N-Body Systems and the Nuclear Shell Model: II

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European Summer University
Strasbourg, June 29^{th}/July 4^{th}-2009

Correlations in nuclei

- for the description of nuclei, mean field is only the starting point
- the two body residual interaction (correlations) is reponsable for the detailled structure of nuclei
- in particular, correlations can induce coherent phenomena i. e. **collectivity**

Correlations in nuclei

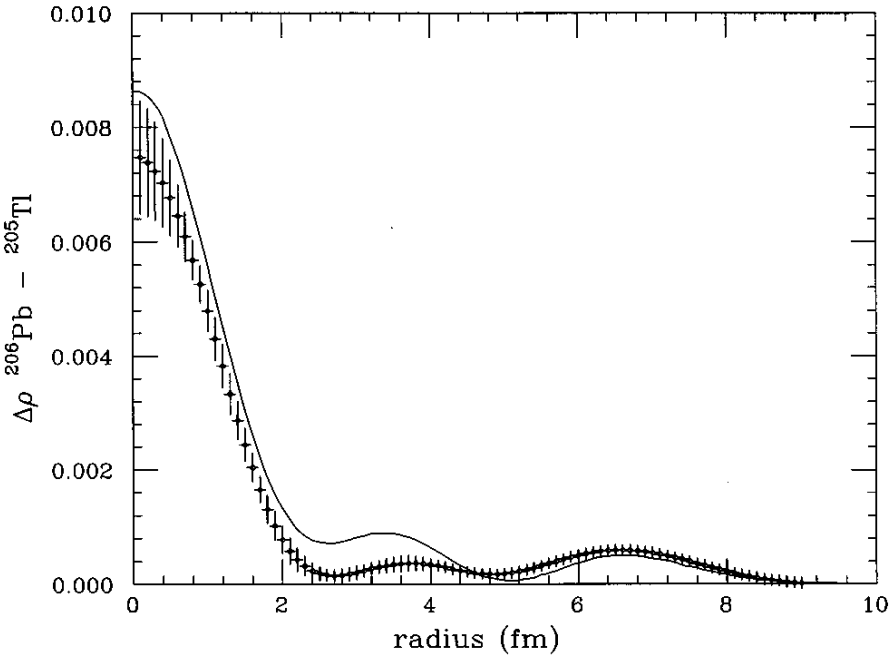
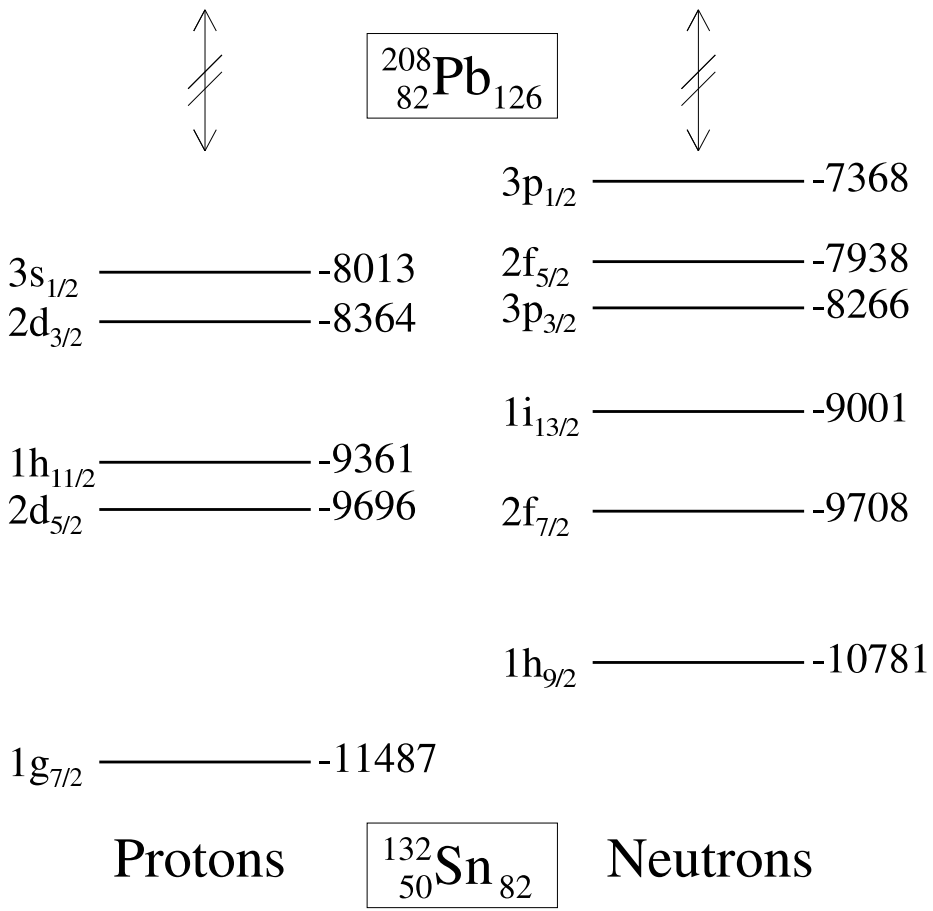


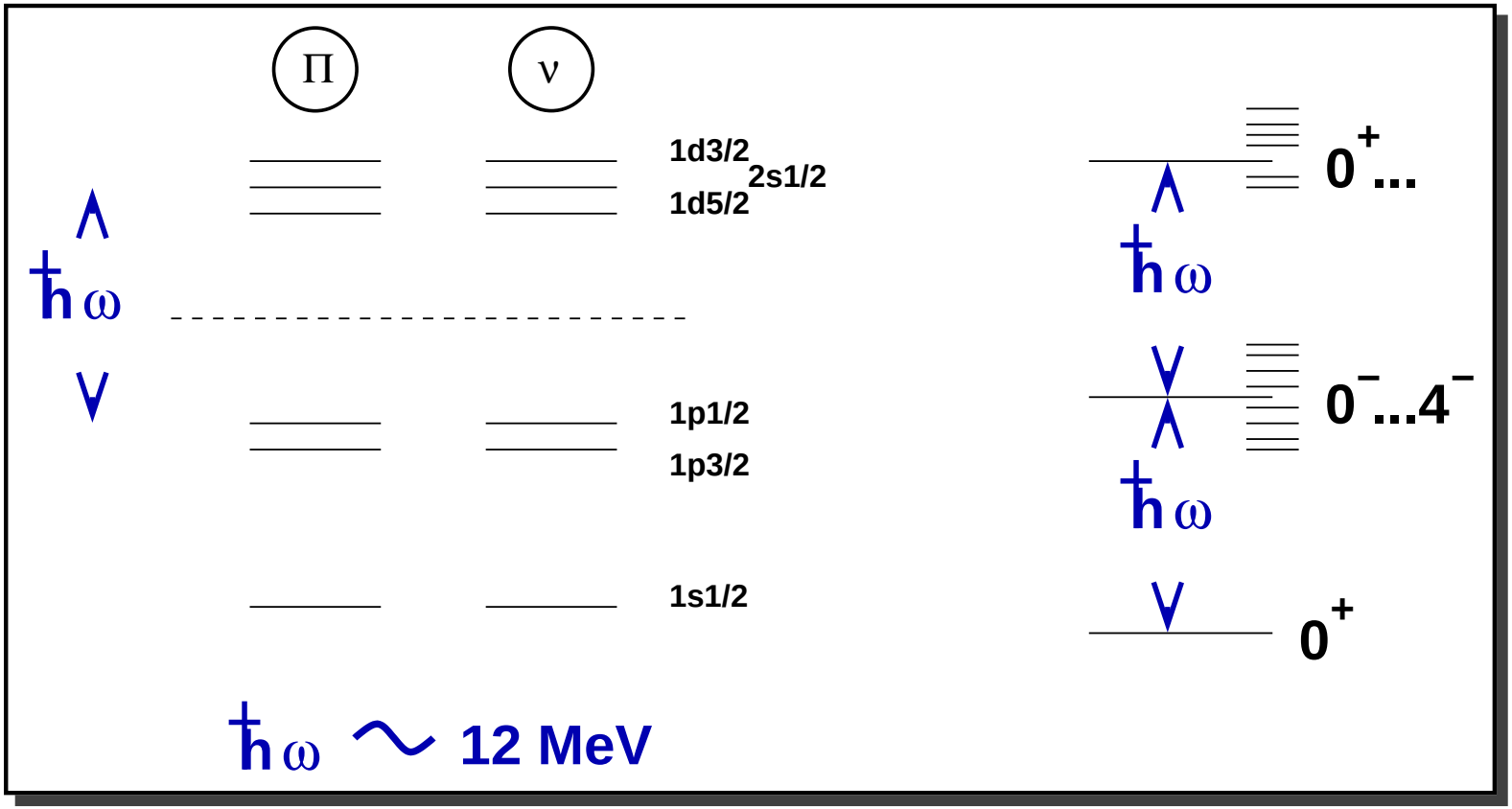
FIG. 3. Density difference between ^{206}Pb and ^{205}Tl. The experimental result of Cavendon *et al.* (1982) is given by the error bars; the prediction obtained using Hartree-Fock orbitals with adjusted occupation numbers is given by the curve. The systematic shift of 0.0008 fm^{-3} at $r \leqslant 4$ fm is due to deficiencies of the calculation in predicting the core polarization effect.



