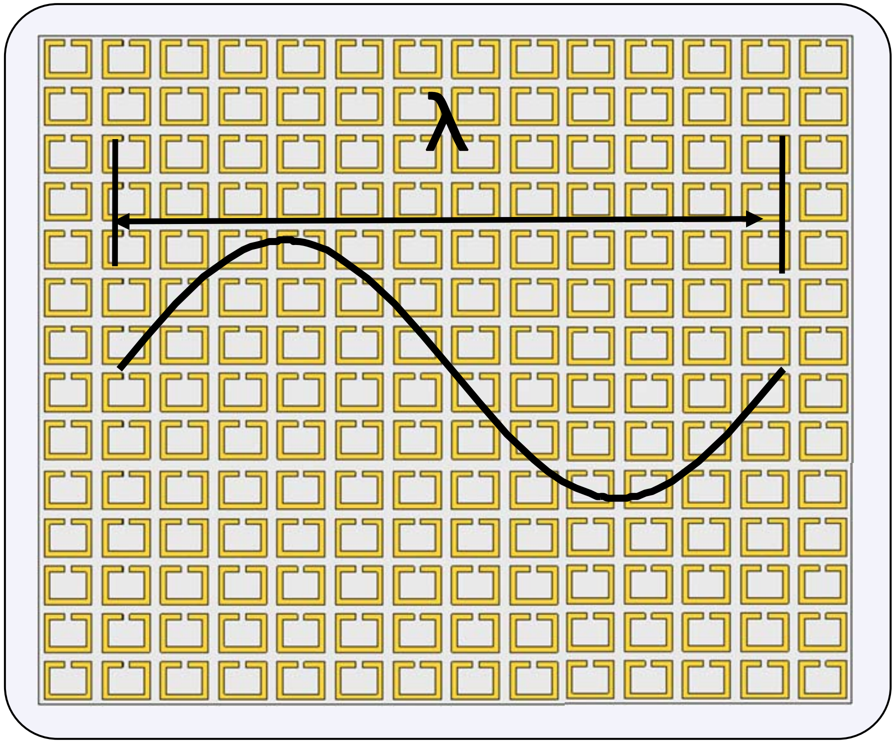








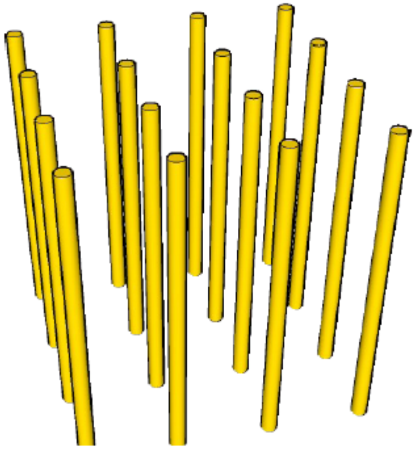
Unit cell dimensions?



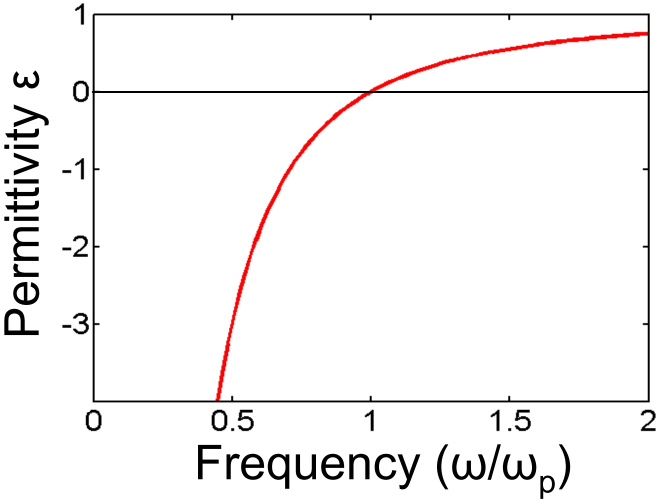
At 10 GHz, λ~3 cm → the unit cell size must be equal to or smaller than 3 mm.

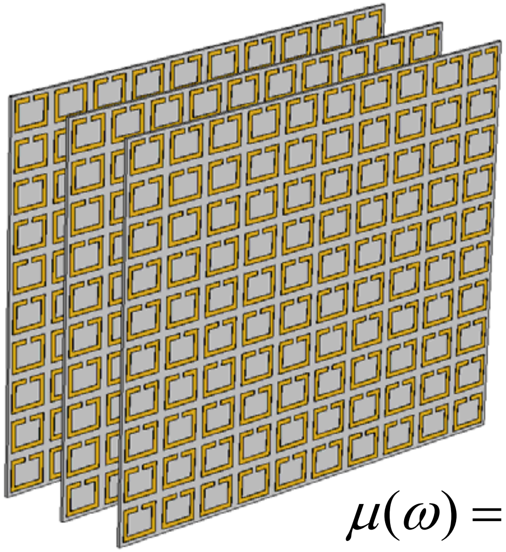
→ fabrication by standard circuit lithography.

Summary

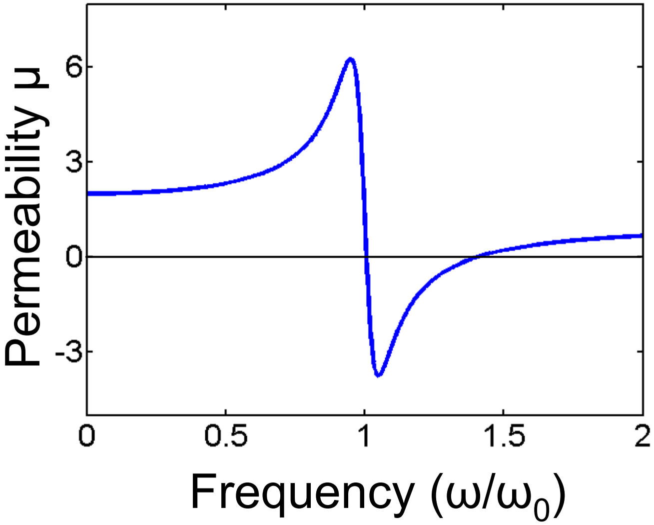


$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$





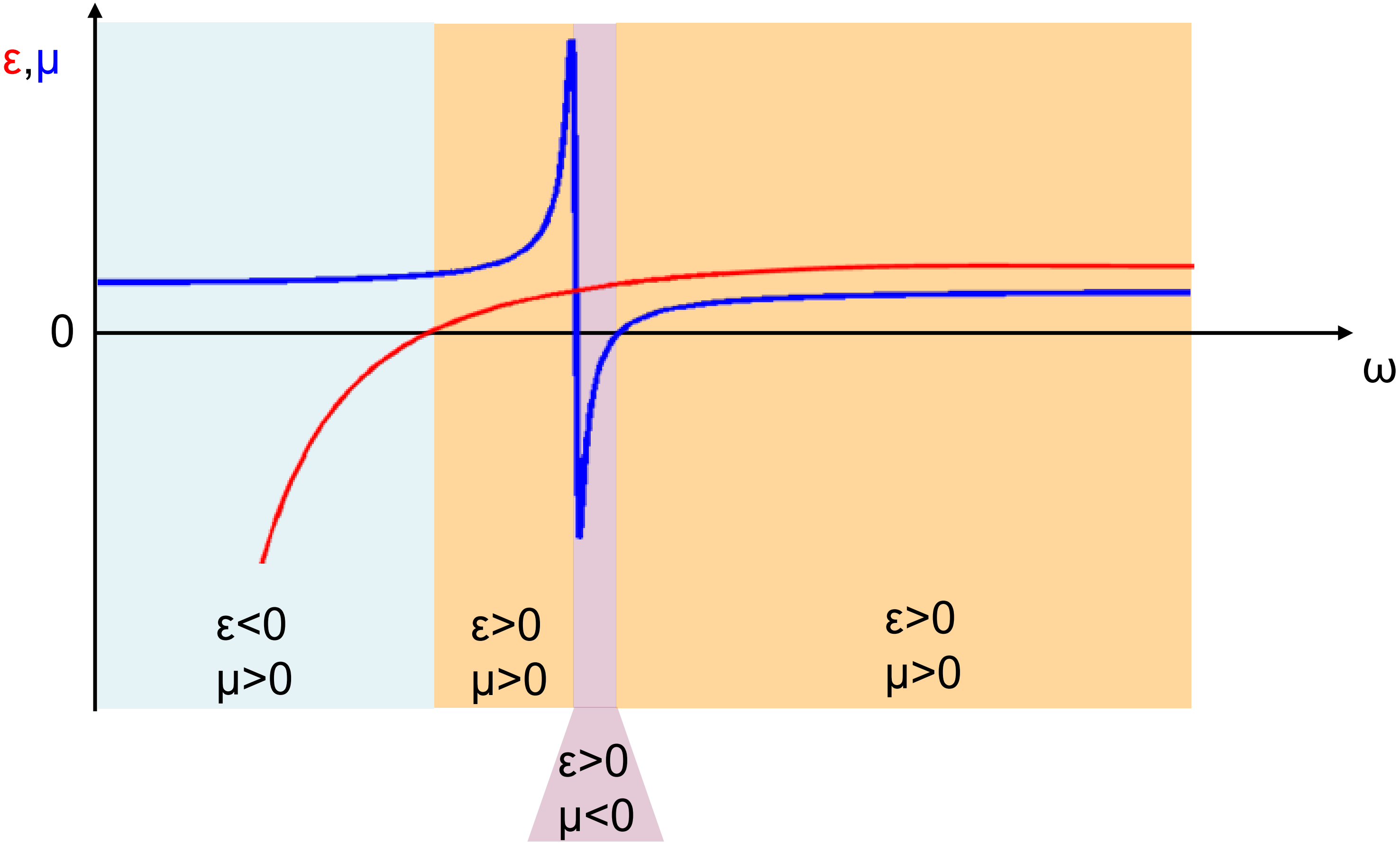
$$\mu(\omega) = 1 - \frac{F\omega_p^2}{\omega^2 - \omega_0^2 + i\Gamma\omega}$$



We have seen two of the most important metamaterial structures. The wire medium provides a negative ε, while the SRR medium provides a negative μ.

What happens if we combine the two architectures together?

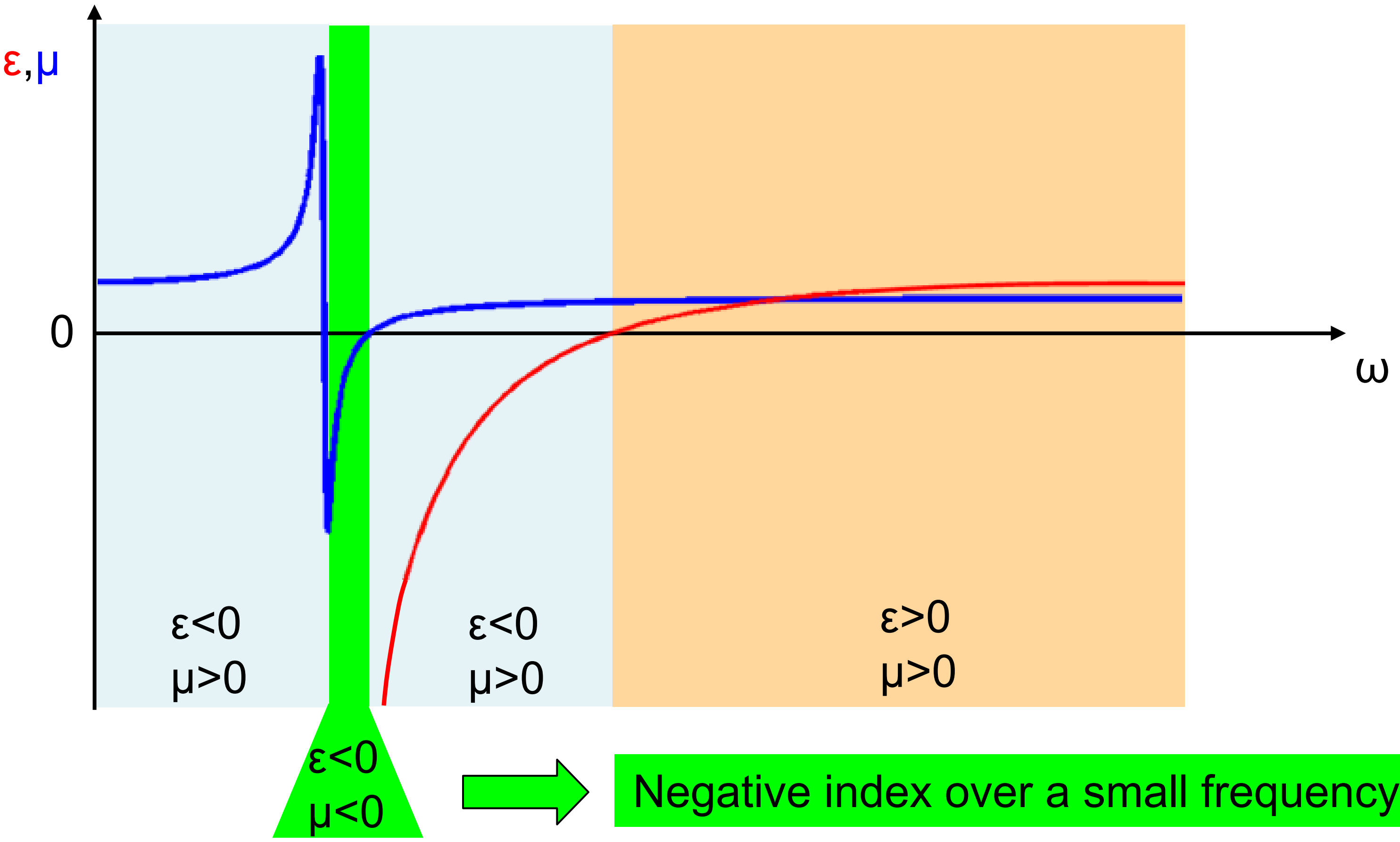
This question has been answered by David R. Smith in 2000:



We have seen two of the most important metamaterial structures. The wire medium provides a negative ε, while the SRR medium provides a negative μ.

What happens if we combine the two architectures together?

