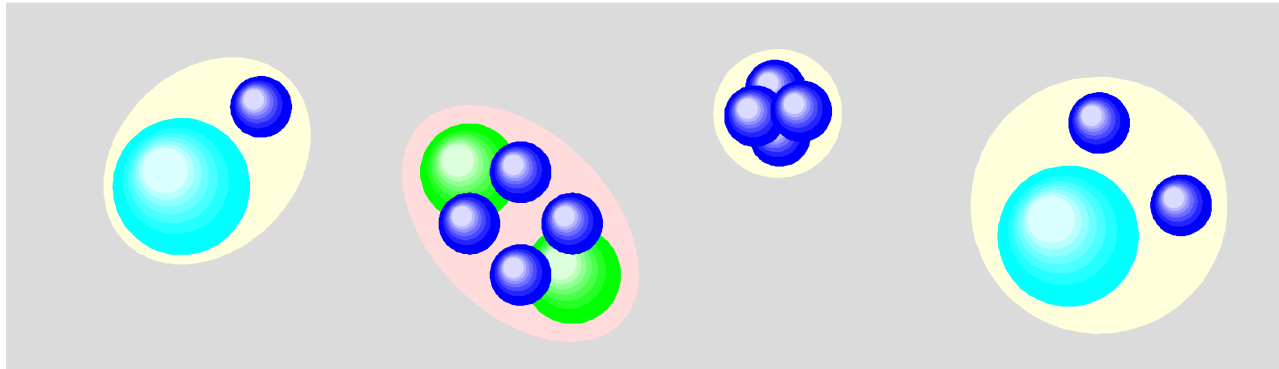


European Summer University

The Secrets of the Atomic Nucleus

Strasbourg, 28 June - 4 July 2009

“Exotic Nuclear Matter: Clusters and Halos in Nuclei”

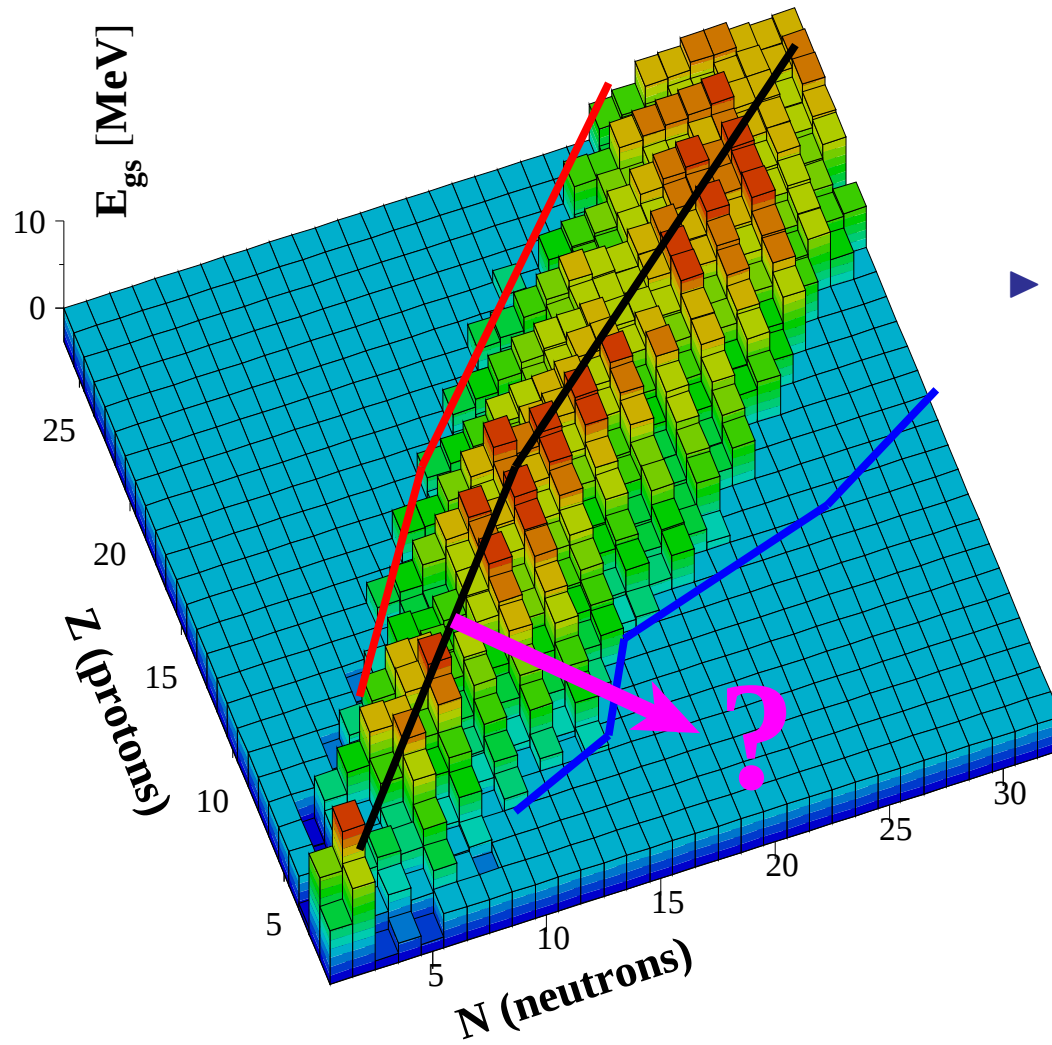


F. Miguel Marqués Moreno

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Neutrons & protons in nuclei



► the valley of stability :

▷ $B = Nm_n + Zm_p - M(N, Z)$

▷ rather a ridge of stability ...

► where are the **drip lines** ?