V. R. Pandharipande, I. Sick and P. K. A. deWitt Huberts, Rev. mod. Phys. **69** (1997) 981

Correlations in nuclei

Example in $^{16}0$:



Correlations in nuclei

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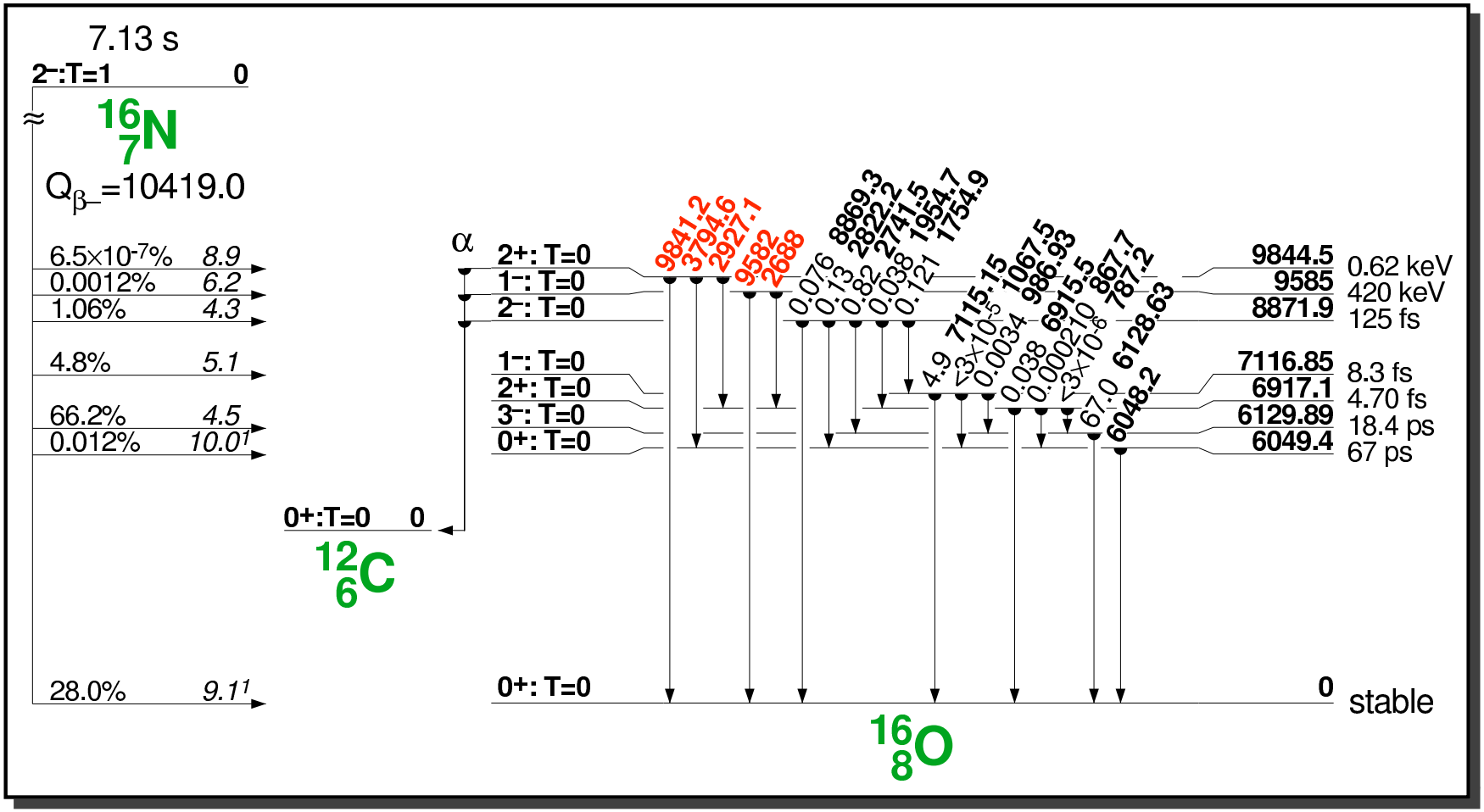