This question has been answered by David R. Smith, in 2000:

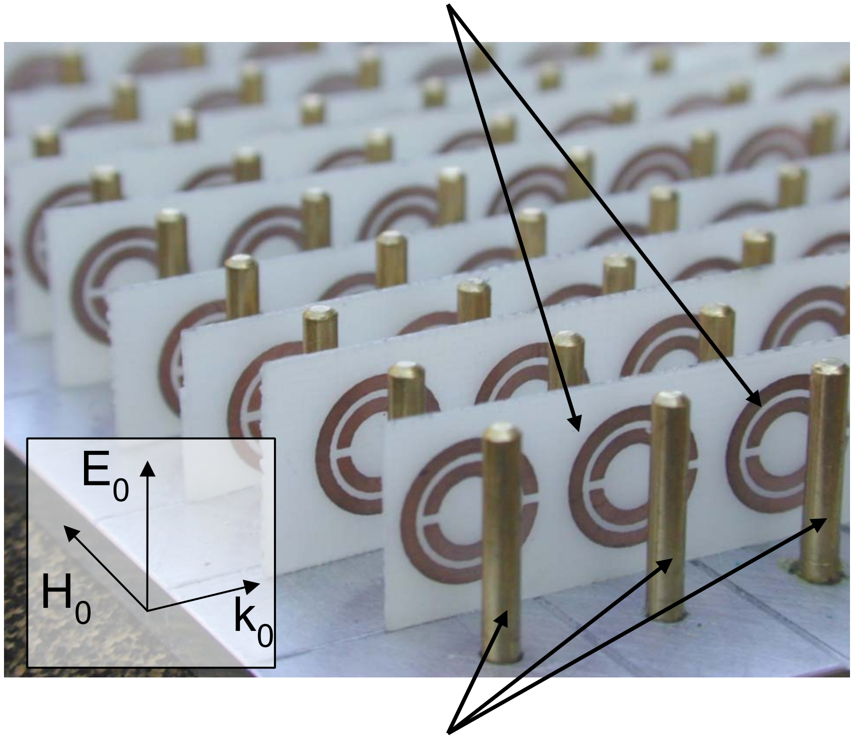


Negative index over a small frequency range!

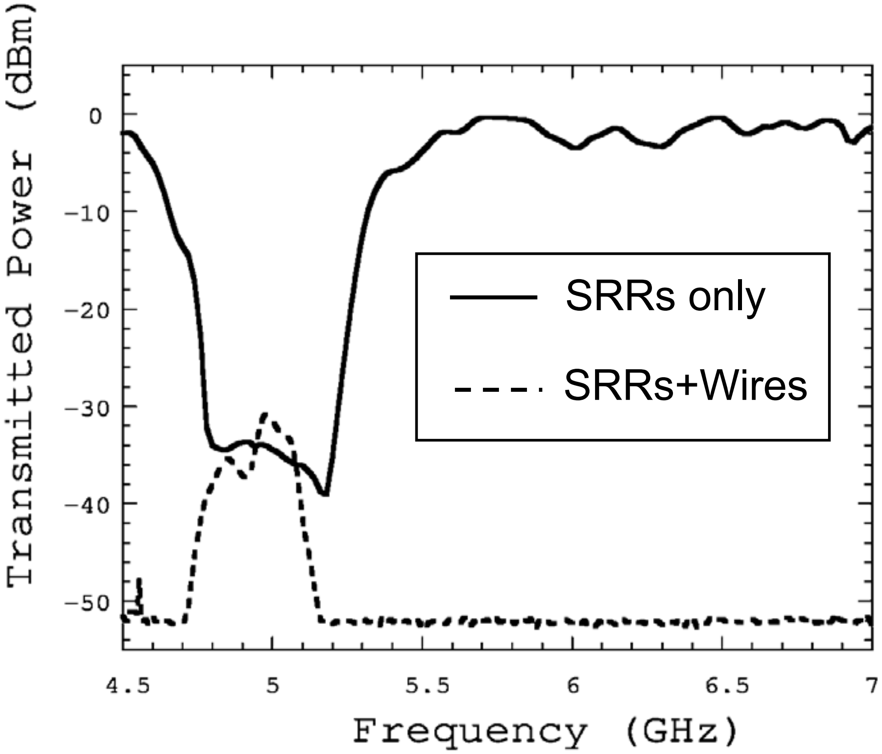
Negative index Metamaterials (NIMs)

In 2000 Smith *et al*. introduced the first NIM

SRRs: provide μ<0

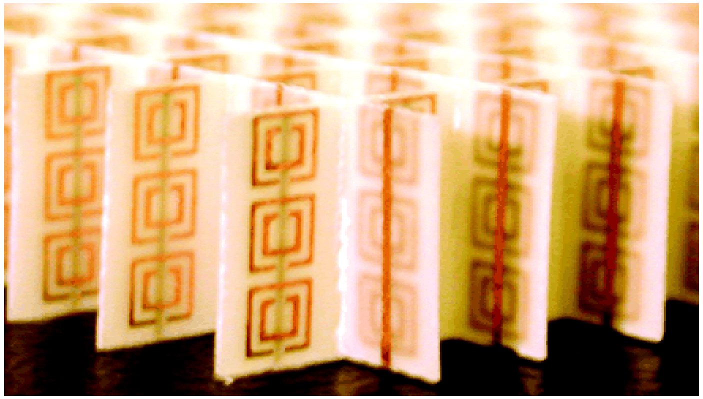


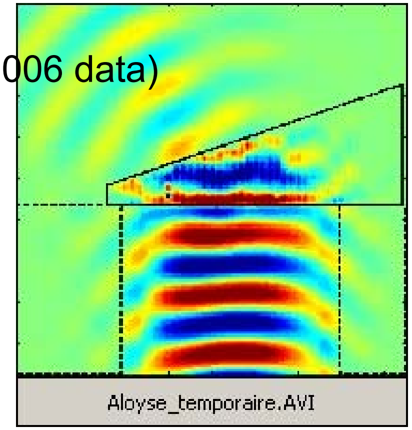
Wires: provide ε<0



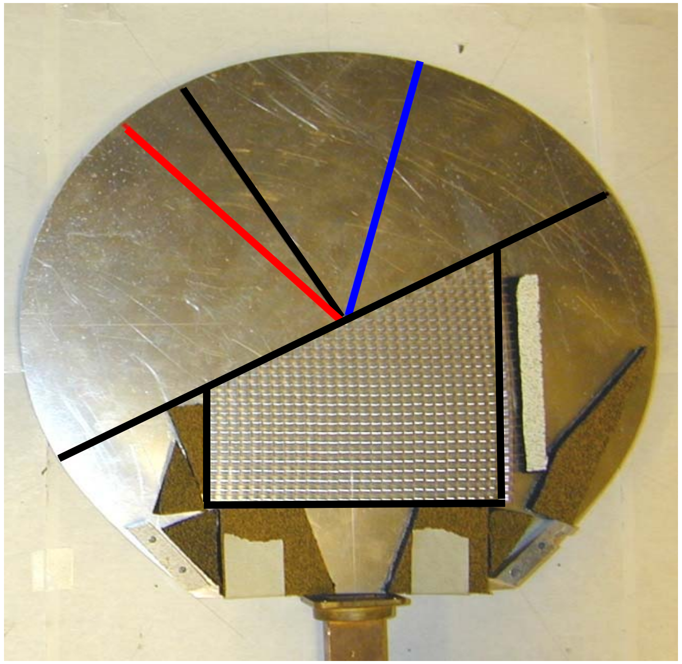
D. R. Smith et al, Phys. Rev. Lett. 84, 4184 (2000).

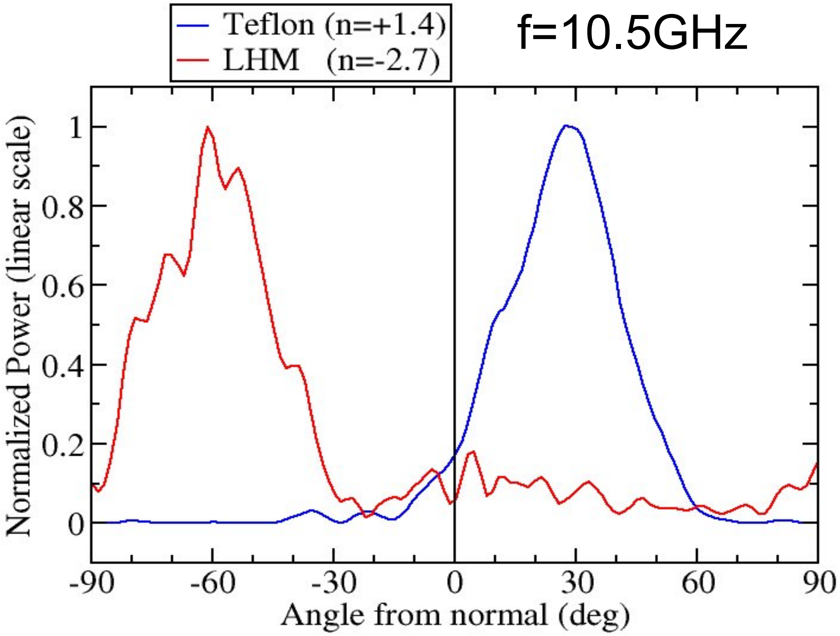
2001: first direct observation of negative refraction





(2006 data)

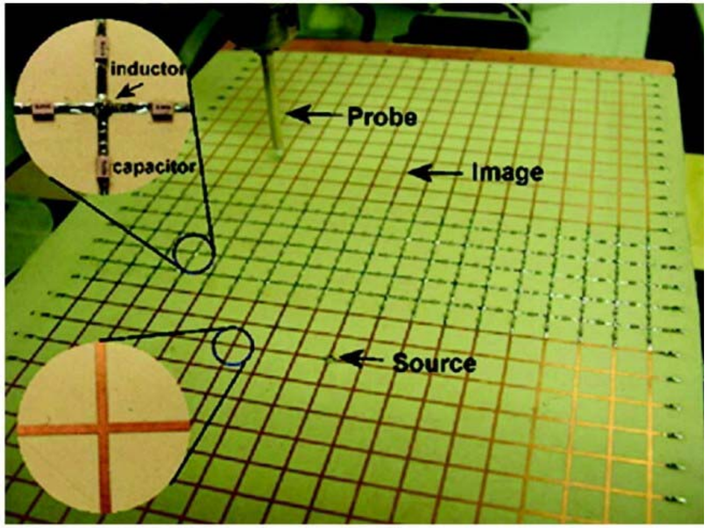


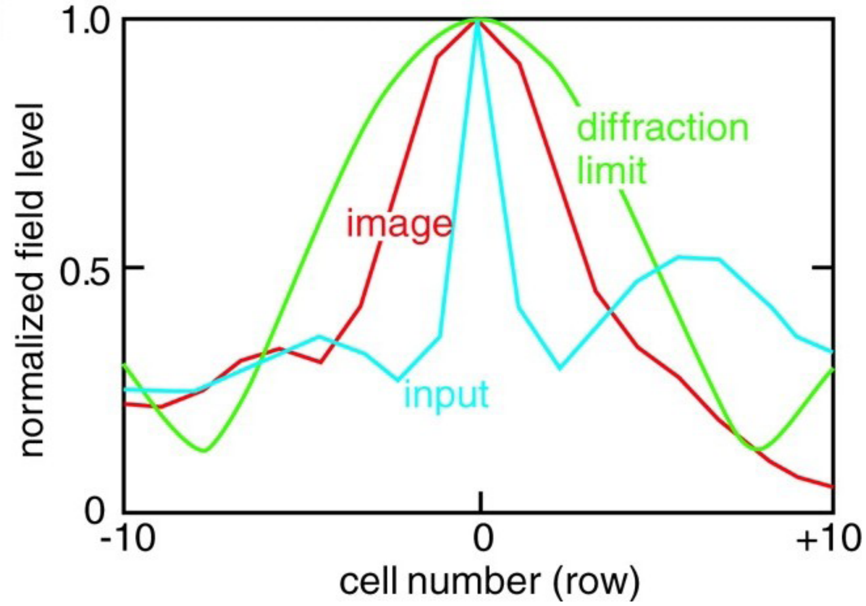


f=10.5GHz

R. Shelby, D. R. Smith, S. Schultz, Science 292, 77 (2001).

2004: first experimental demonstration of subwavelength imaging

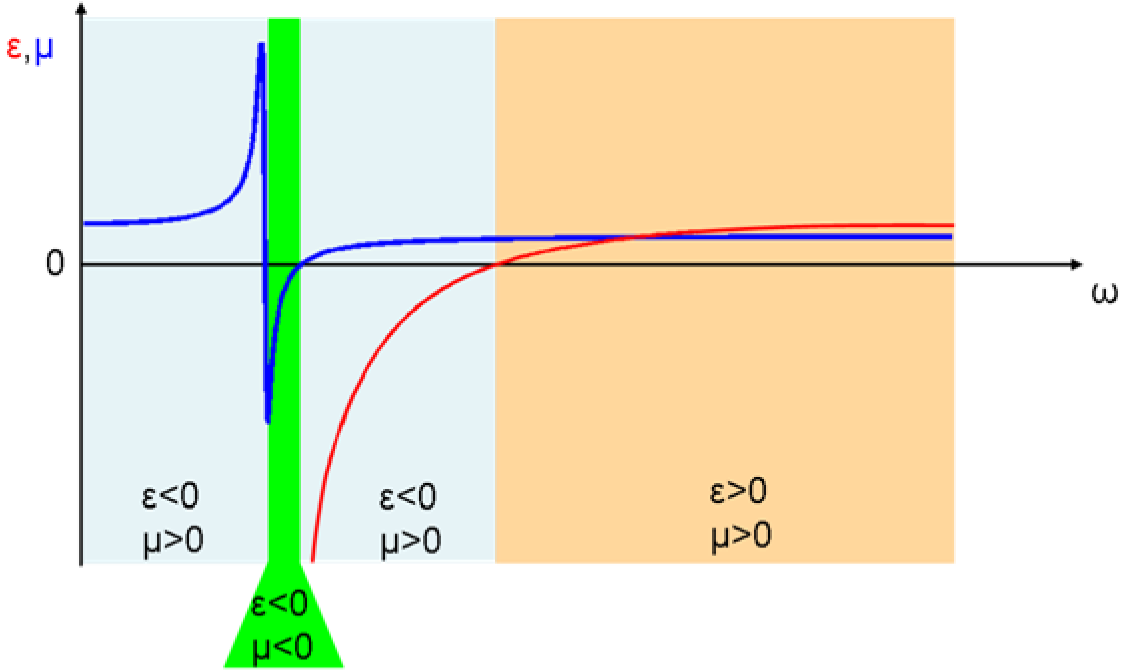




A. Grbic, G. V. Eleftheriades, PRL 92, 117403 (2004).

Limitations

- Magnetic and NIM metamaterials operate over a narrow bandwidth!