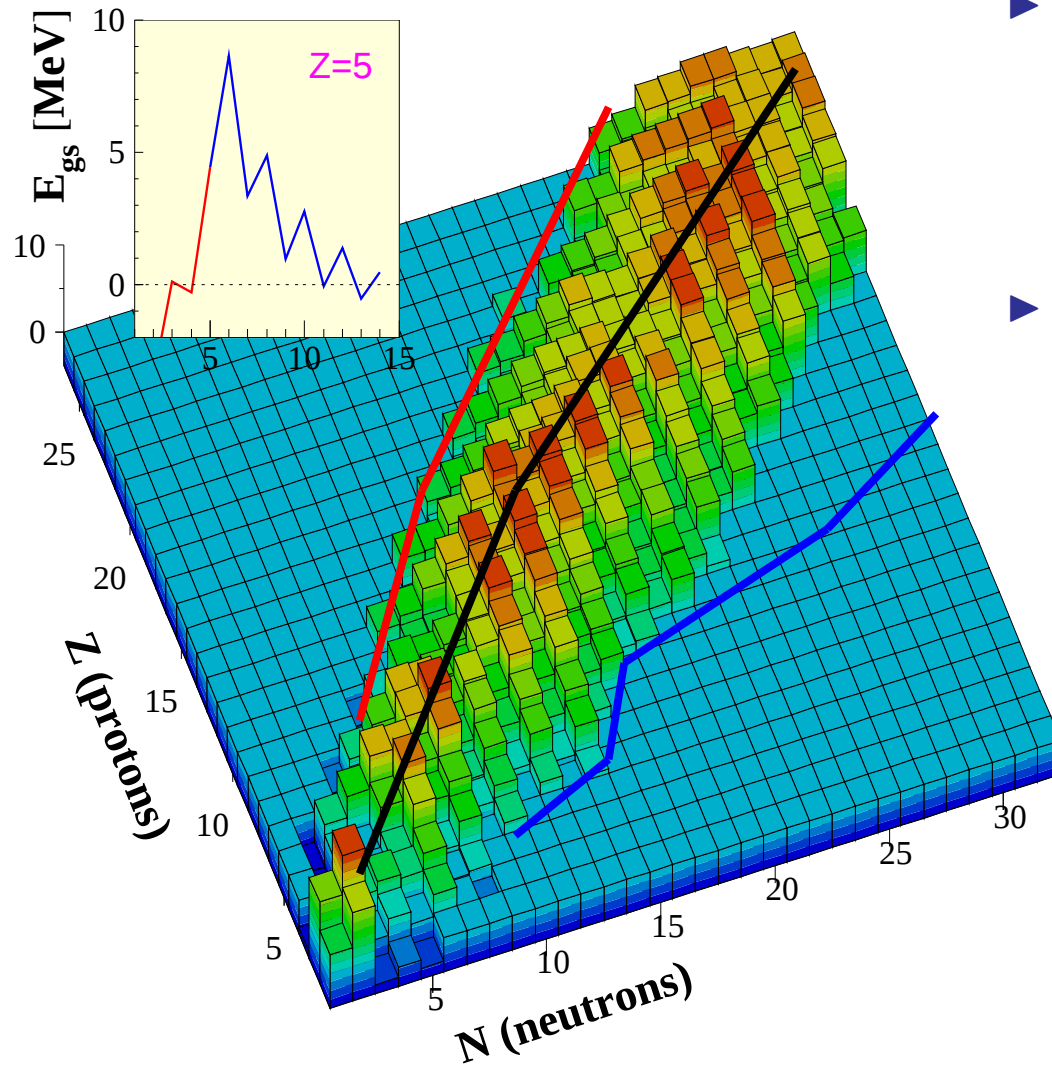
▷ $E_{gs}(N, Z)$

$$= \min \left[\sum M(n_i, z_i) \right] - M(N, Z)$$

► **very light** nuclei :

▷ access to extreme (**N** , **Z**) !!!

Neutrons & protons in nuclei



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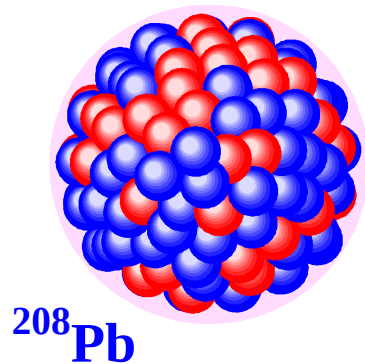
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The liquid drop picture

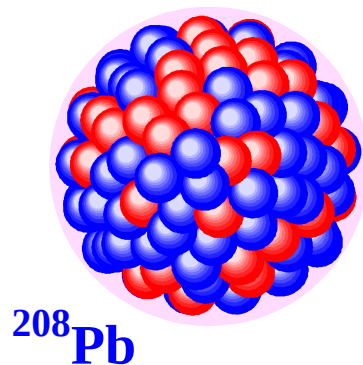
► **are** nuclei liquid drops ?



- ▷ constant density
- ▷ short-range forces
- ▷ saturation
- ▷ deformability
- ▷ well defined surface (tension)
- ▷ mean free path + size...
- ▷ yes, but **quantum** liquid

The liquid drop picture

- **are** nuclei liquid drops ?



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- ▷ short-range forces
- ▷ saturation
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- semi-empirical mass formula :