Correlations in nuclei

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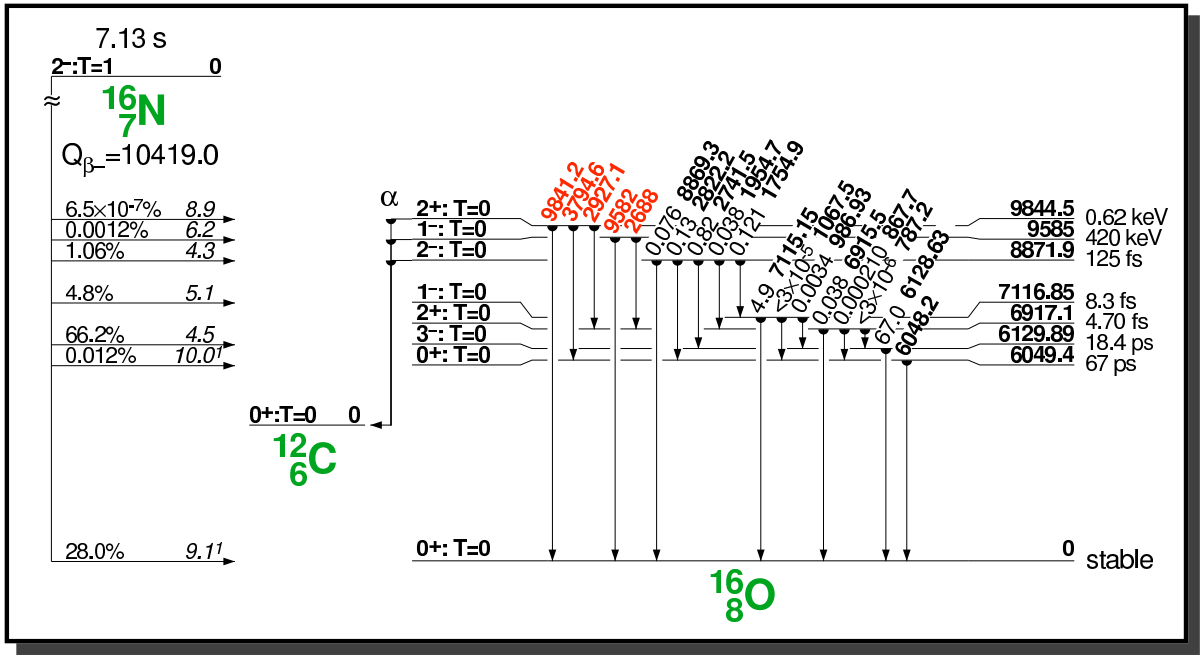
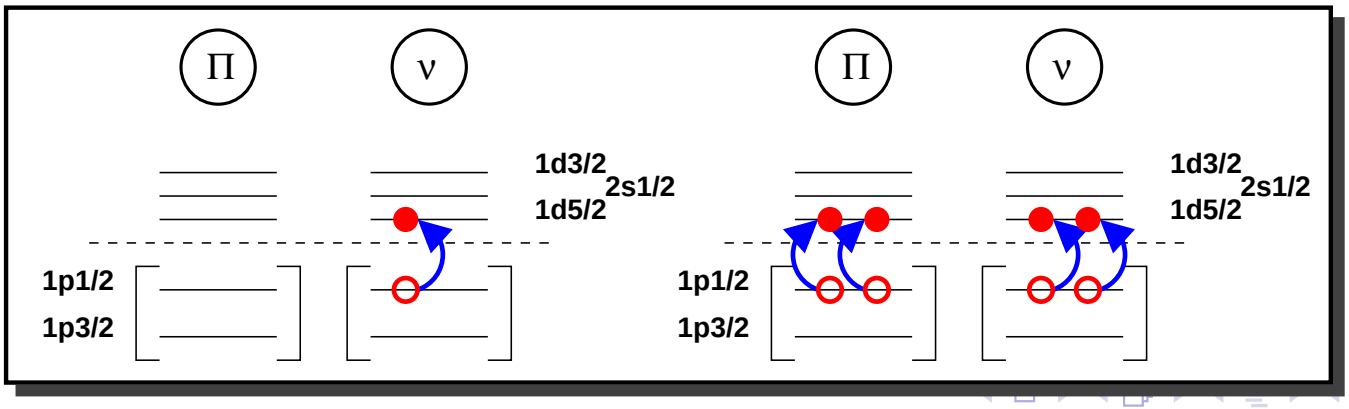
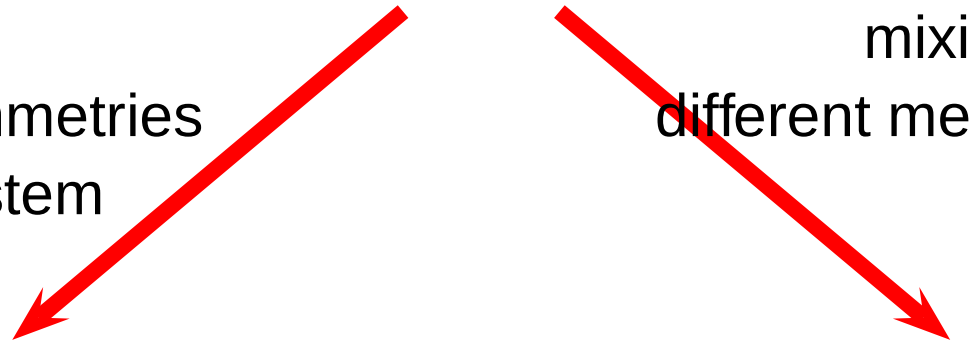




Correlations in nuclei

In order to incorporate the correlations, one has to go beyond mean-field

Spherical mean-field



breaking symmetries of the system

- Hartree-Fock Bogoliubov
- Nilsson
- Deformed Hartree-Fock

mixing different mean-field configuration

- Tamm-Dancoff
- RPA
- Interacting shell-model

Interacting Shell Model

The microscopic description of the nucleus we adopt is that of a non-(explicitely)-relativistic quantum many body system. Therefore we assume:

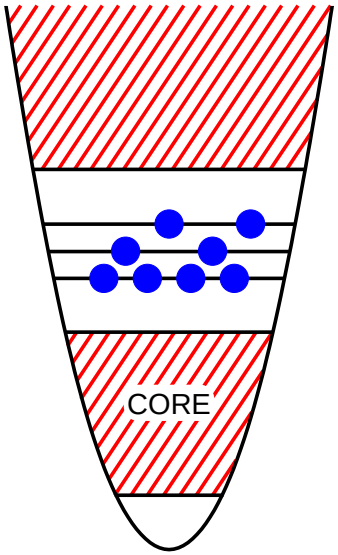
- nucleon velocities small enough to justify the use of non-relativistic kinematics
- hidden meson and quark-gluon degrees of freedom
- two body interactions

Interacting Shell Model

The succes of the independent particle model strongly suggest that the very singular free *NN* interaction can be regularized in the nuclear medium.
For a given number of protons and neutrons the mean field orbitals can be grouped in three blocks.

- Inert core: orbits that are always full.
- Valence space: orbits that contain the physical degrees of freedom relevants to a given property. The distribution of the valence particles among these orbitals is governed by the interaction.
- External space: all the remaining orbits that are always empty.

Shell Model Problem



- Define a valence space
- Derive an effective interaction

$$\mathcal{H}\Psi = E\Psi \rightarrow \mathcal{H}_{\text{eff}}\Psi_{\text{eff}} = E\Psi_{\text{eff}}$$

- Build and diagonalize the hamiltonian matrix.

In principle, all the spectroscopic properties are described simultaneously (Rotational band AND β decay half-life).

Shell model

A shell model calculation needs the following ingredients:

- A valence space
- An effective interaction
- A code to build and diagonalize the hamiltonian matrix

Obviously the last two points limit the choice of the valence space

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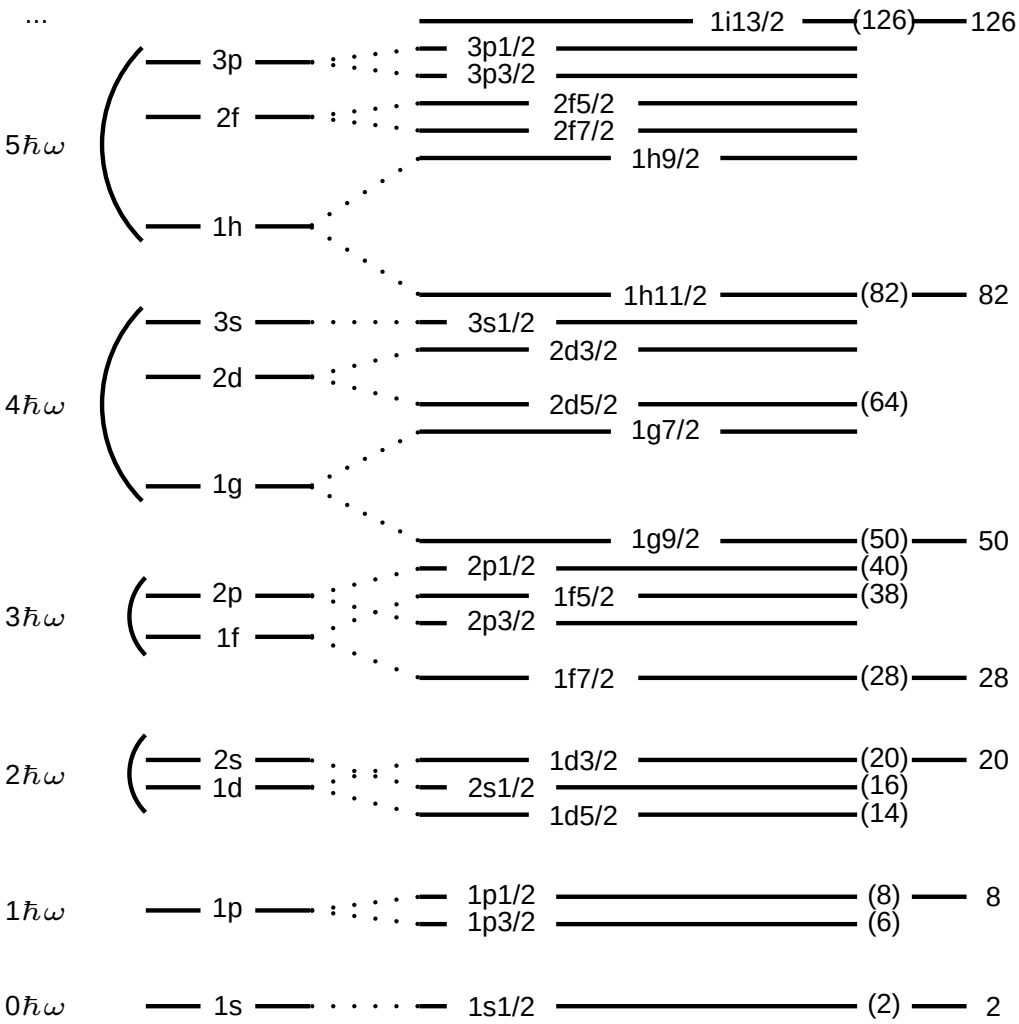
Valence space