The limitation comes from the fact that resonant elements (e.g. SRRs) must be used to obtain unconventional values of ε and μ.

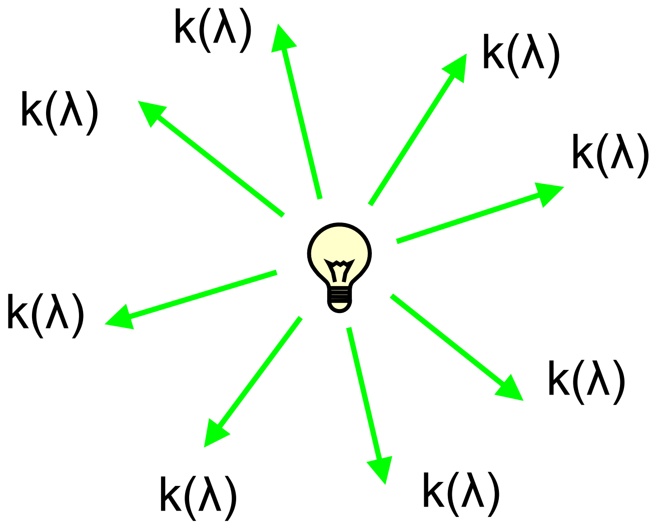
Limitations

- Field homogenization is not that simple!

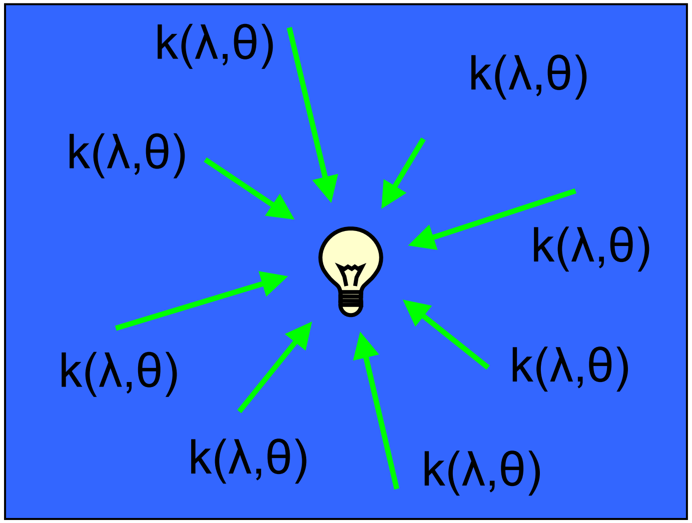
In our analysis of the SRR medium, we have neglected the mutual interactions between neighboring unit cells.

Nontrivial effects happen due to the finite size of the unit cell (spatial dispersion, nonlocal effects...)

Light source in air

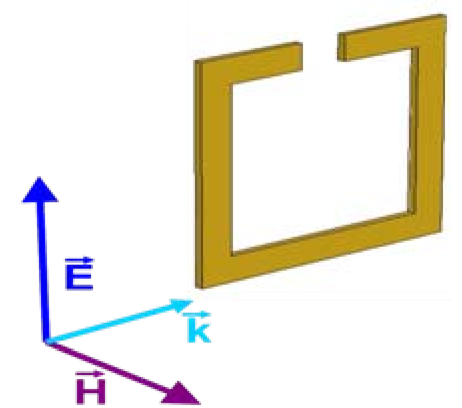


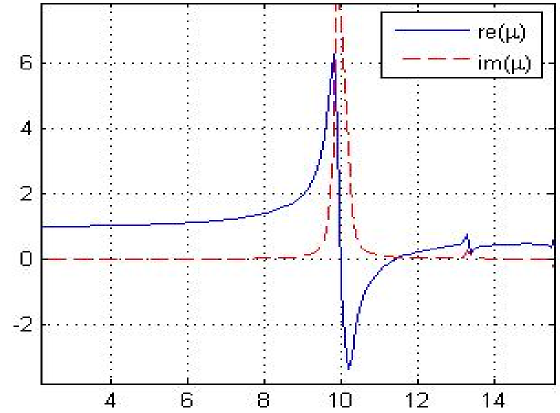
Light source in a NIM metamaterial



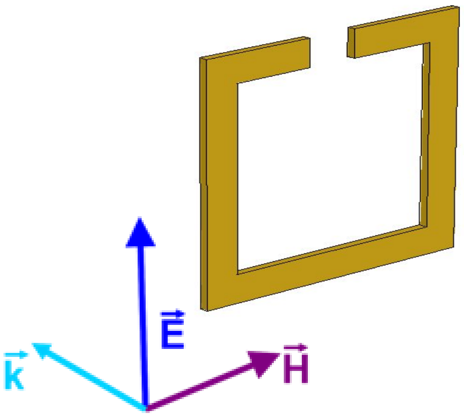
Limitations – Advanced considerations

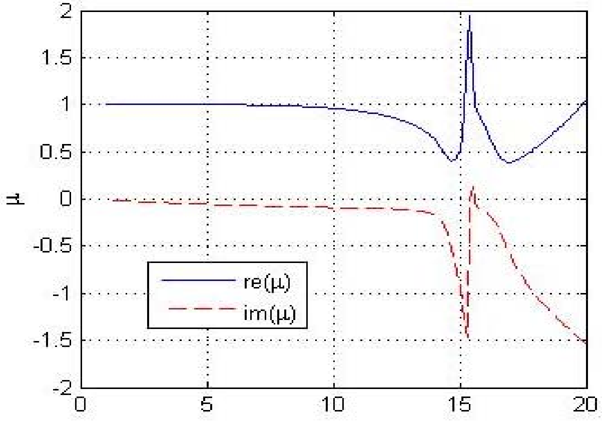
- Most metamaterials are intrinsically anisotropic or bianisotropic media!





$$\cancel{\begin{aligned}\vec{D} &= \varepsilon\,\vec{E}\\ \vec{B} &= \mu\,\vec{H}\end{aligned}}$$



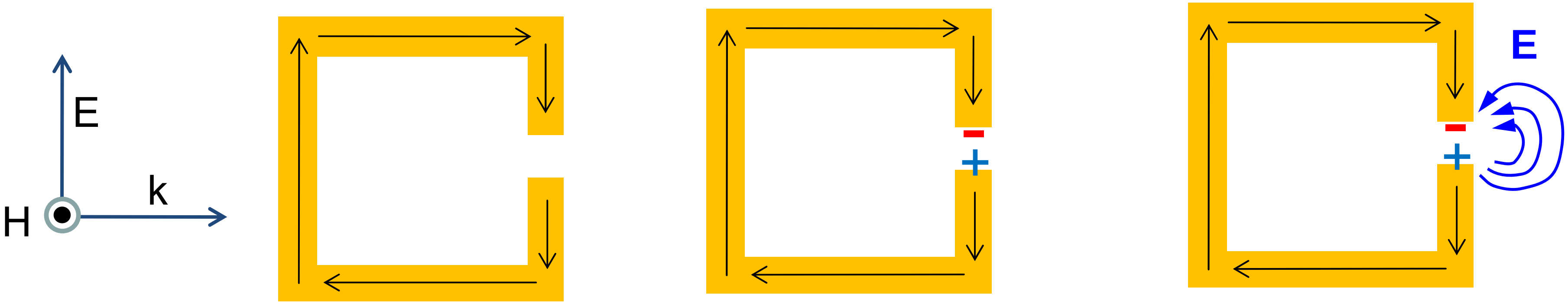


$$\vec{D} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \vec{E}$$

$$\vec{B} = \begin{bmatrix} \mu_{xx} & \mu_{xy} & \mu_{xz} \\ \mu_{yx} & \mu_{yy} & \mu_{yz} \\ \mu_{zx} & \mu_{zy} & \mu_{zz} \end{bmatrix} \vec{H}$$

Limitations – Advanced considerations

- Most metamaterials are intrinsically anisotropic and/or bianisotropic media!



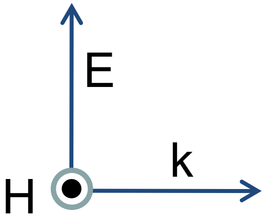
The magnetic field induces a circulating current.

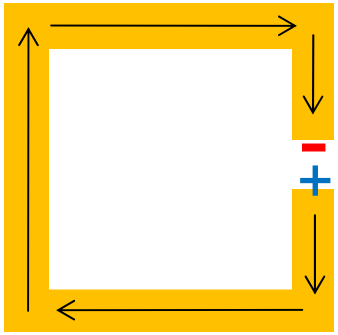
The currents induces a charge separation across the gap.
This is the definition of a dipole moment!

Thus, a fraction of the magnetic field is converted into an electric field.

Advanced considerations

- Most metamaterials are intrinsically anisotropic and/or bianisotropic media!





In other words, SRRs exhibit a coupled magneto-electric response that must be taken into account in the constitutive relations:

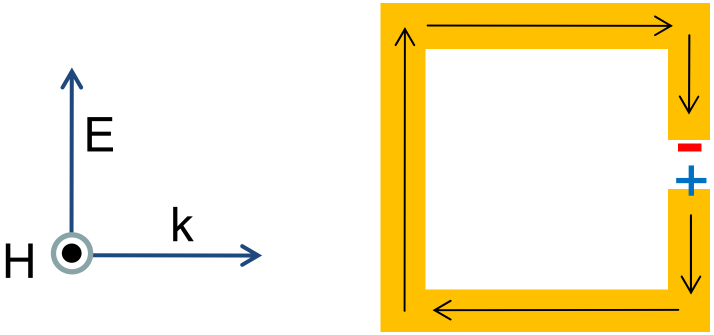
$$\xcancel{\begin{aligned}\vec{D} &= \varepsilon\,\vec{E}\\ \vec{B} &= \mu\,\vec{H}\end{aligned}}$$

$$\vec{D} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \vec{E} + \begin{bmatrix} \zeta_{xx} & \zeta_{xy} & \zeta_{xz} \\ \zeta_{yx} & \zeta_{yy} & \zeta_{yz} \\ \zeta_{zx} & \zeta_{zy} & \zeta_{zz} \end{bmatrix} \vec{H}$$

$$\vec{B} = \begin{bmatrix} \xi_{xx} & \xi_{xy} & \xi_{xz} \\ \xi_{yx} & \xi_{yy} & \xi_{yz} \\ \xi_{zx} & \xi_{zy} & \xi_{zz} \end{bmatrix} \vec{E} + \begin{bmatrix} \mu_{xx} & \mu_{xy} & \mu_{xz} \\ \mu_{yx} & \mu_{yy} & \mu_{yz} \\ \mu_{zx} & \mu_{zy} & \mu_{zz} \end{bmatrix} \vec{H}$$

Advanced considerations

- Most metamaterials are intrinsically anisotropic and/or bianisotropic media!



In other words, SRRs exhibit a coupled magneto-electric response that must be taken into account in the constitutive relations:

$$\boxed{\begin{aligned}\vec{D} &= \varepsilon\ \vec{E}\\ \vec{B} &= \mu\ \vec{H}\end{aligned}}$$

$$\vec{D} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{y} & & \\ \varepsilon_{z} & & \end{bmatrix} \vec{\ } \begin{bmatrix} \zeta_{xx} & \zeta_{xy} & \zeta_{xz} \\ & & \\ & & \end{bmatrix} \vec{H}$$

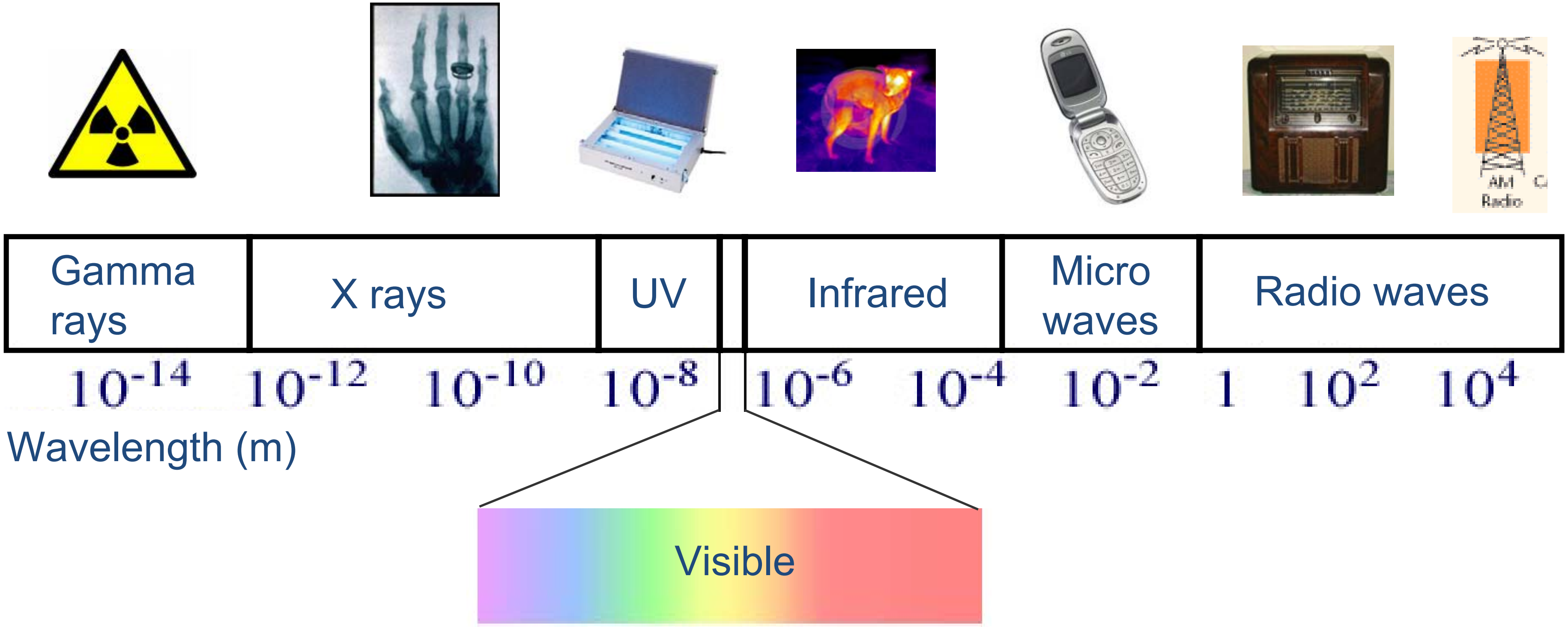
$$\vec{B} = \begin{bmatrix} \xi_{xx} & & \\ \xi_{yx} & & \\ \xi_{zx} & \zeta_{zy} & \zeta_{zz} \end{bmatrix} \begin{bmatrix} & & \\ & & \\ \mu_{zx} & \mu_{zy} & \mu_{zz} \end{bmatrix} \vec{H}$$

With appropriate polarization and propagation direction, the simplified constitutive relations are generally valid

Metamaterials at higher frequencies

So far we have presented experimental results in the microwave regime (frequency ν ~ 10 GHz, wavelength λ ~ 3 cm)

Can we scale these results to higher frequencies?