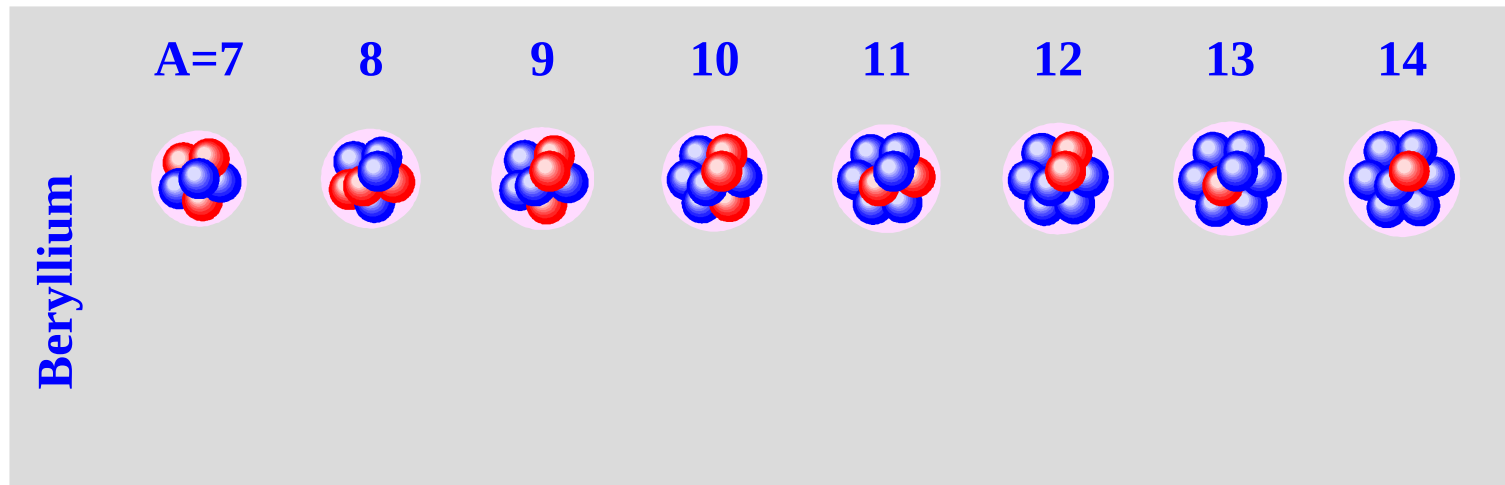
$$M(N, Z) = Nm_n + Zm_p - B(N, Z)$$

$a_v A$	volume
$- a_s A^{2/3}$	surface
$- a_c Z^2/A^{1/3}$	Coulomb
▷ $- a_a (N - Z)^2/A$	asymmetry
$\pm \delta/A^{1/2}$	pairing

...and so the nucleus “behaves” as if :

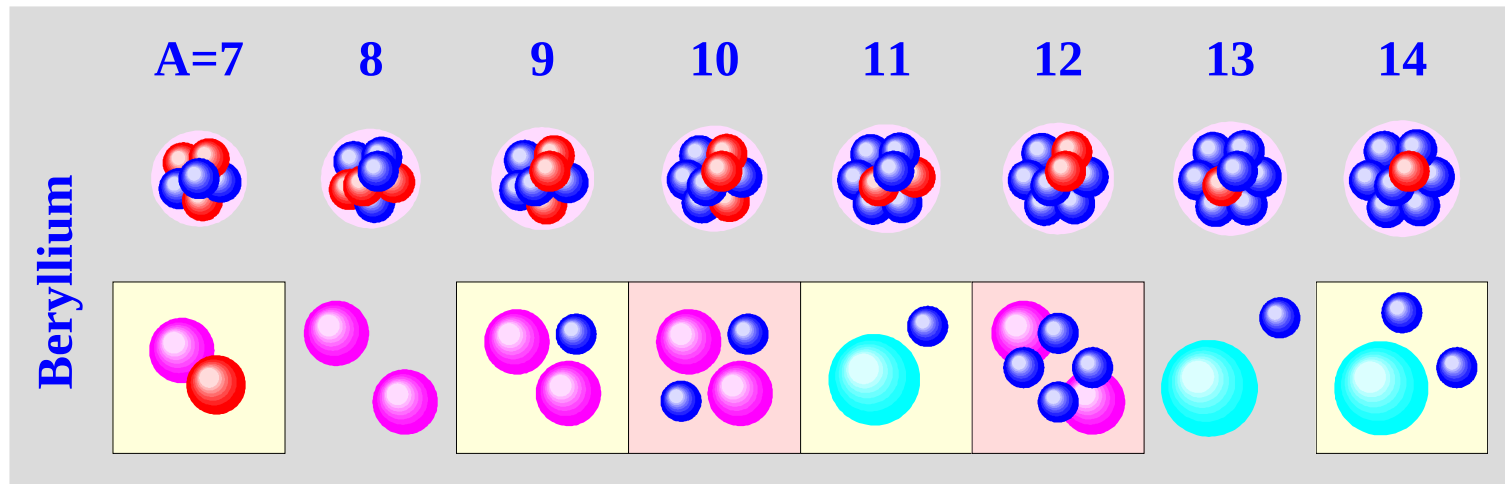
- ▷ limits were **sharp** $[R \sim r_0 A^{1/3}]$
- ▷ volume was **uniform** $[\rho(r) \sim \rho_0]$
- ▷ and **homogeneous** $[\rho_n \approx \frac{N}{Z} \rho_p]$

A complete isotopic chain



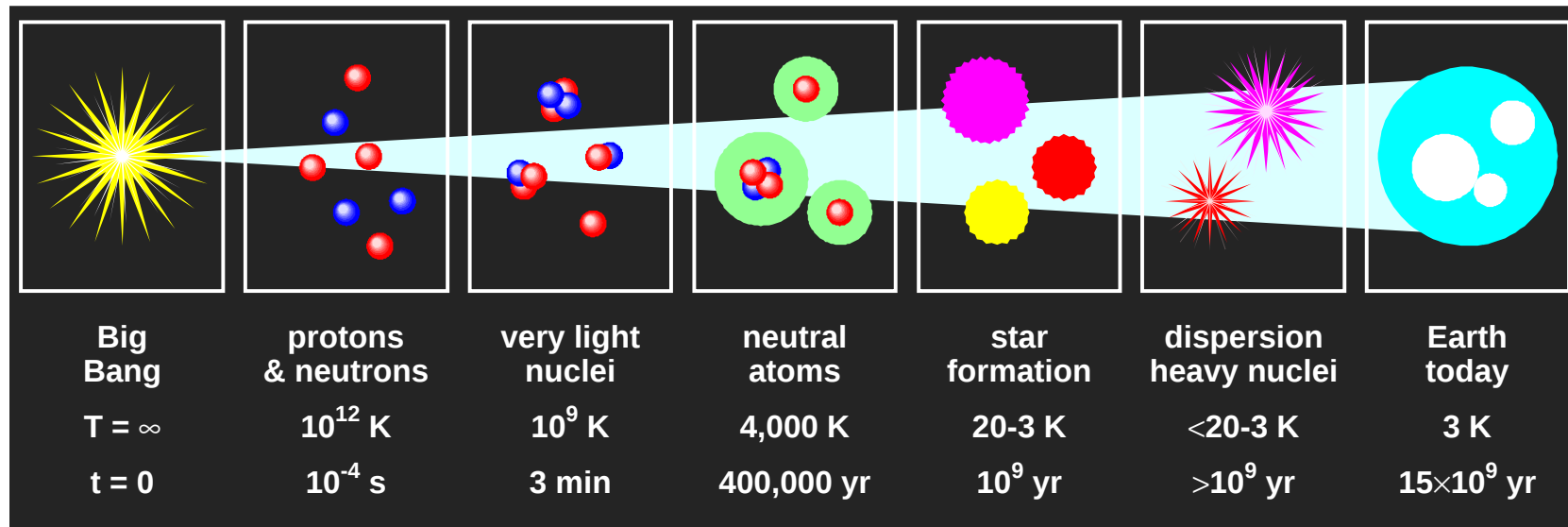
► “normal” size increase : $R = r_0 A^{1/3}$?