Valence space

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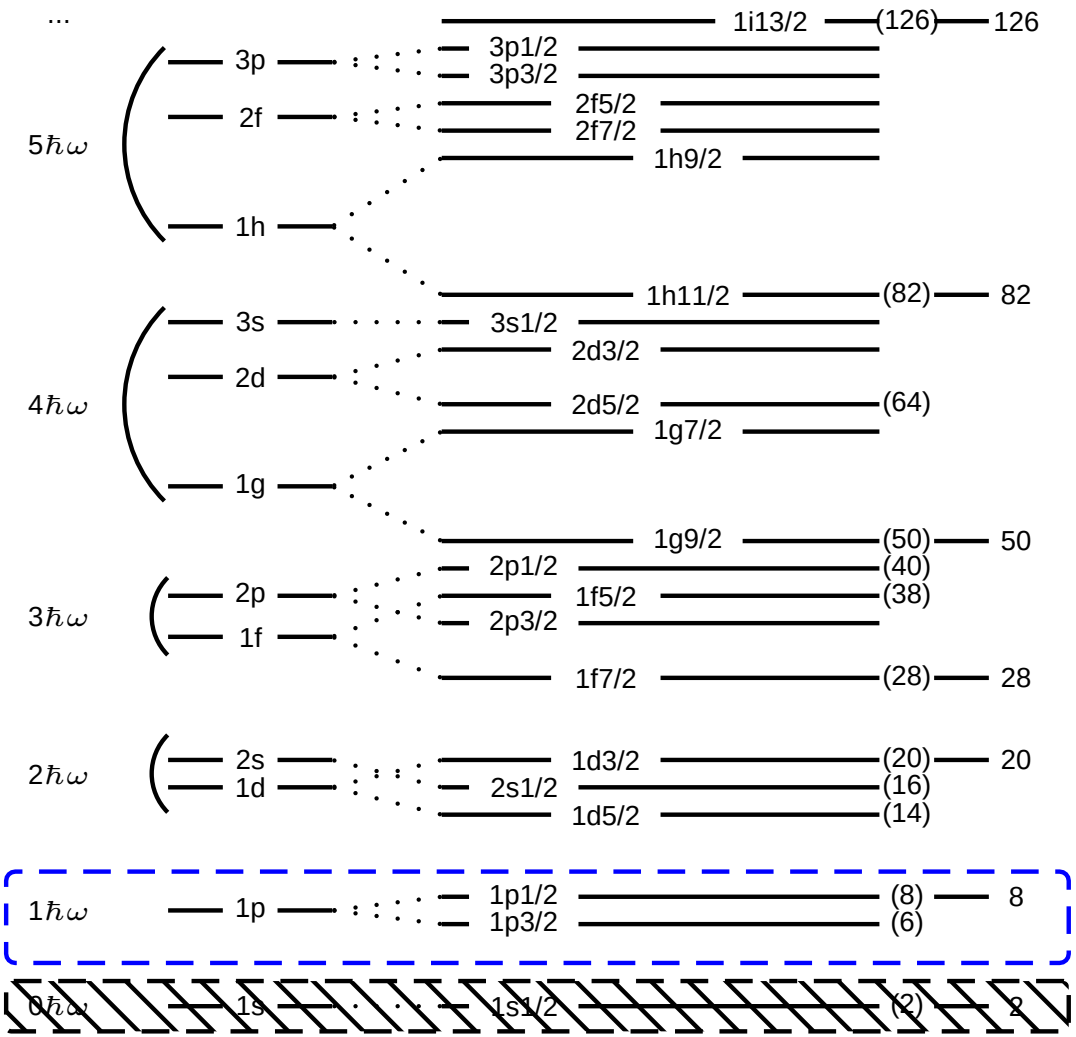
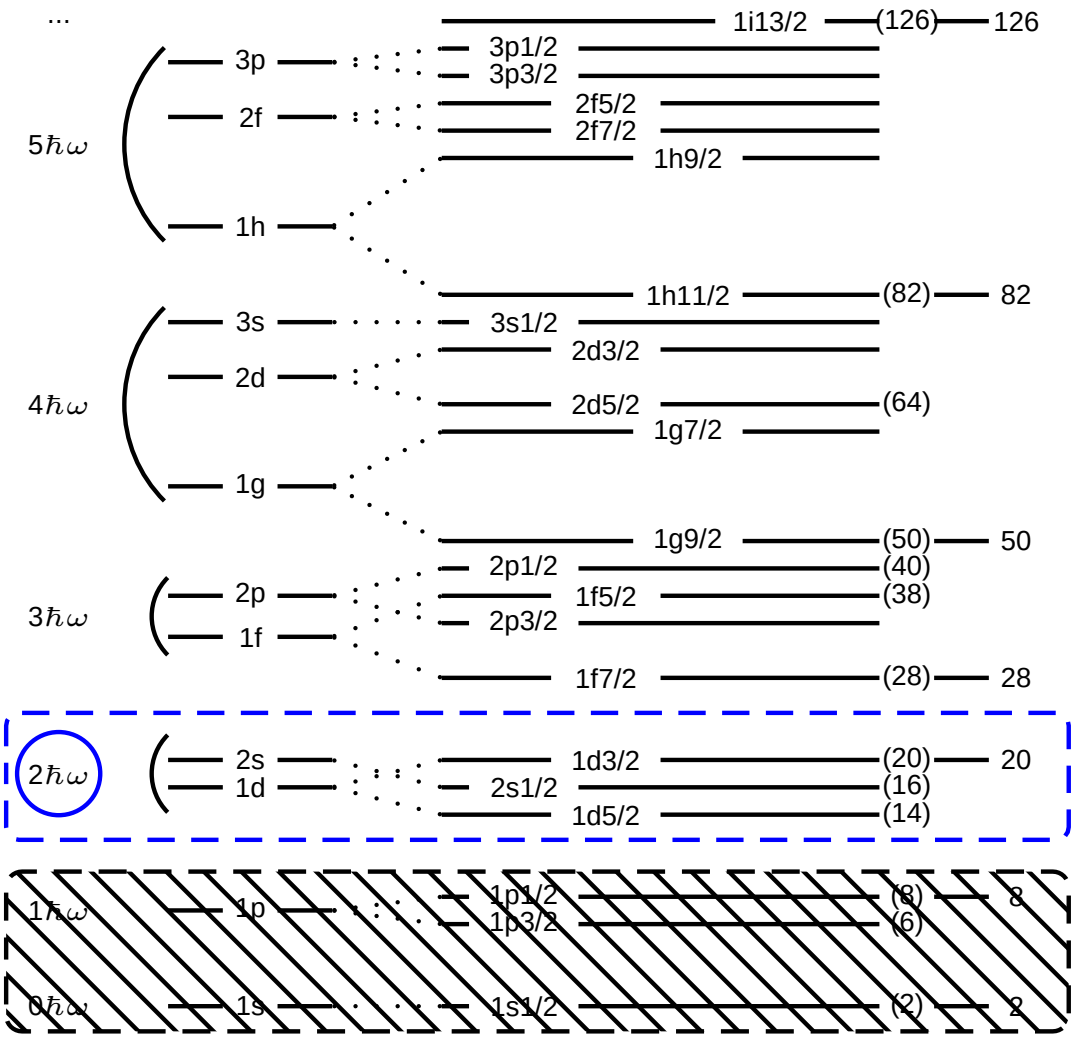