Gamma rays

X rays

UV

Infrared

Micro waves

Radio waves

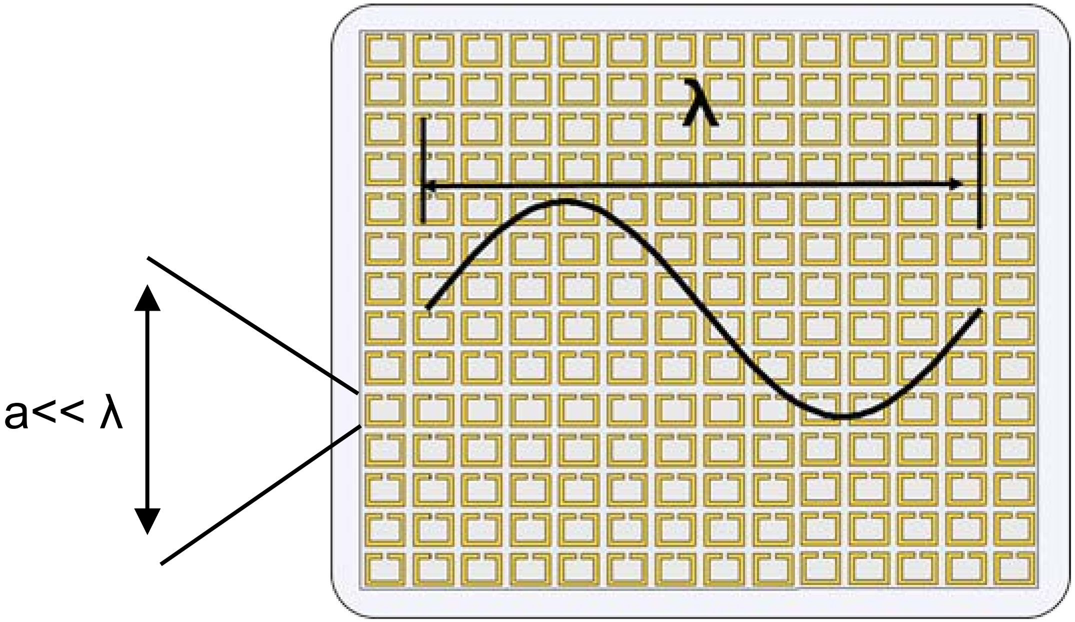
10^{-14} 10^{-12} 10^{-10} 10^{-8} 10^{-6} 10^{-4} 10^{-2} 1 10^{2} 10^{4}

Wavelength (m)

Visible

400 nm 550 nm 800 nm

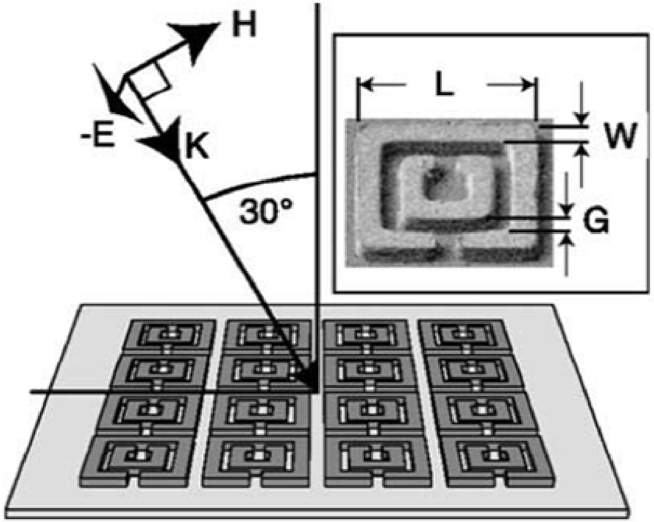
Metamaterials at higher frequencies



As the frequency ν increases, the wavelength λ decreases.

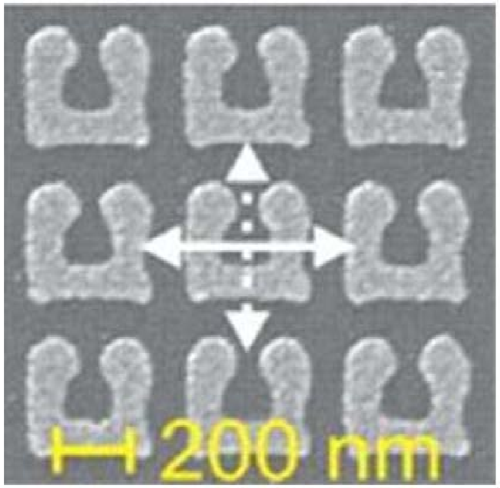
The size of the unit cells must be decreased accordingly.

Metamaterials at higher frequencies

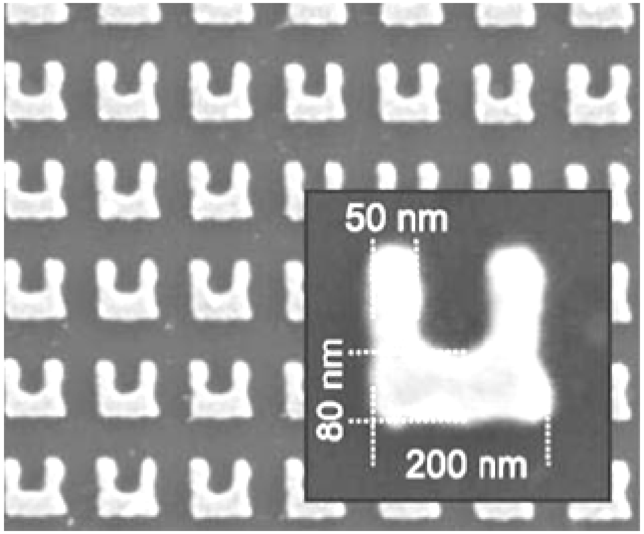


1 THZ (Yen et al., Science 303, 2004)

200 THz (Enkrich, 2005)



100 THz
(Linden et al.,
Science 306, 2004)



The issue of losses at high frequencies (visible to infrared range)

The metamaterial unit cells used in the realization of negative index media are metallic elements.

Metals are almost lossless at low frequencies (in the range spanning radio waves, microwaves and THz radiation).

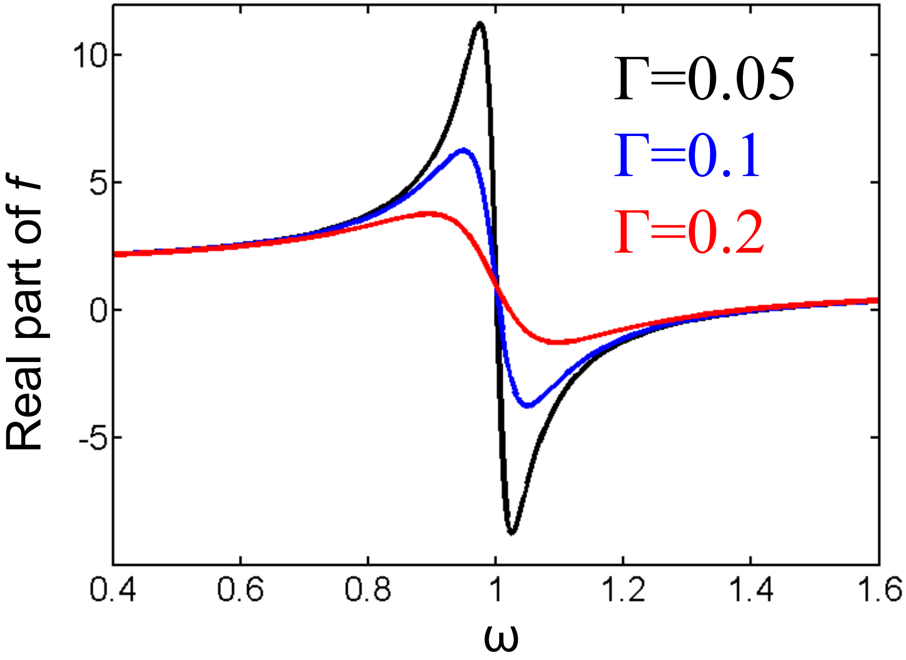
However metals become extremely lossy in the infrared and visible regions. What are the consequences in the metamaterial design?

The issue of losses at high frequencies (visible to infrared range)

A simple example - consider a pure Lorentzian resonance:

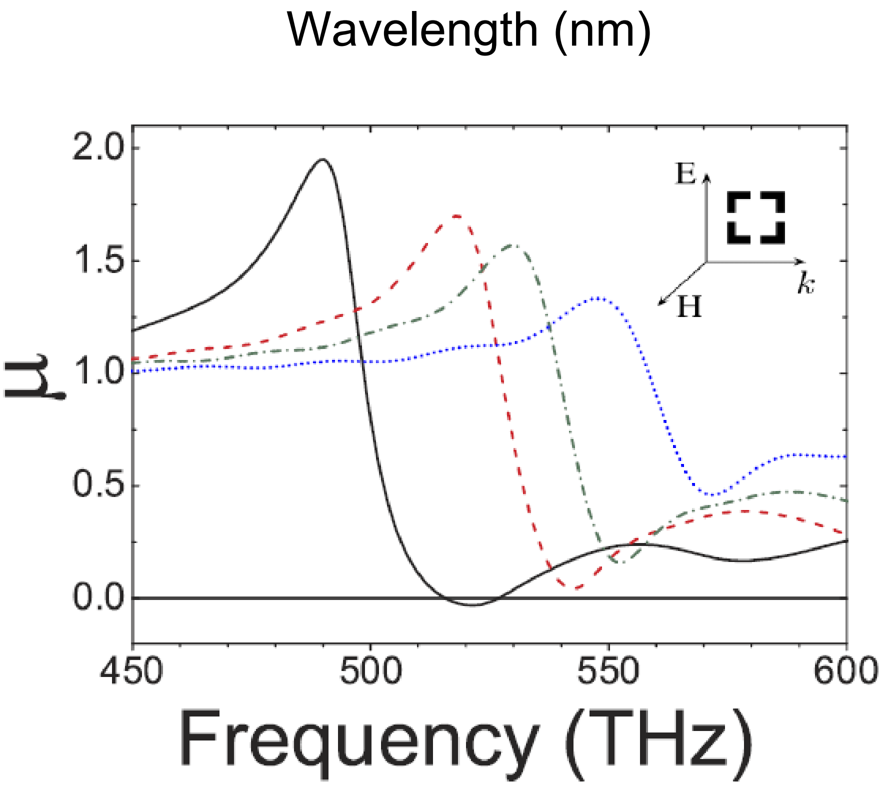
$$f(\omega) = 1 - \frac{1}{\omega^2 - 1 + i\Gamma\omega}$$

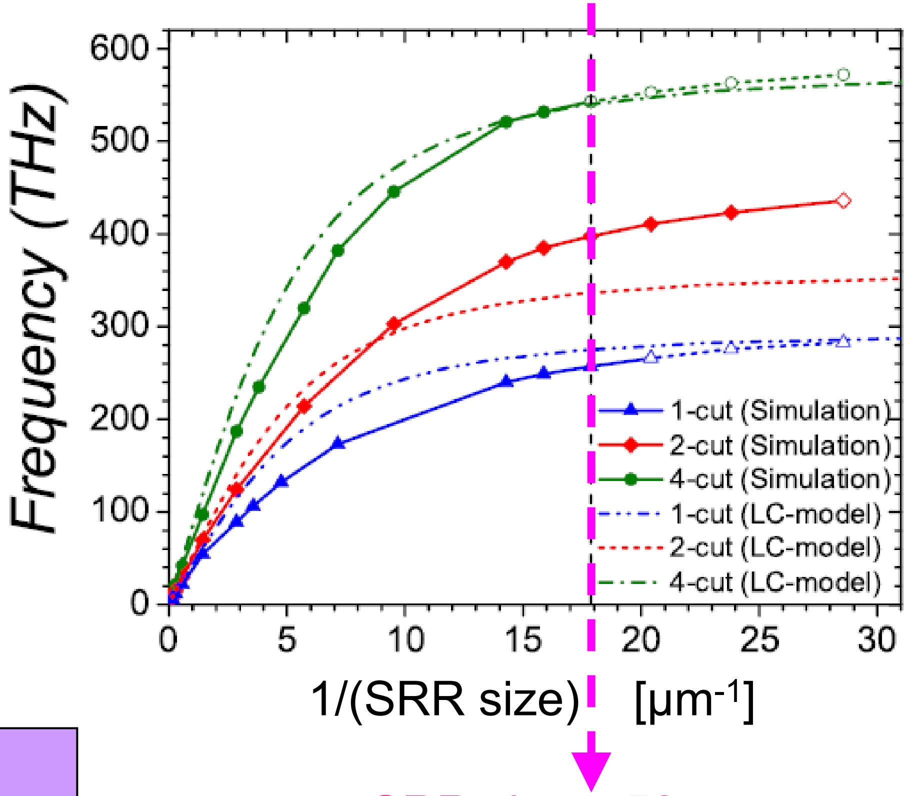
The imaginary part represents the losses of the system