A complete isotopic chain



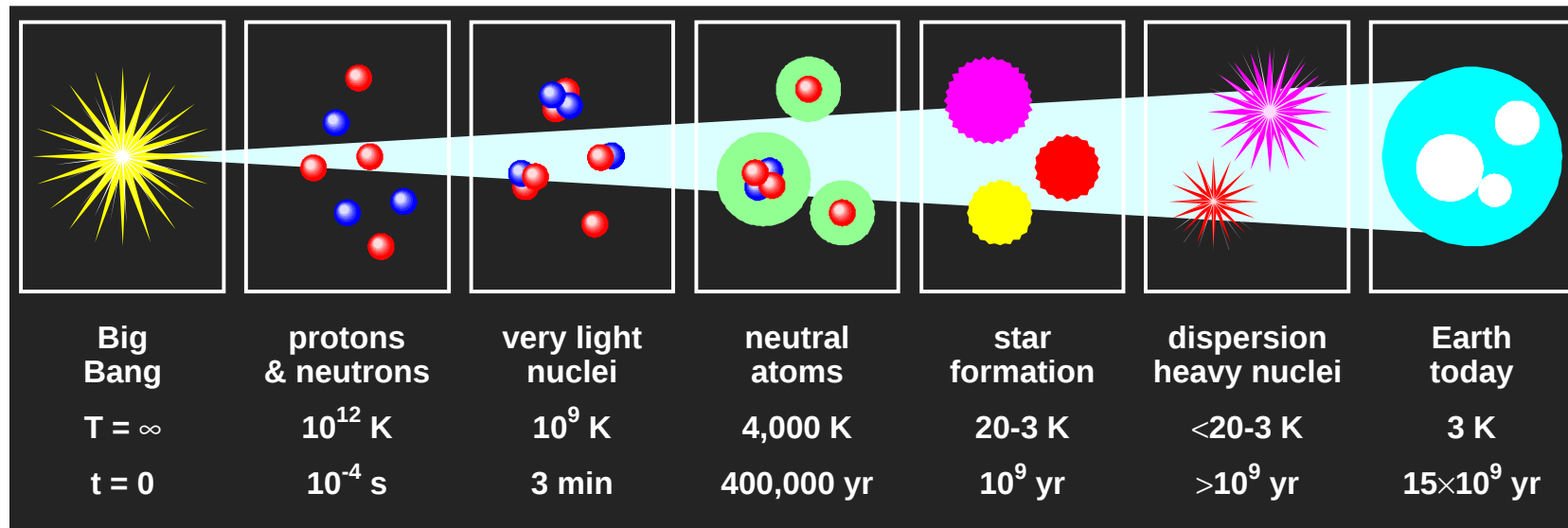
- ▶ many (all?) extreme configurations :
 - ▷ clustering
 - ▷ unbound resonant states
 - ▷ nuclear molecules
 - ▷ neutron haloes
 - ▷ borromean systems
 - ▷ neutral nuclei ?

Expansion of the Universe



- nucleosynthesis :
 - ▷ why [and how] few (N, Z) ?
 - ▷ do unknown combinations exist ?
 - ▷ why **two steps** ?

Expansion of the Universe



- ▶ nucleosynthesis :
 - ▷ why [and how] few (N , Z) ?
 - ▷ do unknown combinations exist ?
 - ▷ why **two steps** ?
- ▶ on Earth only “normal” nuclei left :
 - ▷ study **exotic nuclei** in laboratory !