Valence space

- $4 \leq A \leq 16$ p shell

  Cohen-Kurath interaction

Valence space



Valence space

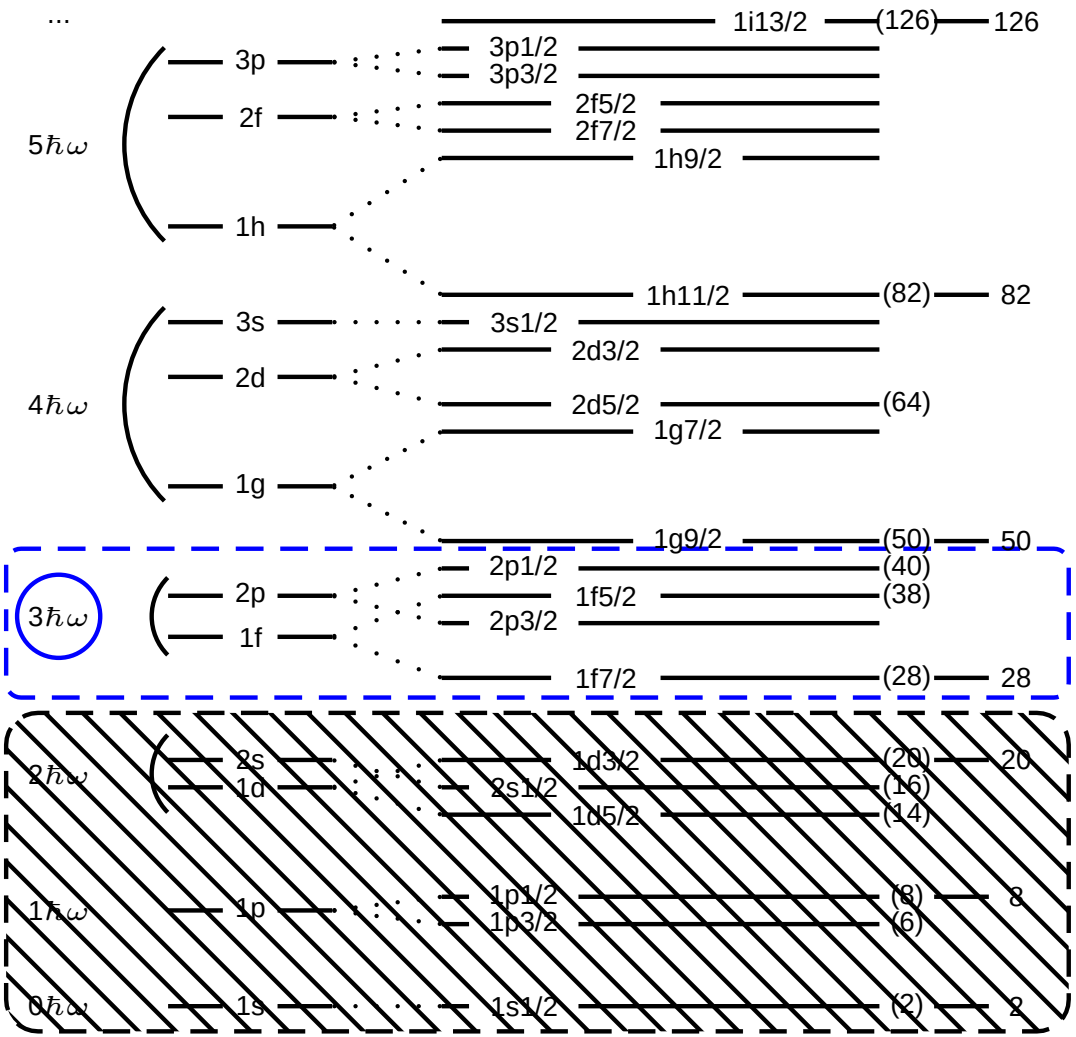
- $4 \leq A \leq 16$ p shell

  Cohen-Kurath interaction

- $16 \leq A \leq 40$ sd shell

  USD interaction

Valence space



Valence space

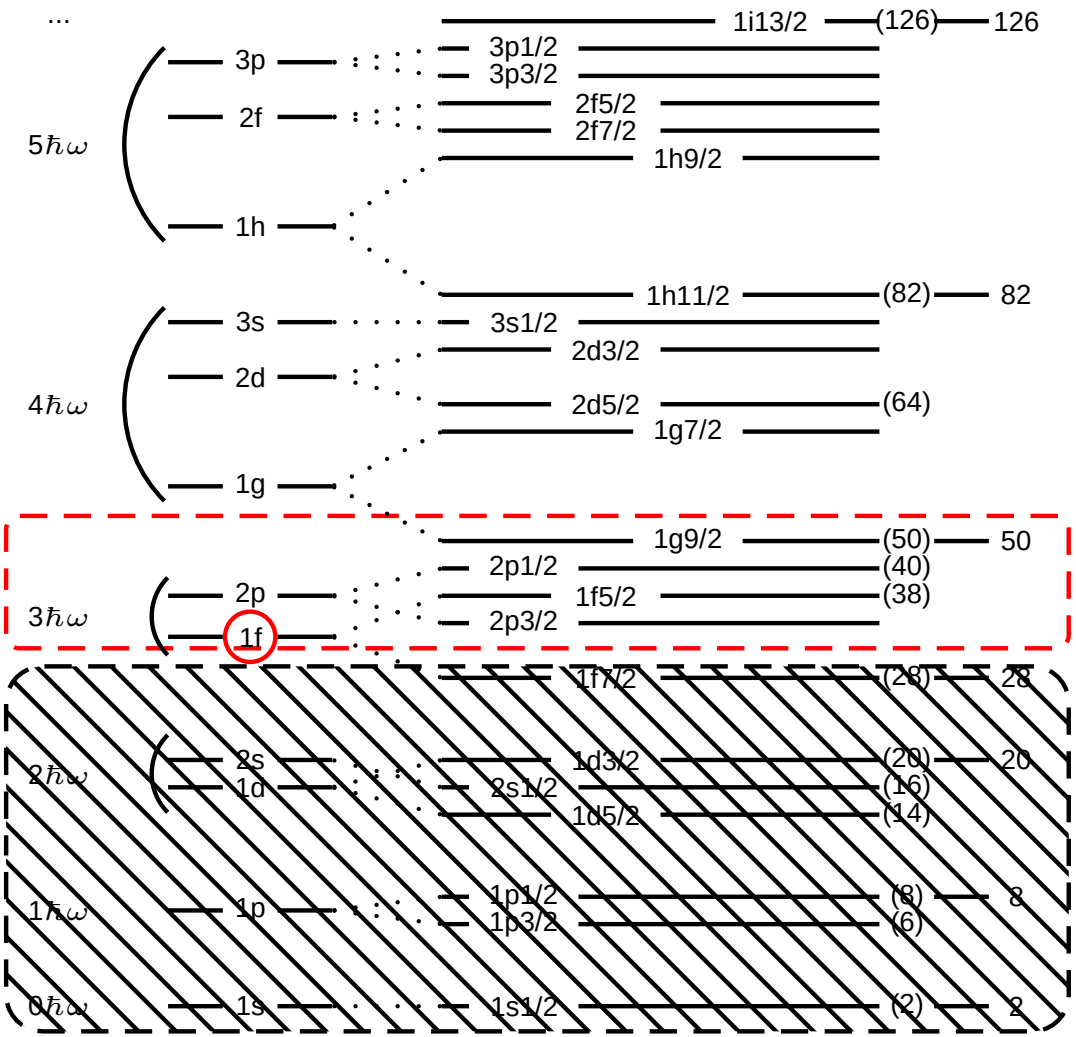
- $4 \leq A \leq 16$ p shell

  Cohen-Kurath interaction
- $16 \leq A \leq 40$ sd shell