Breakdown of linear scaling

Consider the magnetic response of an array of SRR resonators:





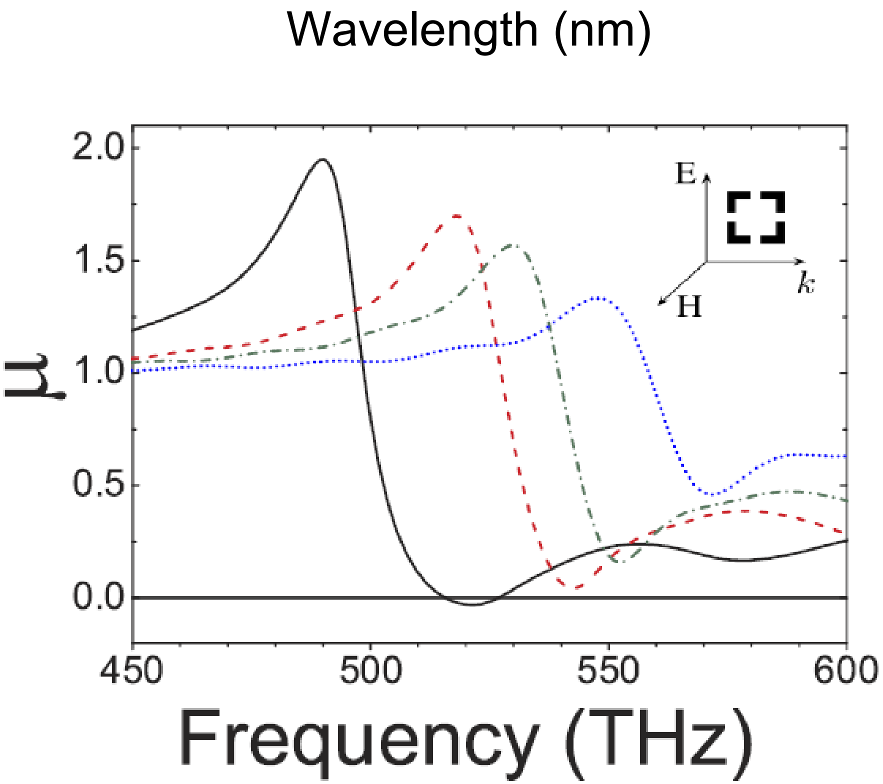
Due to resistive losses, the SRR response saturates

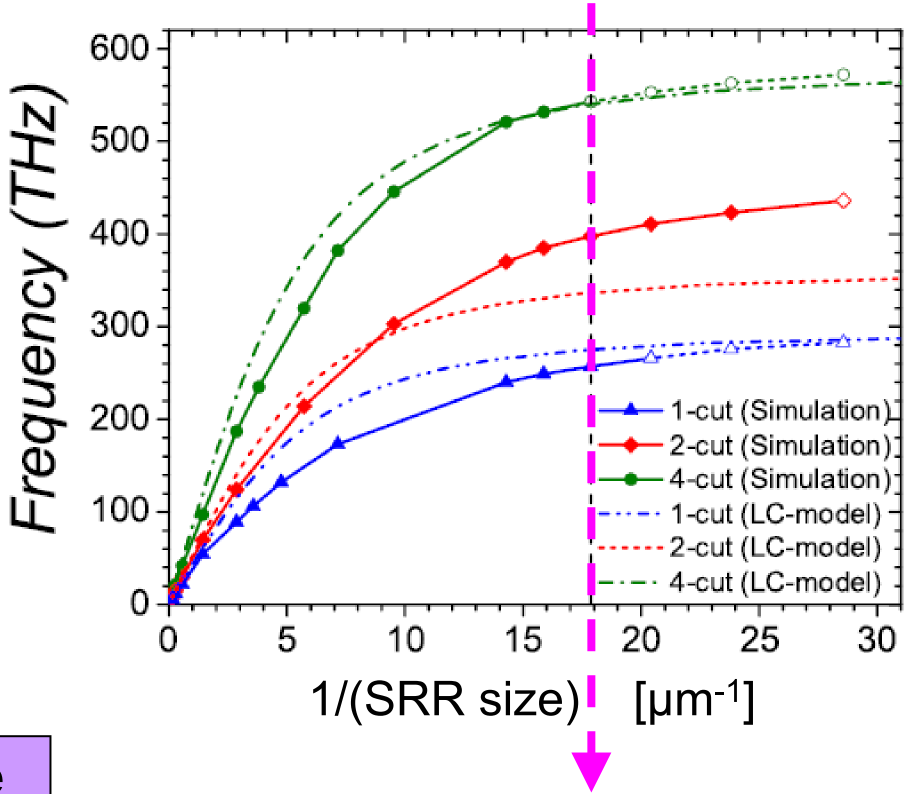
SRR size ~ 50 nm

Zhou et al., PRL 95 223902 (2005)

Breakdown of linear scaling

Consider the magnetic response of an array of SRR resonators:





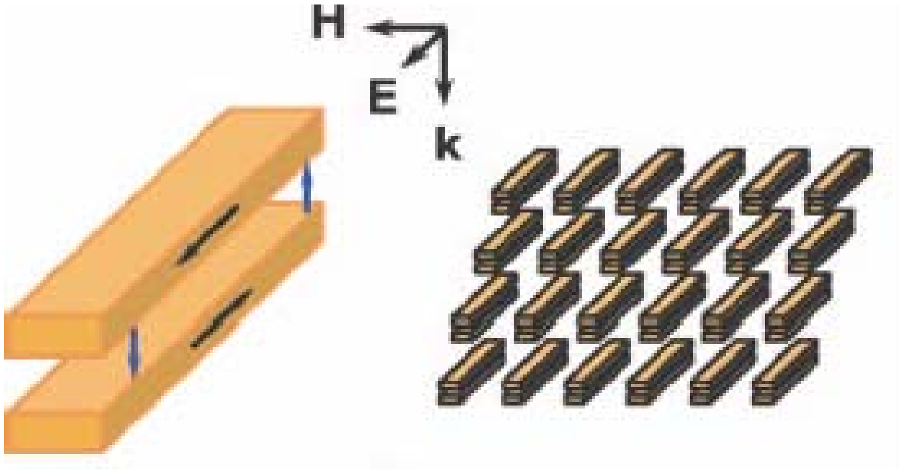
Resonances at optical wavelengths are surface plasmon modes

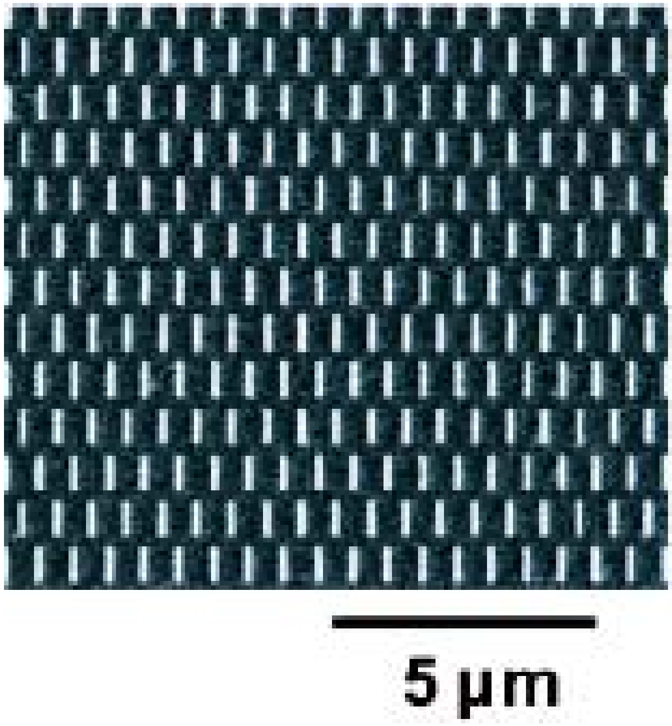
SRR size ~ 50 nm

Zhou et al., PRL 95 223902 (2005)

Artificial magnetism at optical wavelengths

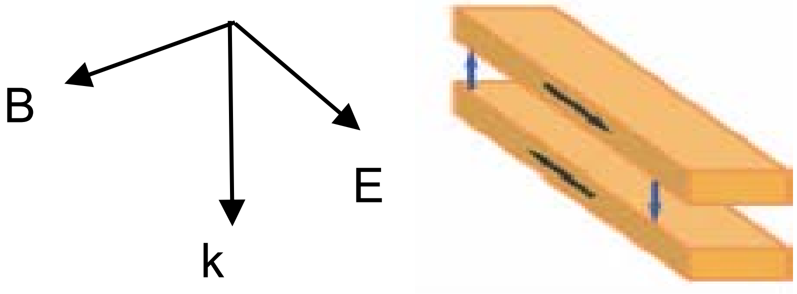
Is there a better geometry than planar SRRs to obtain μ<0?

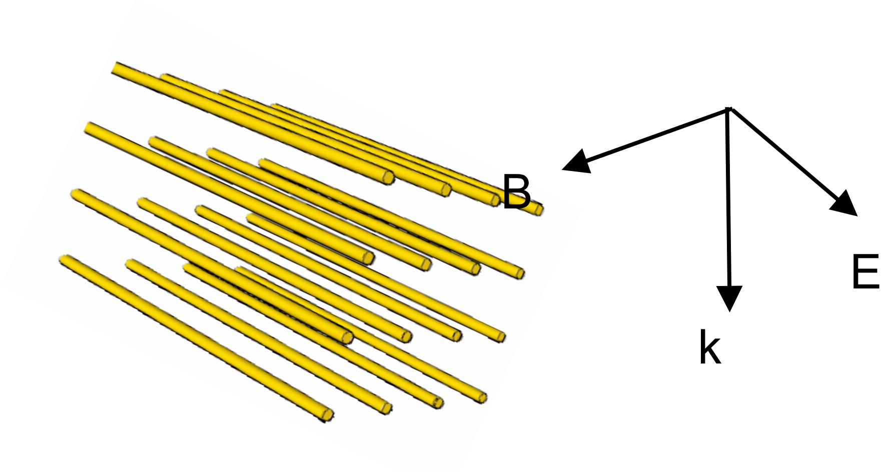




Shalaev et al., Opt. Lett. 30, 3356 (2005)

Negative refraction at optical wavelengths

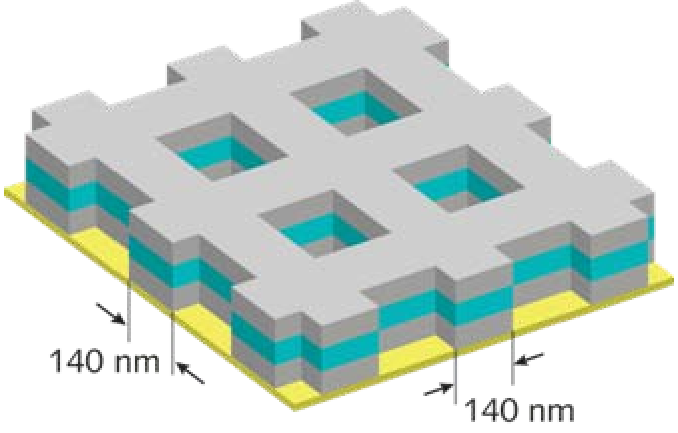




Pairs of metallic nanorods: provide μ<0

Wire medium: provides ε<0

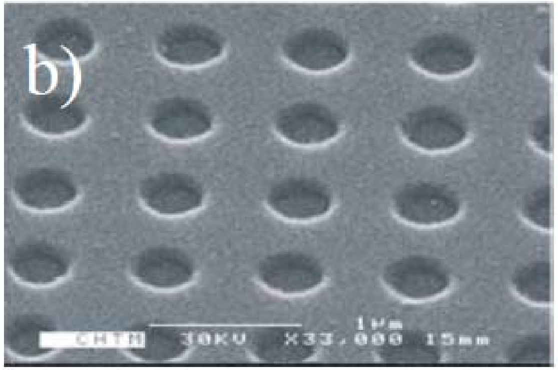
Combine the two structures:
one obtains a metamaterial « fishnet » providing n<0