Extraterrestrial nuclei on Earth

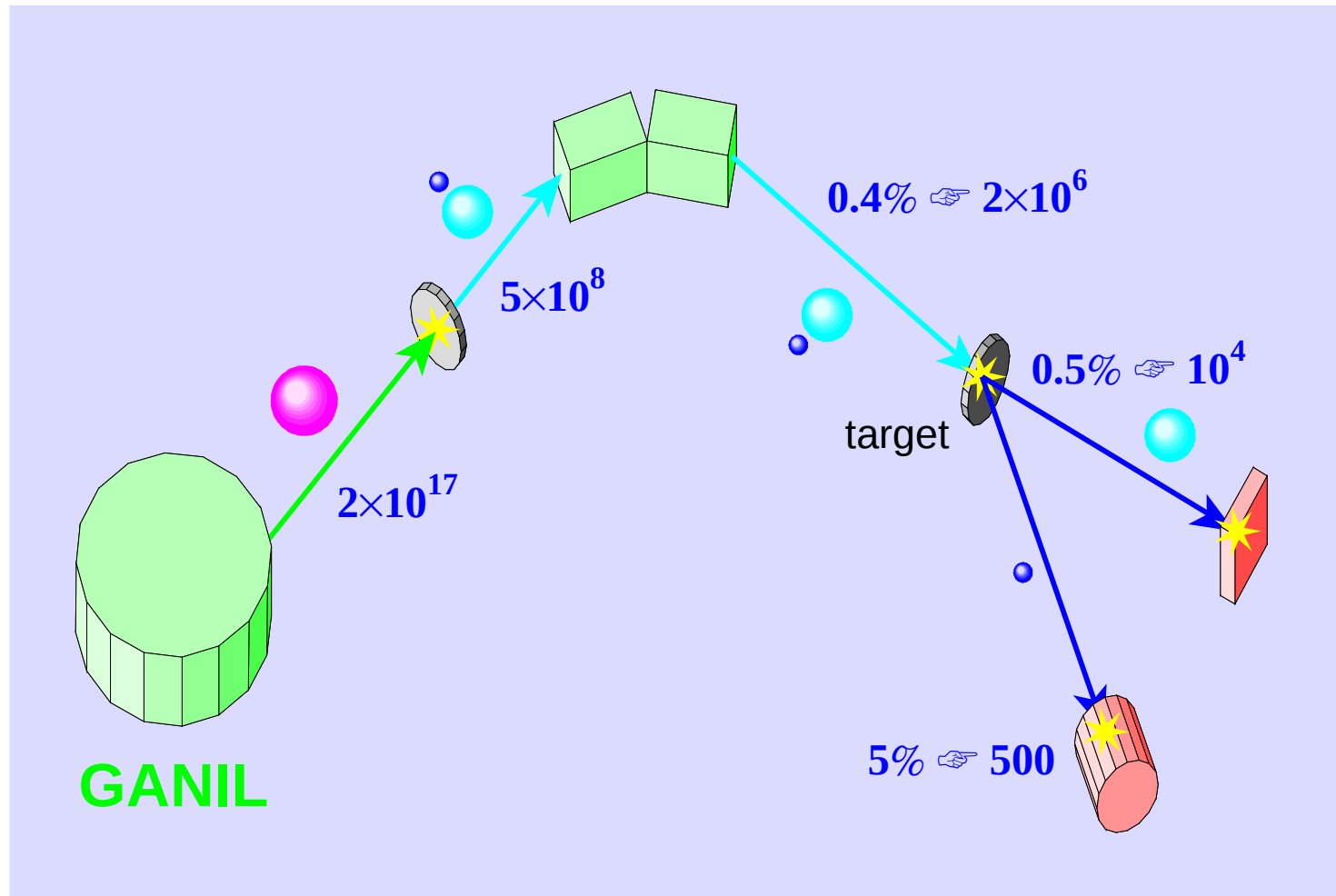
► $^{19}\text{C} \rightarrow ^{18}\text{C} + n$ experiment @ GANIL :

▷ add $7n$ to ^{12}C ?? study it in less than 49 ms ???