  USD interaction
- $40 \leq A \leq 80$ pf shell

  KB3, GXPF1 interactions

Valence space



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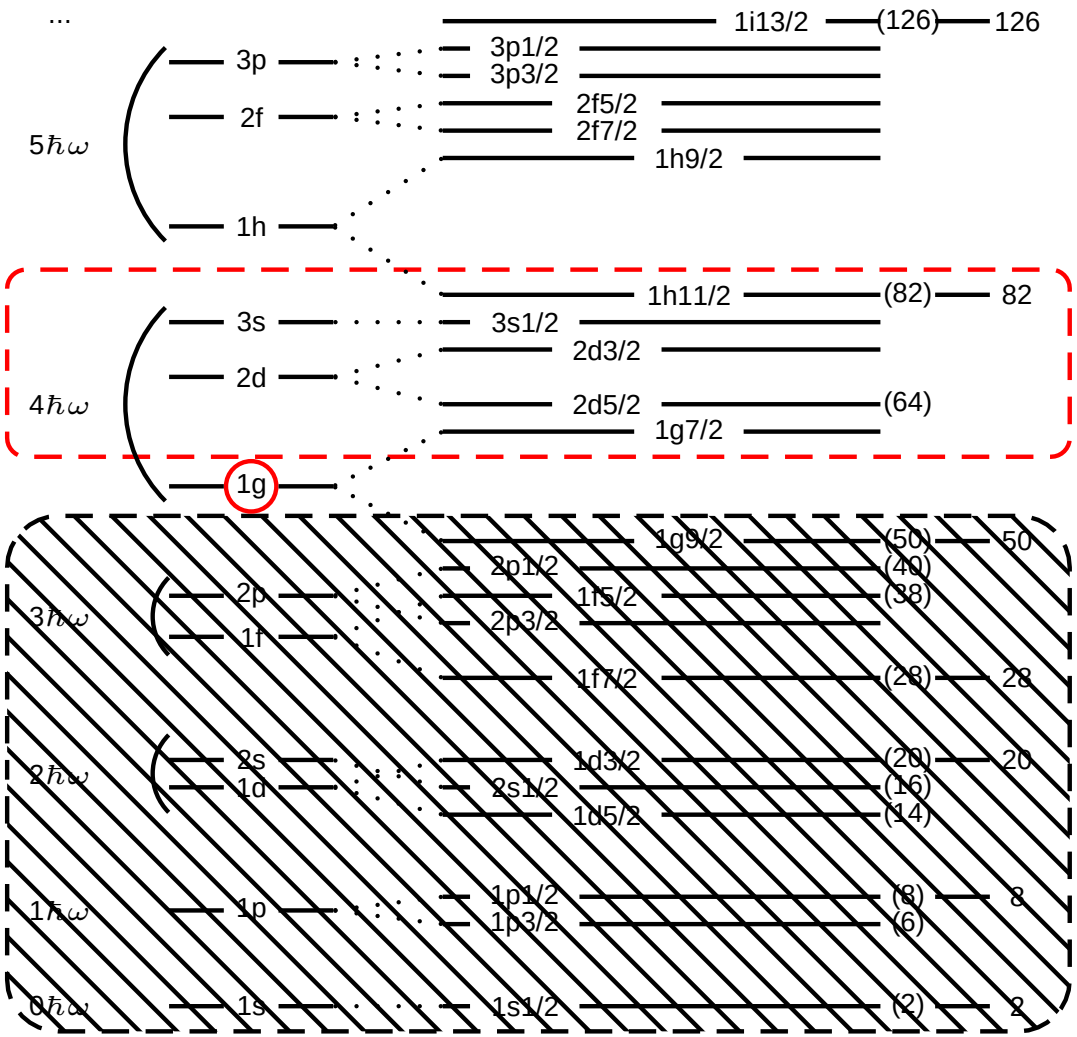
- **Heavier nuclei :**
  Spin-orbite shell closures

  28, 50, 82, 126

  Transition between ^{40}Ca and ^{100}Sn

Valence space



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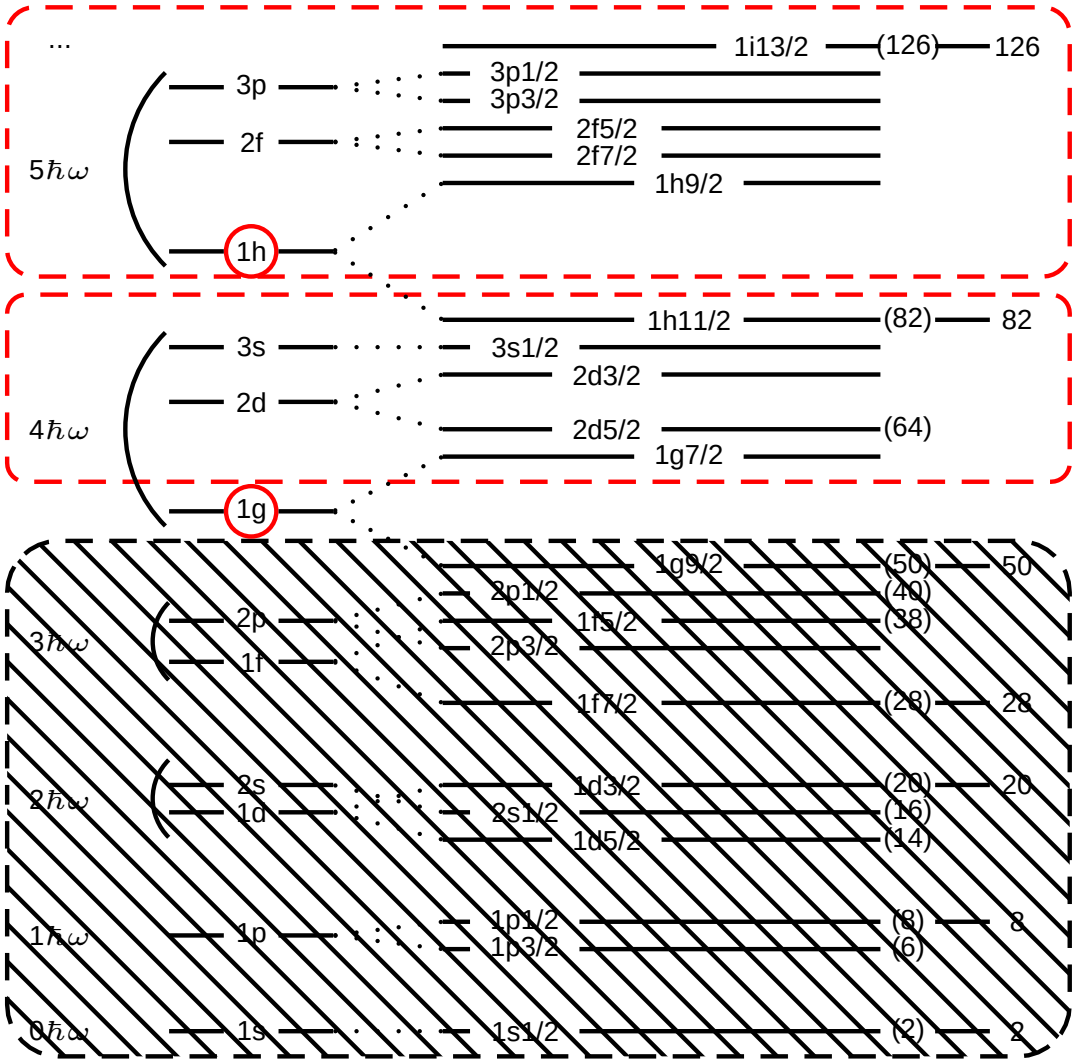