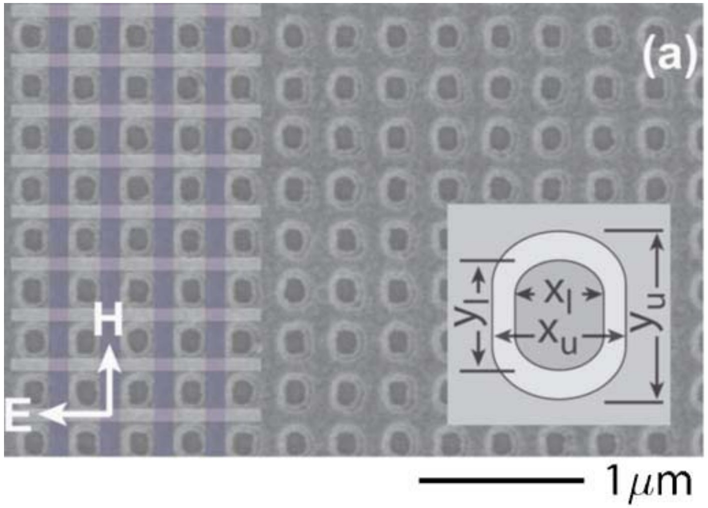
⬜ Metal

🟦 Dielectric

Fishnet metamaterial: Experimental samples

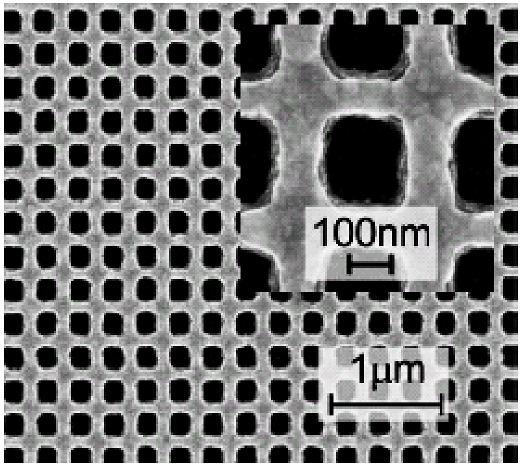


S Zhang PRL (2005)
n=-2+4i @ λ=2 nm



Chettiar et al., Opt. Lett. 2007

n=-1+i @ λ=800 nm



Dolling et al., Opt. Lett. 2006

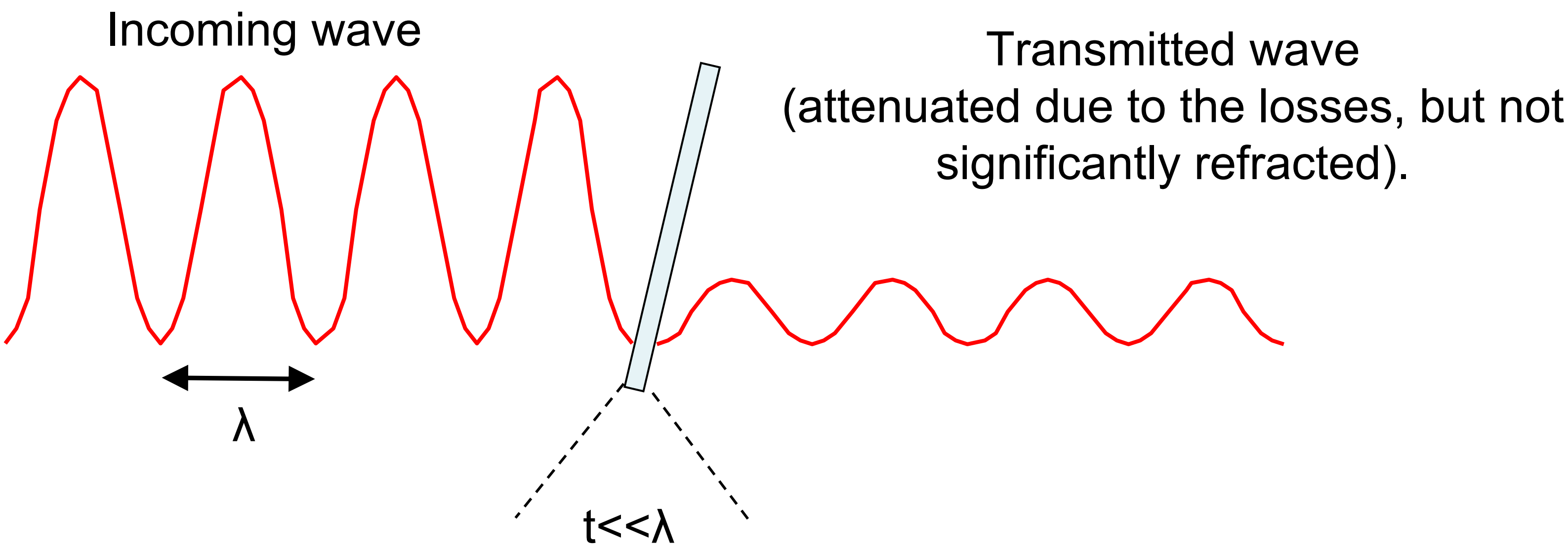
n=-1+0.3i @ λ=1.4 nm

Theses samples are planar structures fabricated by e-beam lithography

How to measure the index of refraction of optical NIM metamaterials?

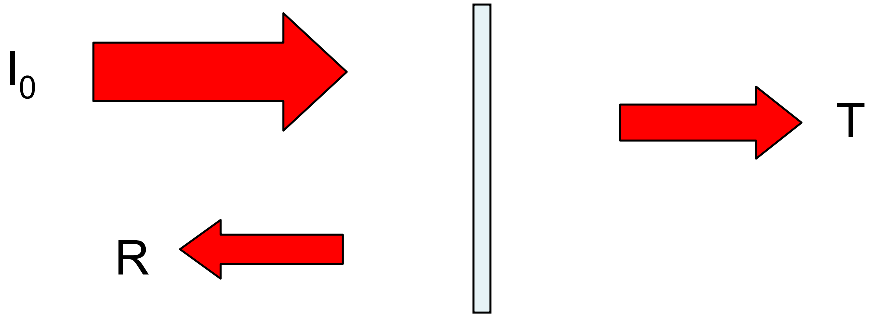
All the samples shown in the previous slide are planar structures of sub-wavelength thickness *t*.

These structures are too thin to refract light significantly so a direct measurement of their index is not possible.

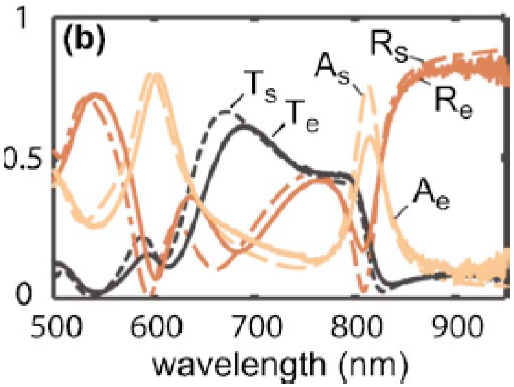


How to measure the index of refraction of optical NIM metamaterials?

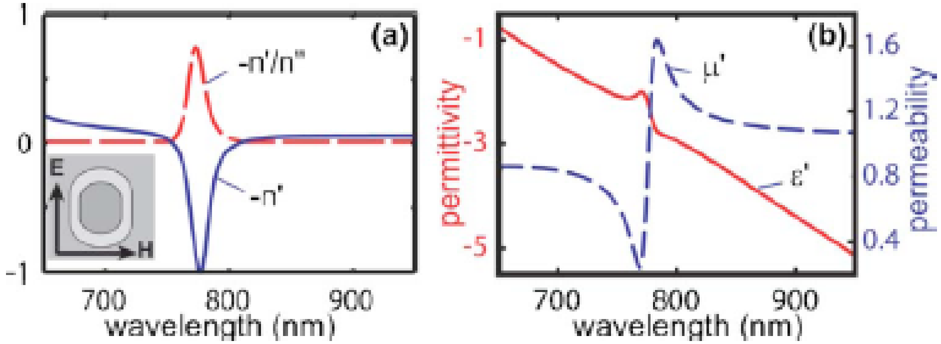
1) Measure optical reflection and transmission



2) Prove that the results agree with numerical simulations



3) Calculate ε and μ from the numerical model



From Chettiar et al., Opt Lett. 2007

Fishnet metamaterial: Summary

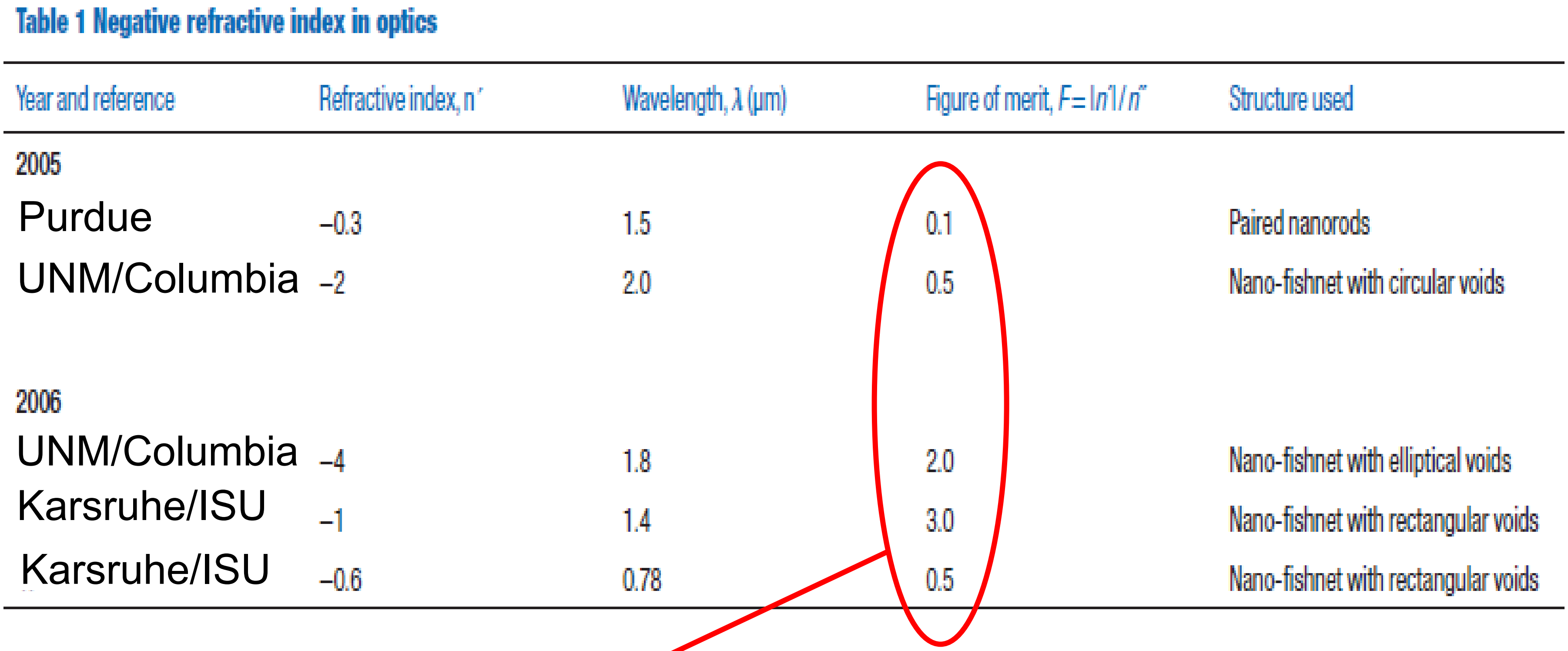


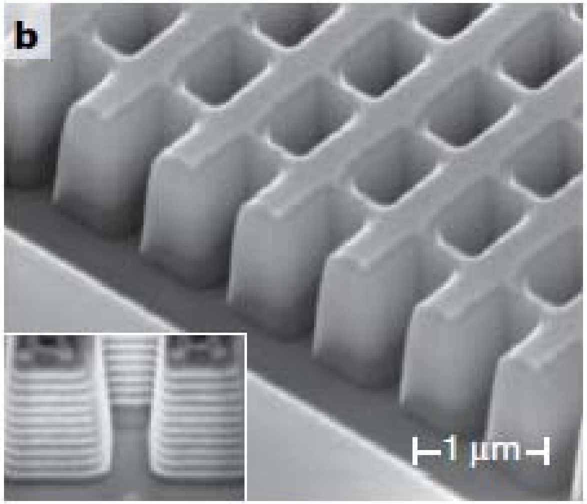
Table 1 Negative refractive index in optics

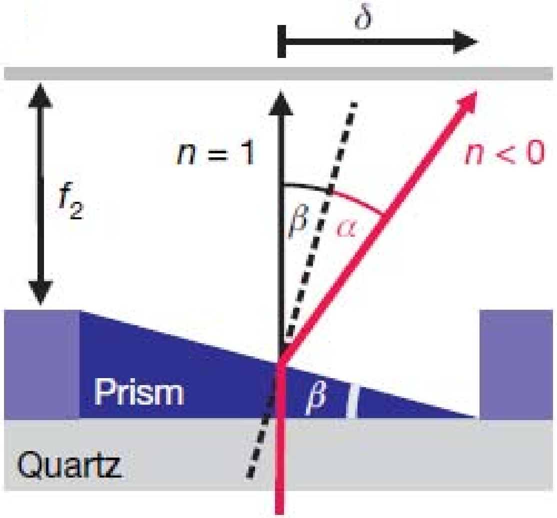
Year and reference	Refractive index, n'	Wavelength, λ (μm)	Figure of merit, $F = \|n'\| / n''$	Structure used
2005				
Purdue	−0.3	1.5	0.1	Paired nanorods
UNM/Columbia	−2	2.0	0.5	Nano-fishnet with circular voids
2006				
UNM/Columbia	−4	1.8	2.0	Nano-fishnet with elliptical voids
Karsruhe/ISU	−1	1.4	3.0	Nano-fishnet with rectangular voids
Karsruhe/ISU	−0.6	0.78	0.5	Nano-fishnet with rectangular voids