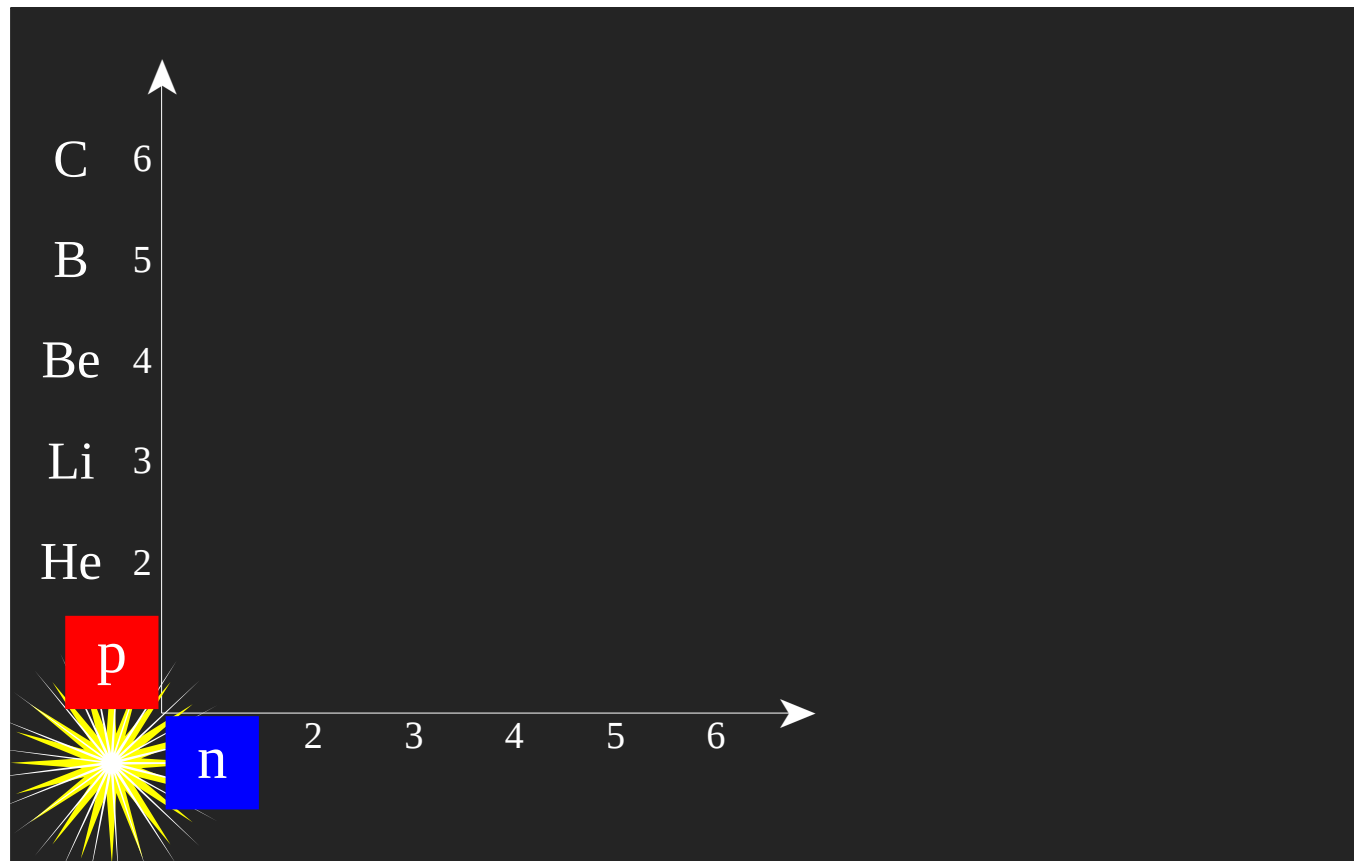
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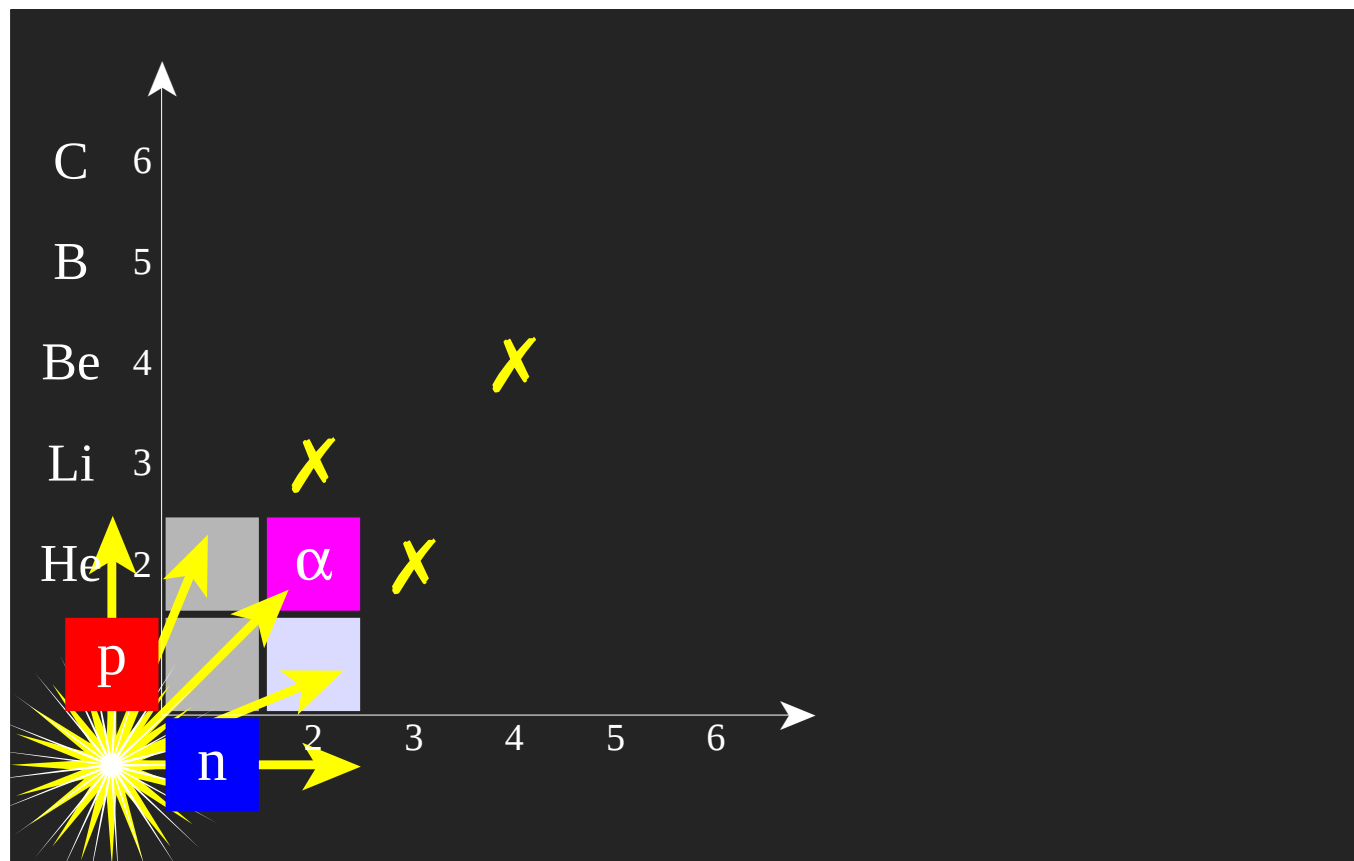