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The interaction in second quantization

The hamiltonian can be written as:

$$\mathcal{H} = \sum_{k=1}^{A} T(k) + \sum_{k<l}^{A} W(k,l)$$

Introducing a mean field $\sum_{k}^{A} U(k)$, it can be written as:

$$\mathcal{H} = \underbrace{\sum_{k=1}^{A} \{T(k) + U(k)\}}_{\mathcal{H}^{(0)}} + \underbrace{\{\sum_{k<l}^{A} W(k,l) \; - \; \sum_{k=1}^{A} U(k)\}}_{V}$$

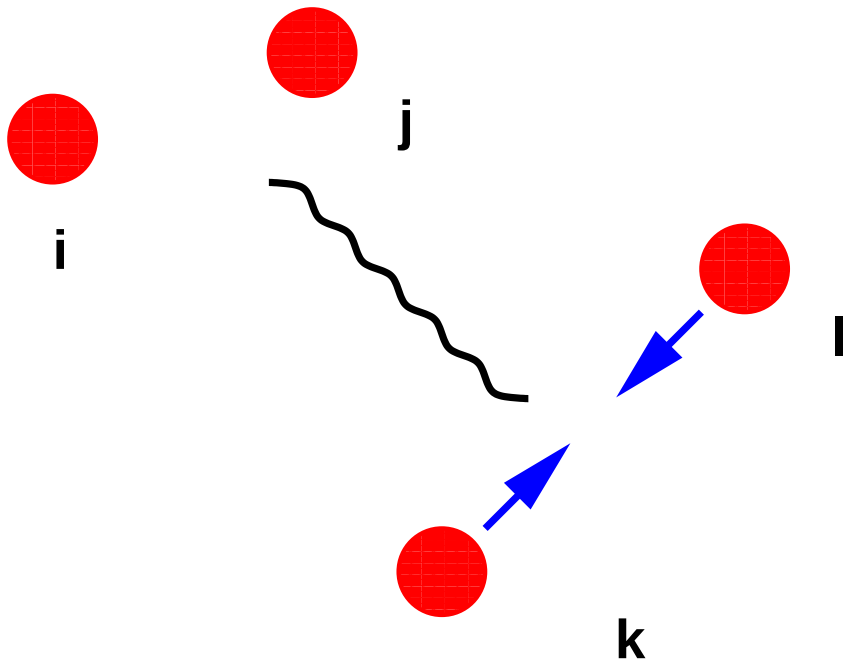
The basis states are eigenvectors of the mean field and the two body hamiltonian is diagonalized in this basis.

The interaction in second quantization

And in second quantization

$$\mathcal{H} = \sum_{ij}^{A} \langle i|T|j\rangle a_i^\dagger a_j + \frac{1}{4} \sum_{ijkl}^{A} \langle ij|V|kl\rangle a_i^\dagger a_j^\dagger a_l a_k$$

Two nucleons interact and are scattered from states k and l, to states iand j with an amplitude V_{ijkl}



Shell model

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computation of matrix elements

The so called m scheme basis is commonly used:
a) The symmetries of the hamiltonian are not taken into account explicitly. The basis is composed of Slater determinants made from the valence orbits $a_i^\dagger|0\rangle$

$$|\Phi_I\rangle = \prod_{i=nljm\tau} a_i^\dagger|0\rangle \;=\; a_{i1}^\dagger ... a_{iA}^\dagger|0\rangle$$

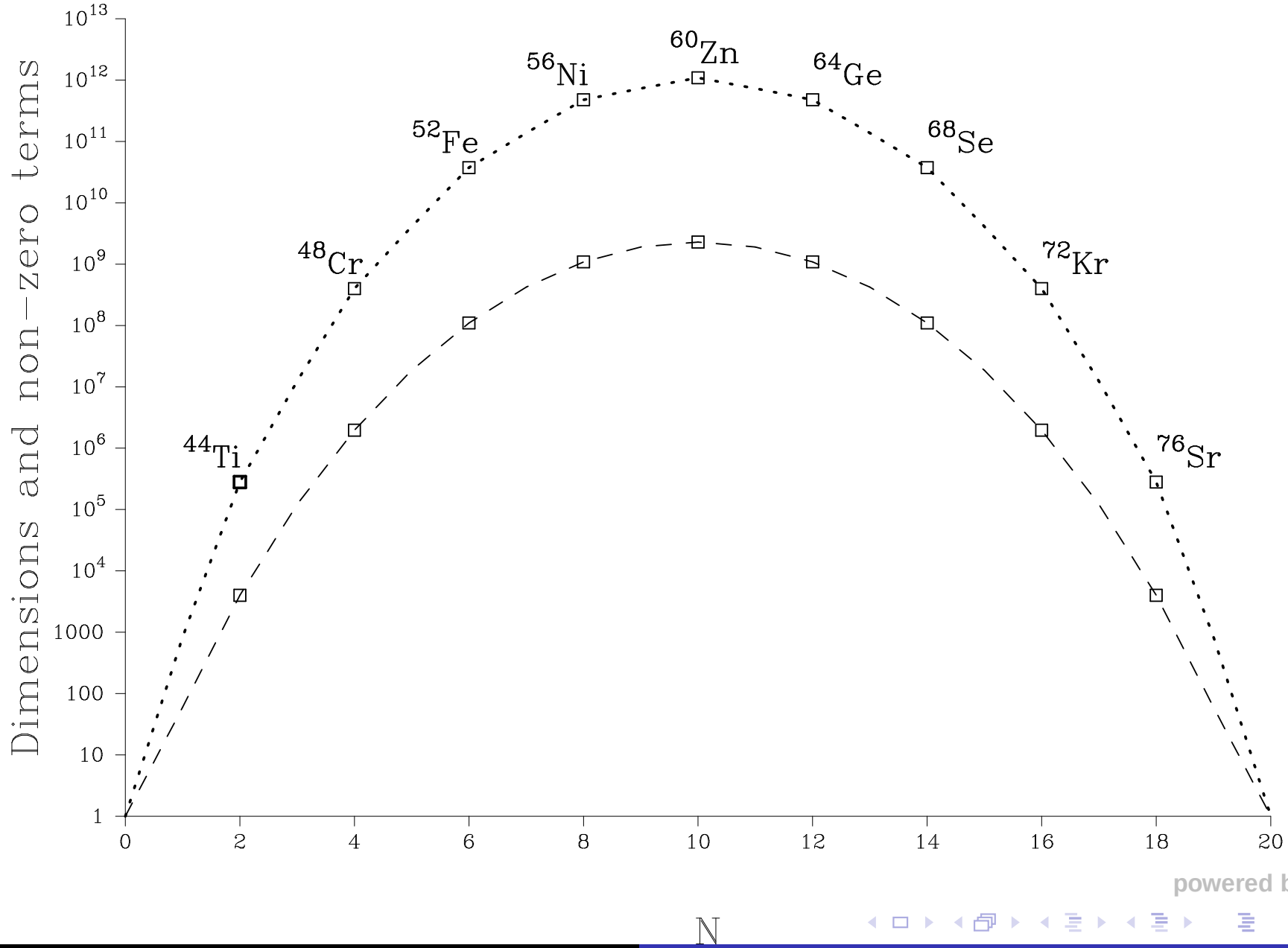
The physical states come out of the diagonalization of the Hamiltonian matrix