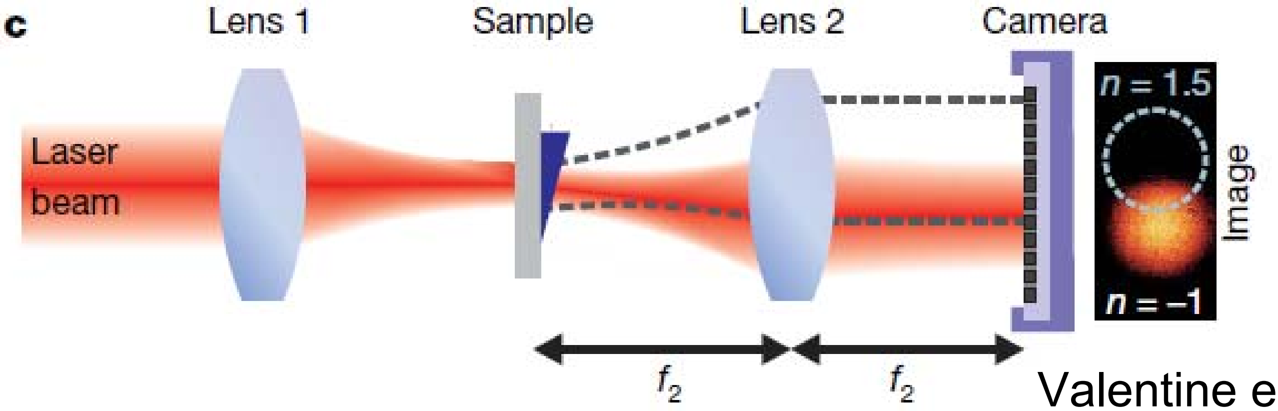
High resistive losses

V. Shalaev, Nature Photonics 1, 41 (2007)

Direct observation of negative refraction in a 3D fishnet structure





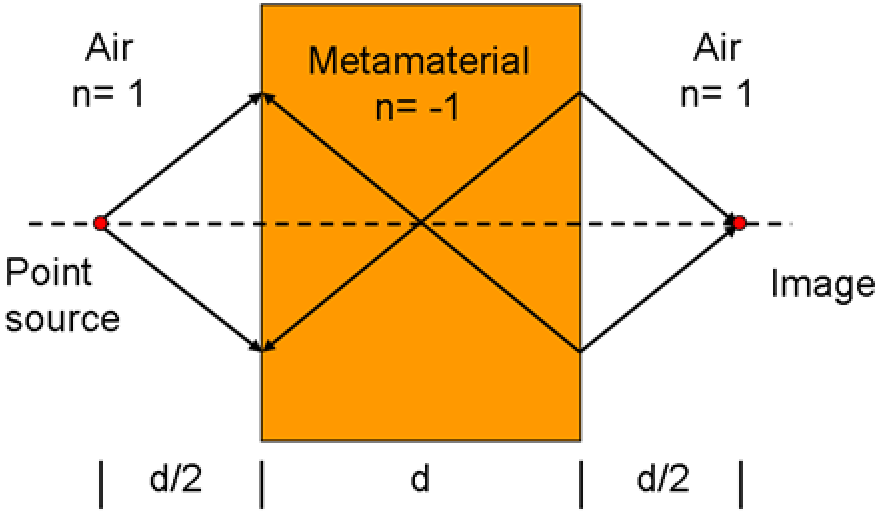


λ~1750 nm

Valentine et al., Nature 2008

Subwavelength imaging at optical wavelengths

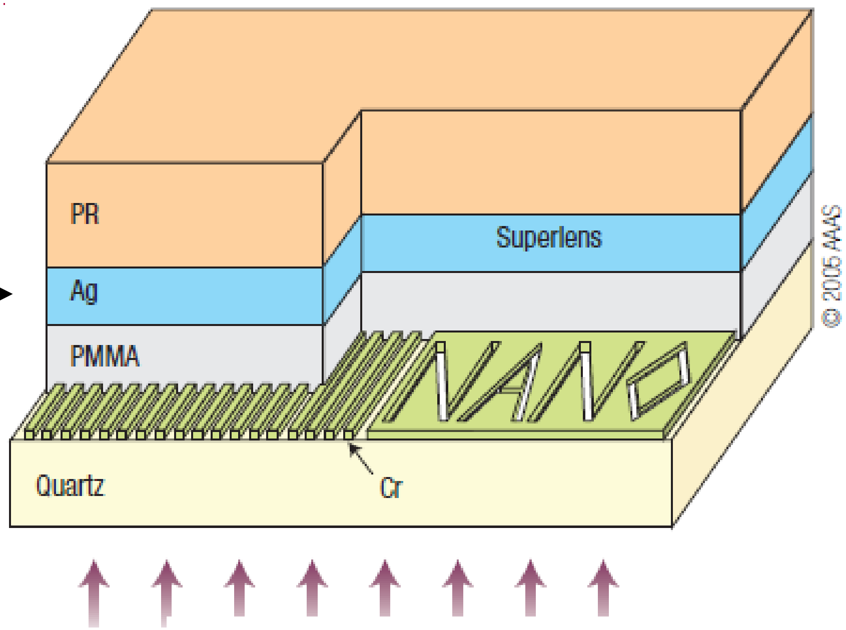
In the first part of the lecture, we saw that a slab of lossless NIM acts as a « perfect » lens.



However, current optical NIM metamaterials are extremely lossy, thus destroying the sub-diffraction resolution.

<u>Solution</u>: use an optically thin slab of metal (ε<0, μ>0) in the quasistatic limit.

Optical near-field superlens



Photoresist

Silver slab with sub-λ thickness

Object

365 nm illumination

Object: linewidth=40 nm

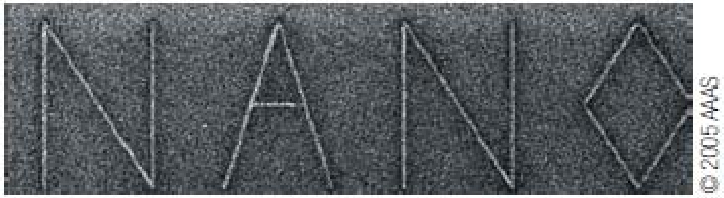
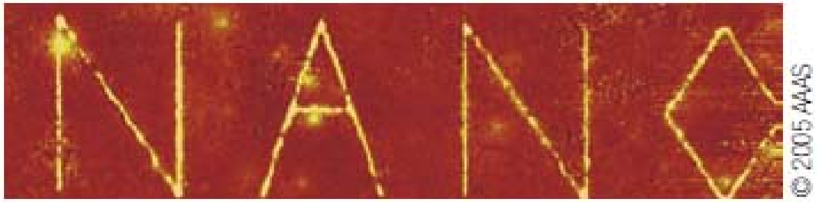
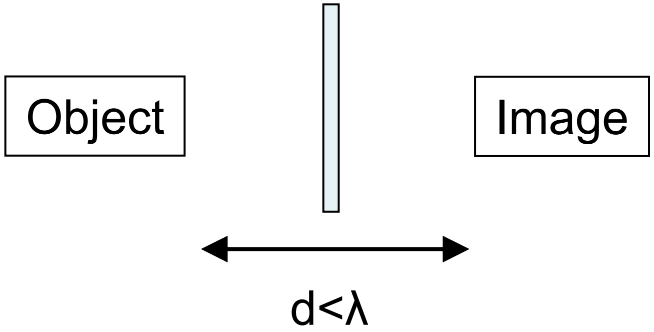


Image printed in photoresist: linewidth=89 nm



Far-field imaging with sub-λ resolution

Images created by the « perfect lens » and by the optical thin Ag slab superlens are separted by small (usually subwavelength) distances from the object.

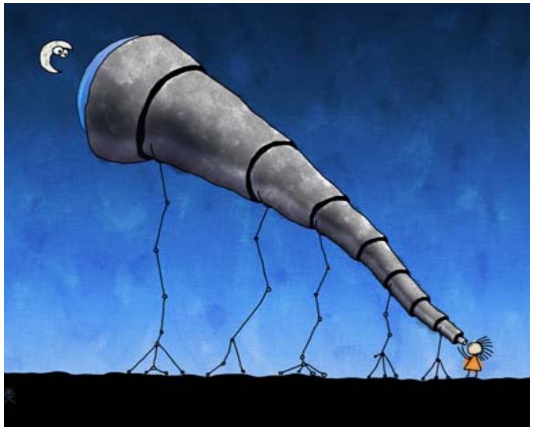


Object

Image

d<λ

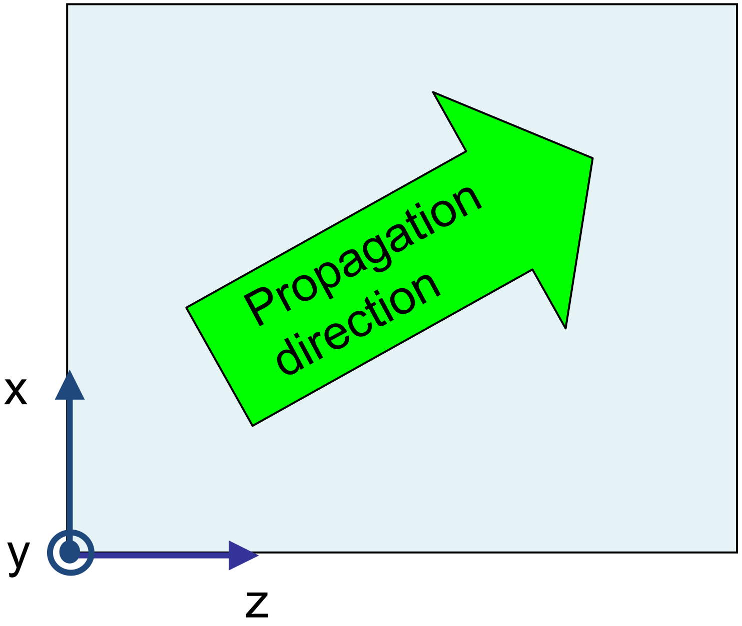
This is a serious limitation compared to conventional lenses



Is it possible to combine the advantages of the two approaches?

The optical hyperlens

Consider a plane wave propagating in the following anisotropic medium:



$$\varepsilon = \begin{pmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix} \qquad \mu = \begin{pmatrix} \mu_x & 0 & 0 \\ 0 & \mu_y & 0 \\ 0 & 0 & \mu_z \end{pmatrix}$$

We assume that a plane wave polarized along the y axis propagates in the x-z plane:

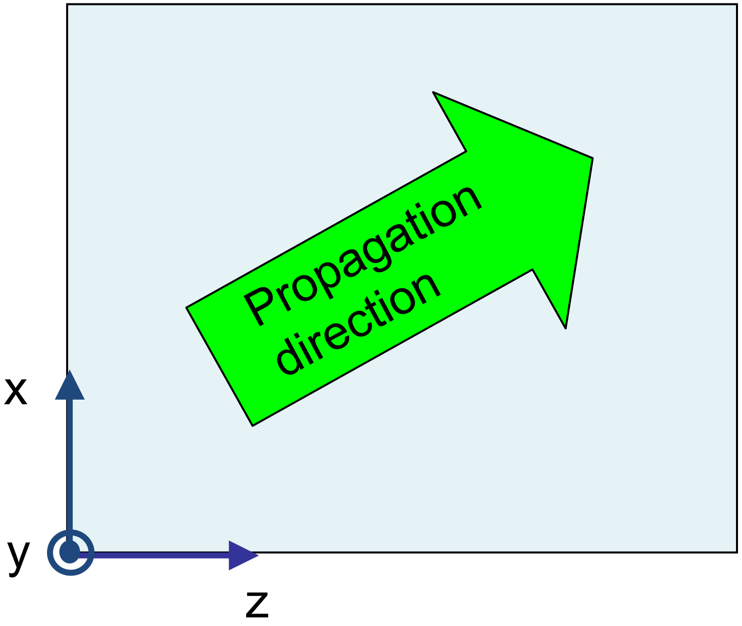
$$\vec{E} = [0, Ey, 0]\exp[i(k_x x + k_z z - \omega t)]$$

By inserting these expressions in Maxwell's equations, it can be shown that k, ε and μ are related by the following relation:

$$k_z^2 = \varepsilon_y \mu_x \frac{\omega^2}{c^2} - \frac{\mu_x}{\mu_z} k_x^2$$

The optical hyperlens

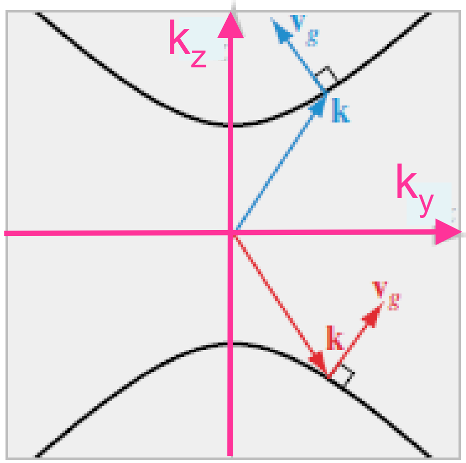
Consider a plane wave propagating in the following anisotropic medium:



$$k_z^2 = \varepsilon_y \mu_x \frac{\omega^2}{c^2} - \frac{\mu_x}{\mu_z} k_x^2$$

$$\text{if } \varepsilon_y \mu_x > 0 \text{ and } \mu_x / \mu_z < 0,$$

then k_x and k_z are always real (even those wavevectors with: $\|k\| > 2\pi / \lambda$)

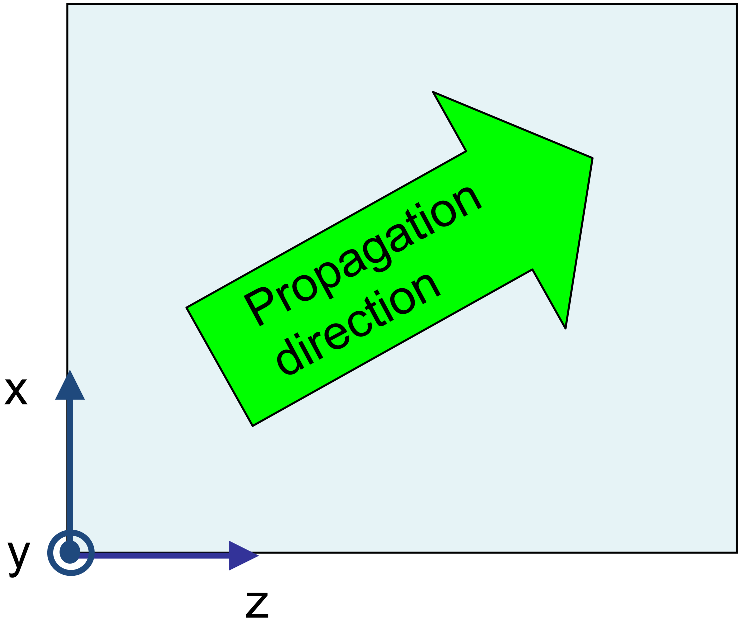


Such medium is called « indefinite » or « hyperbolic »

Schurig and Smith, PRL 90 077405 (2003)

The optical hyperlens

Consider a plane wave propagating in the following anisotropic medium:



$$k_z^2 = \varepsilon_y \mu_x \frac{\omega^2}{c^2} - \frac{\mu_x}{\mu_z} k_x^2$$

$$\text{if } \varepsilon_y \mu_x > 0 \quad \text{and} \quad \mu_x / \mu_z < 0,$$

then k_x and k_z are always real (even those wavevectors with: $\|k\| > 2\pi / \lambda$)

Consequence: even the near-field distribution of an object can propagate in a hyperbolic medium!

Schurig and Smith, PRL 90 077405 (2003)

Far-field imaging with sub-λ resolution: the optical hyperlens

How to construct a hyperbolic medium?

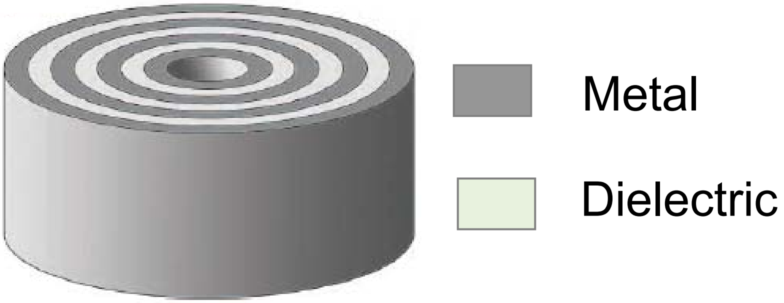
<u>At microwave frequencies</u>: use SRRs and wires (Smith and Schurig, 2003)

<u>In the visible range</u>: use alternating layers of metals and dielectrics (Podolskiy & Narimanov, 2005)

How to magnify an object with sub-diffraction resolution?