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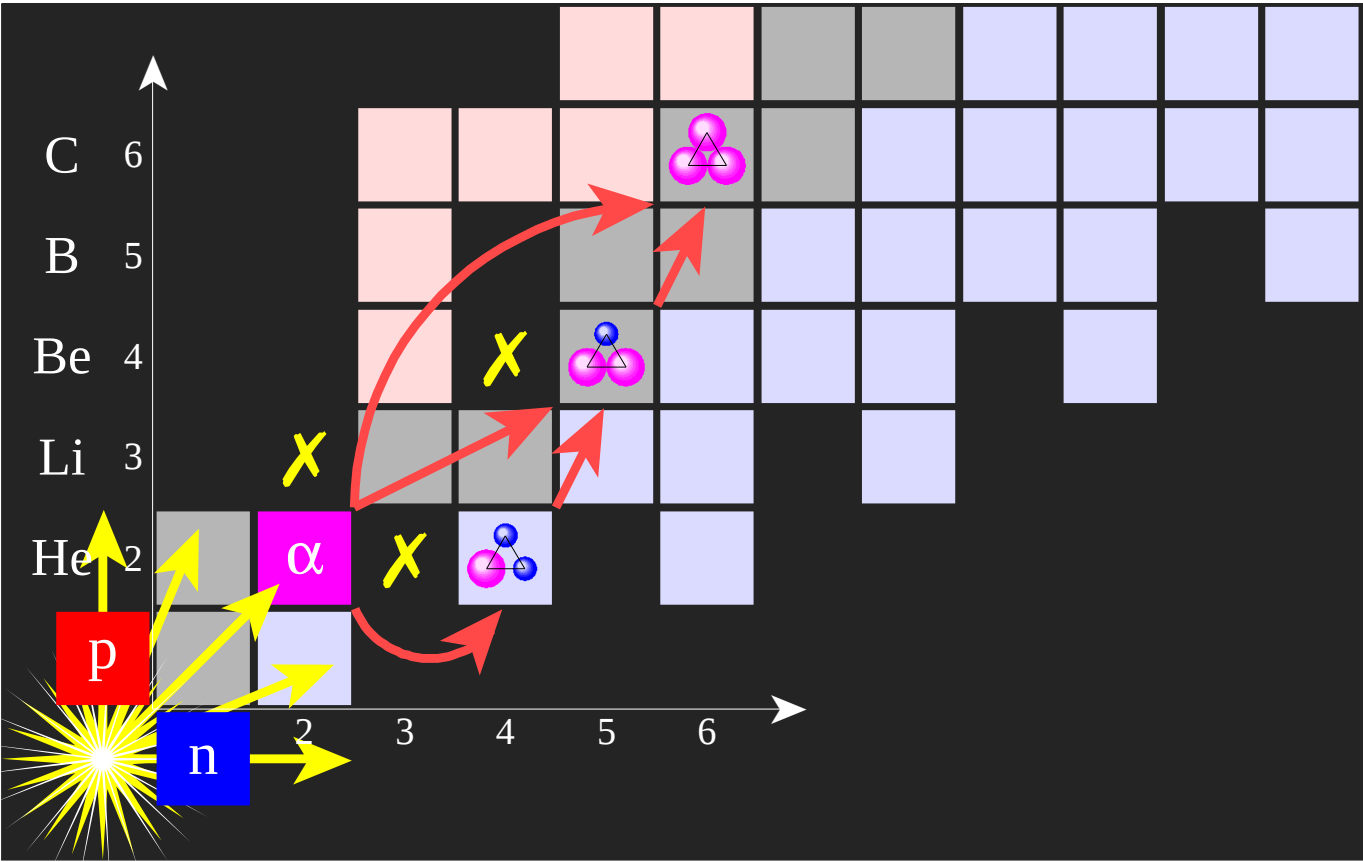
Nuclei in the nucleus