$$\langle\Phi_I|H_{eff.}|\Phi_J\rangle$$

The drawback is that the size of the matrices is maximal:

$$\mathrm{D} \sim \binom{d_\pi}{p} \cdot \binom{d_\nu}{n}$$

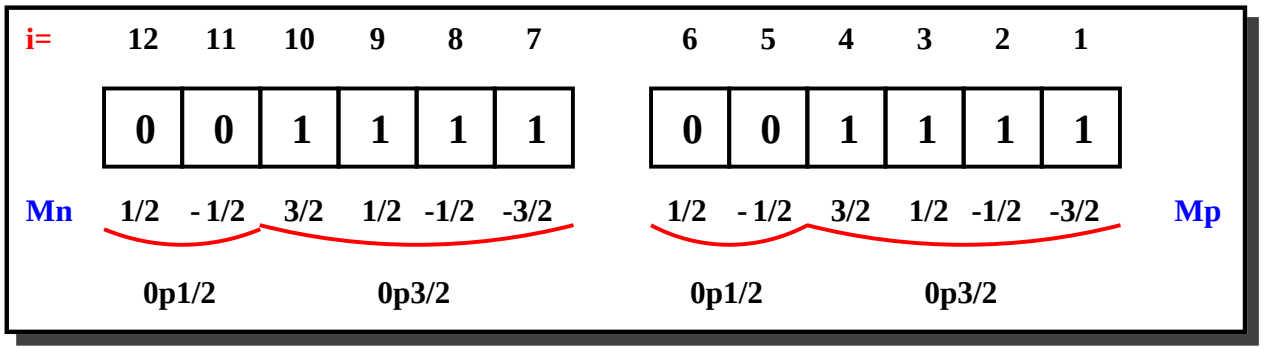
Dimensions in *pf* shell



Computing the matrix elements

The advantage is that the Hamiltonian matrix is sparse and ... the matrix elements are very easy to compute. We represent a Slater determinant by a machine word, where each state is a bit (0 empty 1 occupied)

Example : ^{12}C in the p-shell



$$\equiv a^\dagger_{10} a^\dagger_9 a^\dagger_8 a^\dagger_7 \; b^\dagger_4 b^\dagger_3 b^\dagger_2 b^\dagger_1 \; |0\rangle$$

In this example the Slater determinant is a 12 bits word.
The action of the Hamiltonian on such an object is very simple:

Computing the matrix elements

Let $|I\rangle$ be a basis function; the action of a two body term $|I\rangle$, $a_i^\dagger a_j^\dagger a_k a_l |I\rangle$ carries:

- an amplitude $\pm V_{ijkl}$,
  if k and l are occupied and i and j are empty or equal to k, l
- or zero amplitude otherwise

If the result is not zero it produces another state $|J\rangle$

Computing the matrix elements

Back to ^{12}C

$a^\dagger_{12} b^\dagger_6 a_9 b_1$ | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |

$=$ | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 |

$= a^\dagger_{12} a^\dagger_{10} a^\dagger_8 a^\dagger_7 \; b^\dagger_6 b^\dagger_4 b^\dagger_3 b^\dagger_2 \; |0\rangle$

In practice,

- all the machine words corresponding to all the basis states are generated $\{|I\rangle\}$ I=1, ...N
- and a loop is made on all the two body operators $a^\dagger_i a^\dagger_j a_k a_l$

Computing matrix elements

- The resulting states $|J\rangle$ are clasified (identified) in the list of Slater determinants and the matrix element $\mathcal{H}_{IJ}$ is

$$\langle J|\mathcal{H}|I\rangle \;=\; \pm V_{ijkl}$$

This is the Glasgow method (Whitehead, (1977))

Symmetries of the hamiltonian

The information related to the two body interaction is fully contained in the matrix elements:

$$\langle (j_1 j_2)^{JT}_{MT_z} | V | (j_3 j_4)^{JT}_{MT_z} \rangle$$

Symmetries of the hamiltonian have the following consequences:

- at fixed J (T) values, the matrix elements corresponding to all possible values of M (T_z) are equal
- matrix elements between different J (T) values are vanishing

Shell Model Hamiltonian

The hamiltonian can be written as:

$$\mathcal{H} = \sum_{i}^{A} \epsilon_i a_i^\dagger a_j + \frac{1}{4} \sum_{ijkl}^{A} \langle ij|V^{res.}|kl\rangle a_i^\dagger a_j^\dagger a_l a_k$$

In order to express the number of particles operators $n_i = a_i^+ a_i \propto (a_i^+ \tilde{a}_i)^0$,

→ particle-hole recoupling :

$$V = \sum_{\lambda\tau} W_{ikjl}^{\lambda\tau} \left[(a_i^+ \tilde{a}_k)^{\lambda\tau} (a_j^+ \tilde{a}_l)^{\lambda\tau} \right]^{00}$$

$$W_{ikjl}^{\lambda\tau} \propto \sum_{JT} V_{ijkl}^{JT} \begin{Bmatrix} i & k & \lambda \\ j & l & \lambda \\ J & J & 0 \end{Bmatrix} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & \tau \\ \frac{1}{2} & \frac{1}{2} & \tau \\ T & T & 0 \end{Bmatrix}$$

Monopole Hamiltonian

$\mathcal{H}_m$ corresponds only to the terms $\lambda\tau$ =00 and 01 which implies that $i = j$ and $k = l$ and writes as

$$\mathcal{H}_m = \sum_i n_i \epsilon_i + \sum_{i \leq j} n_i . n_j \, V_{ij}$$

and $V_{ij}^T = \frac{\sum_J V_{ijij}^{JT} [J]}{\sum_J [J]}$

Important property (to admit): for closed-shell configurations

$$\langle CS \pm 1 | \mathcal{H} | CS \pm 1 \rangle = \langle CS \pm 1 | \mathcal{H}_m | CS \pm 1 \rangle$$

Monopole Hamiltonian

The monopole Hamiltonian is responsible for the Evolution of Shell Gaps in nuclei with filling of protons/neutrons

The Shell Gap evolves linearly with the filling of the shell

This turns out to be crucial for Shell Gaps far from the stability line, in nucleosynthesis astrophysical processes ...

Summary

- at first order, Independent Particle Motion in nuclei
- But detailled description requires to go beyond this description
- one way is the Interacting Shell Model :
  - relies on shell closures
  - uses m-scheme basis
  - diagonalizes the Residual two-body interaction
- More to be explained in the following presentation by Olivier Sorlin about Shell Evolution and Monopole Hamiltonian ...