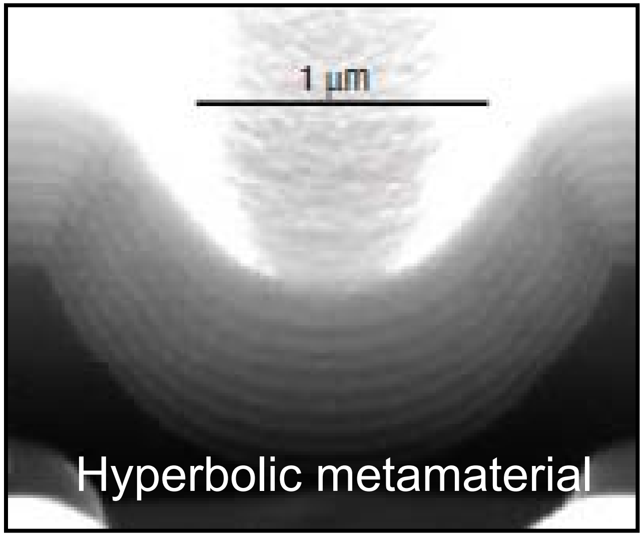
Use cylindrical geometry

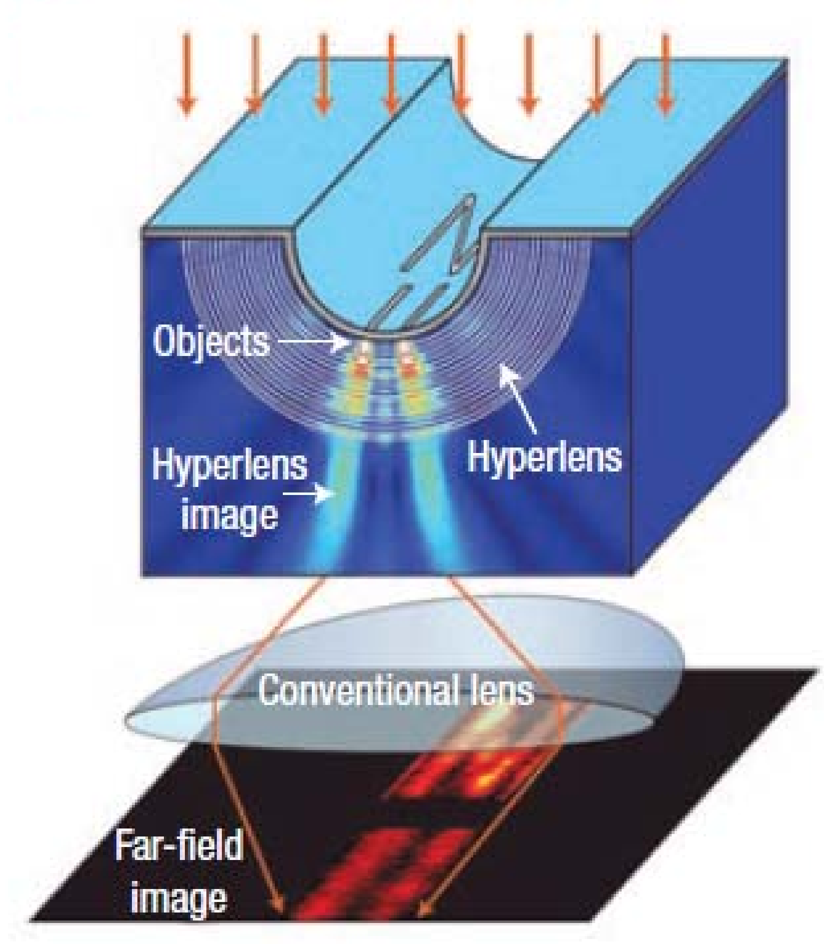
Jacob et al., Opt Express 14, 8247 (2006);
Salandrino et al., PRB 74 075103 (2006).



Far-field imaging with sub-λ resolution: the optical hyperlens

Experimental demonstration using stacks of Ag and Al_2O_3





Liu, Z. *et al. Science* **315,** 1686 (2007)

Beyond negative index metamaterials…

The optical hyperlens is an example of a composite structure with properties other than ε<0 and or μ<0.

The metamaterial approach is not limited to NIM structures! Remember that the most general constitutive relations that can be obtained are:

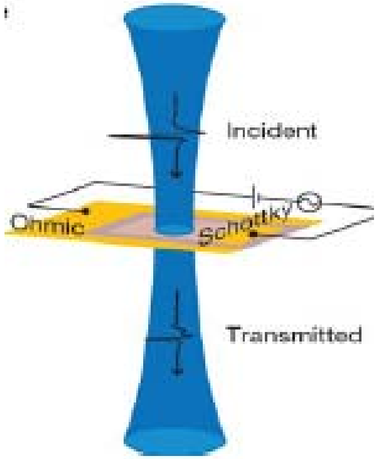
$$\vec{D} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \vec{E} + \begin{bmatrix} \zeta_{xx} & \zeta_{xy} & \zeta_{xz} \\ \zeta_{yx} & \zeta_{yy} & \zeta_{yz} \\ \zeta_{zx} & \zeta_{zy} & \zeta_{zz} \end{bmatrix} \vec{H}$$

$$\vec{B} = \begin{bmatrix} \xi_{xx} & \xi_{xy} & \xi_{xz} \\ \xi_{yx} & \xi_{yy} & \xi_{yz} \\ \xi_{zx} & \xi_{zy} & \xi_{zz} \end{bmatrix} \vec{E} + \begin{bmatrix} \mu_{xx} & \mu_{xy} & \mu_{xz} \\ \mu_{yx} & \mu_{yy} & \mu_{yz} \\ \mu_{zx} & \mu_{zy} & \mu_{zz} \end{bmatrix} \vec{H}$$

In fact, metamaterials offer a unique opportunity—the ability to independently control every tensor component of the constitutive relations.

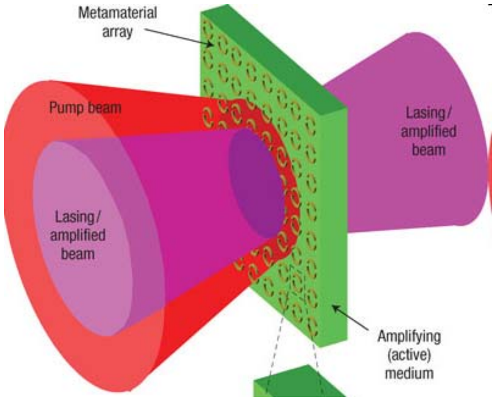
Beyond negative index metamaterials…

Ultra-fast switches



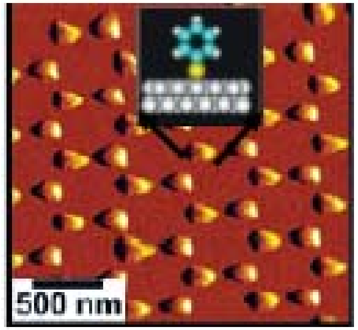
Chen et al.,
Nature (2006)

Photonic components



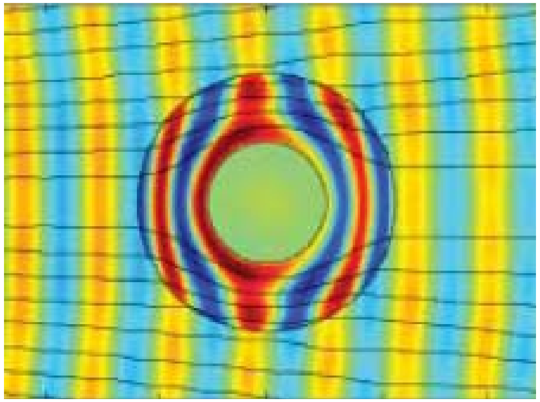
Zheludev et al., Nature Photon. (2008)

Sensing



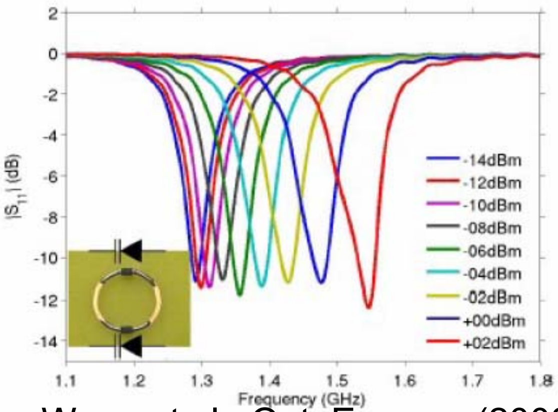
Van Duyne et al., MRS bulletin (2005)

Invisibility cloaks

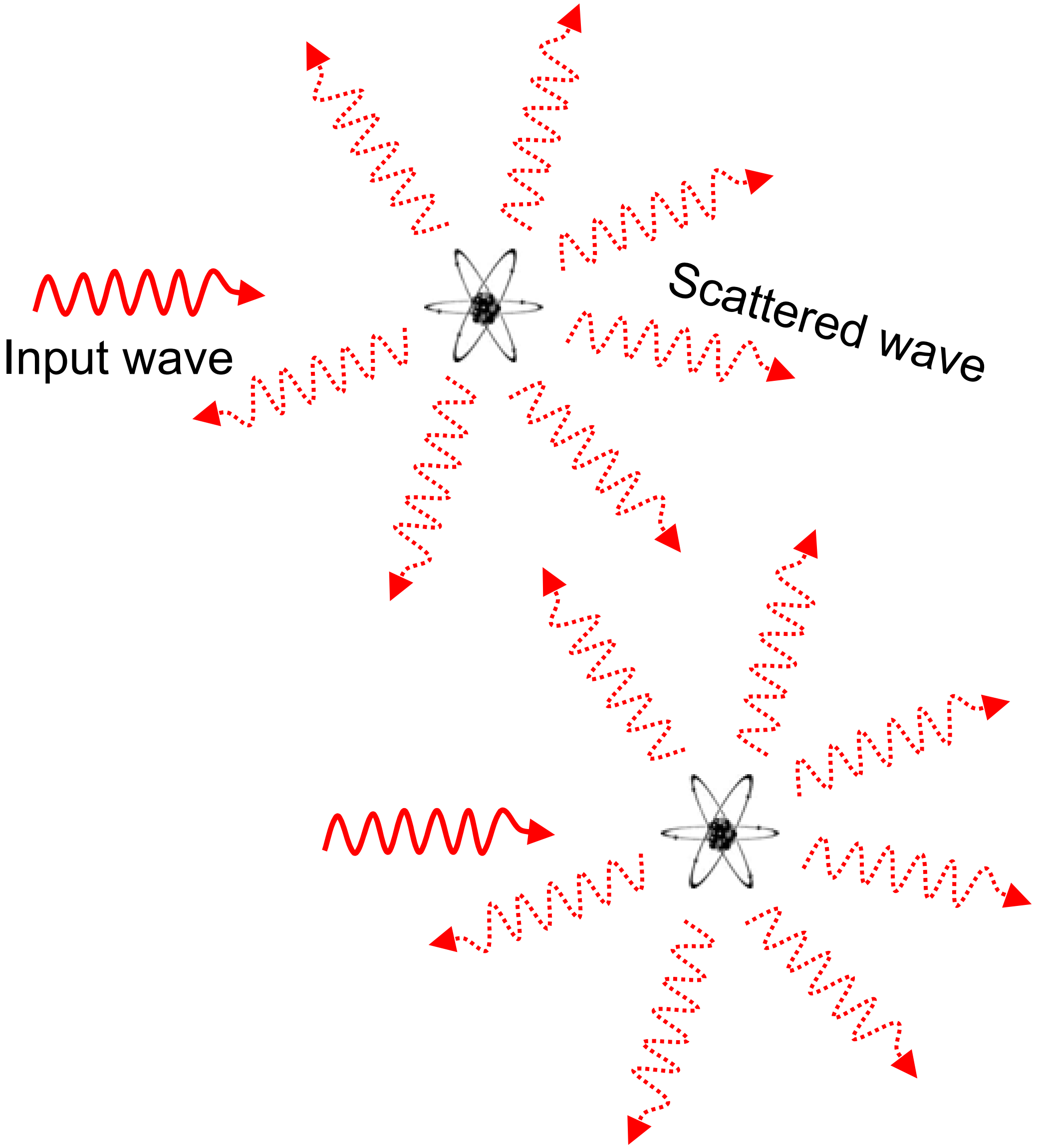


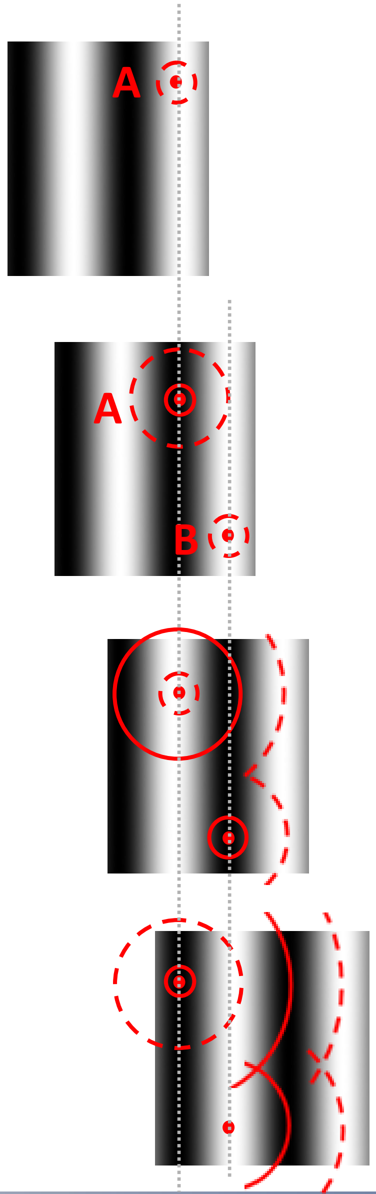
Pendry et al., Science (2006)
Leonhardt et al., Science(2006)

Enhanced nonlinear effects



Wang et al., Opt. Express (2008)


Input wave
Scattered wave


A
A
B

Huygens-Fresnel principle