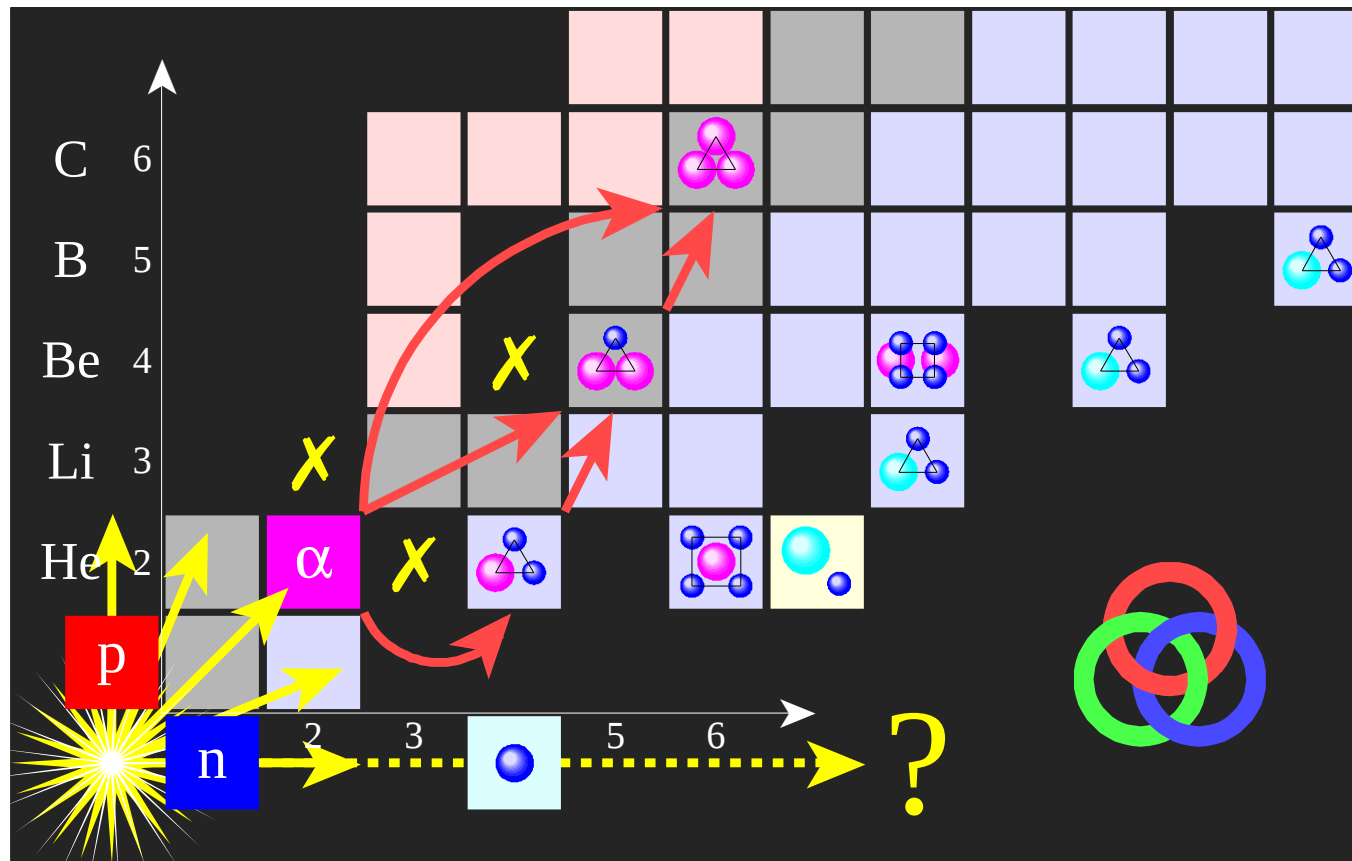
Nuclei in the nucleus



Nuclei in the nucleus



Nuclei in the nucleus

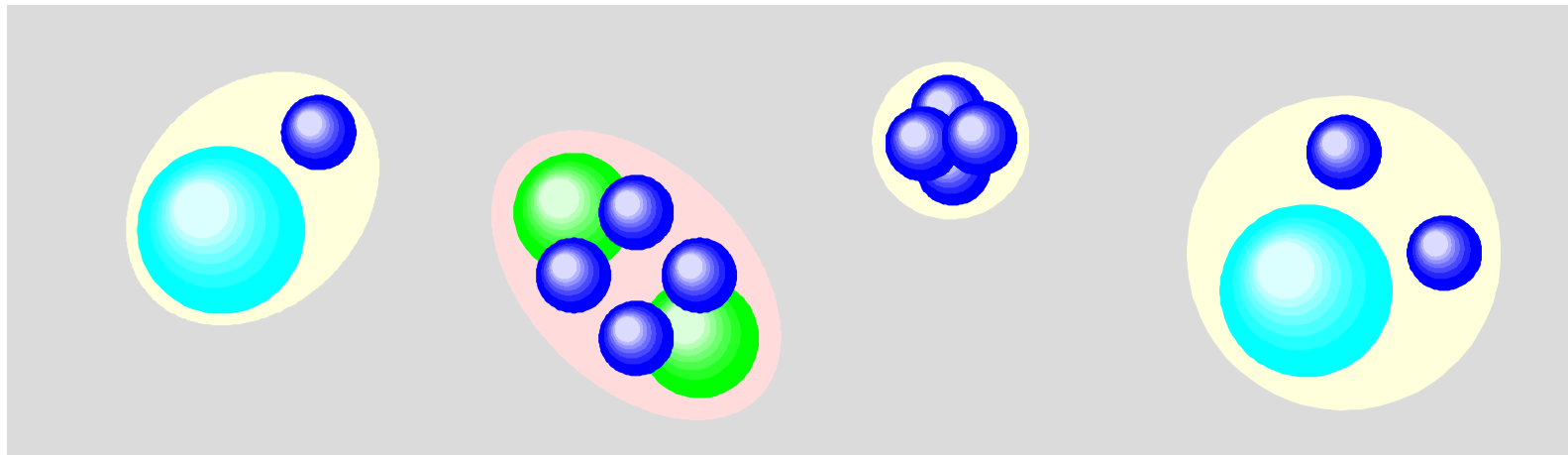


- ▷ N nucleons $\equiv$ few (**correlated**) clusters !
- ▷ formation of carbon (and $A > 12$)
- ▷ are nuclei **sharp/uniform/homogeneous** ???

(I) The Nuclear Halo

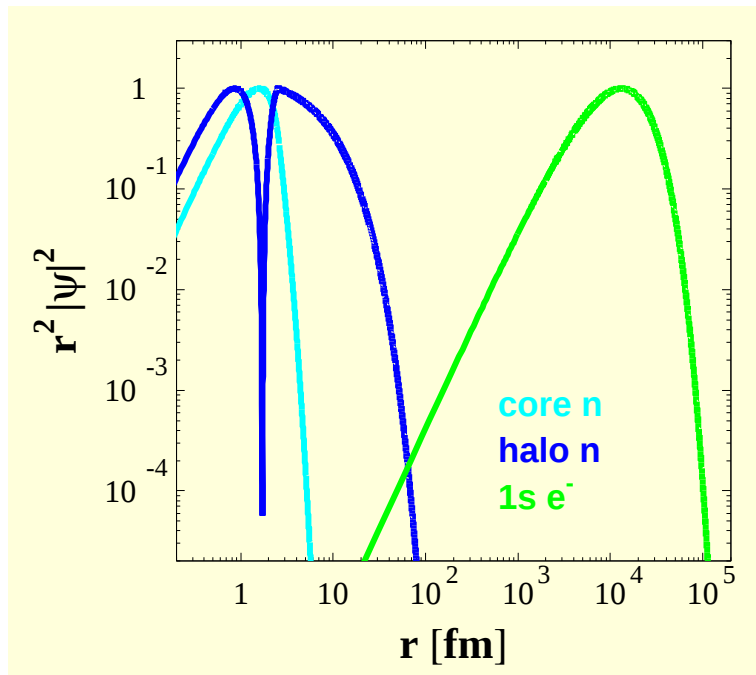
(II) Nuclear Molecules

(III) Neutron Clusters ?



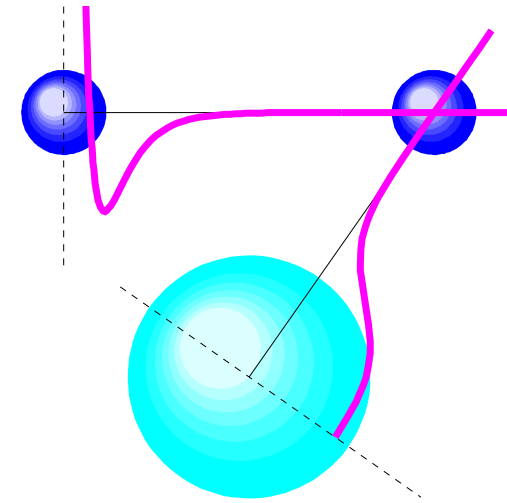
Halo : an exaggerated picture ?

► the size of the halo [Be atom] :



- ▷ still negligible at atomic level ...
- ▷ a potentially **huge** nuclear effect !
[...and **theoretical** problem!]

► the geometry of the halo [^{11}Li] :



- ▷ rms distances $\gg$ binary ranges !
- ▷ classically forbidden configurations
-  picture : not exaggerated at all !!!

A quantum liquid

► particle-wave duality :

▷ wavelength $\sim$ nuclear radius

► the nucleon wave function :

$$H|\Psi\rangle = E|\Psi\rangle$$

▷ Ψ vanishes outside the nucleus

▷ but $|\Psi|^2 \neq 0$ for being outside ...

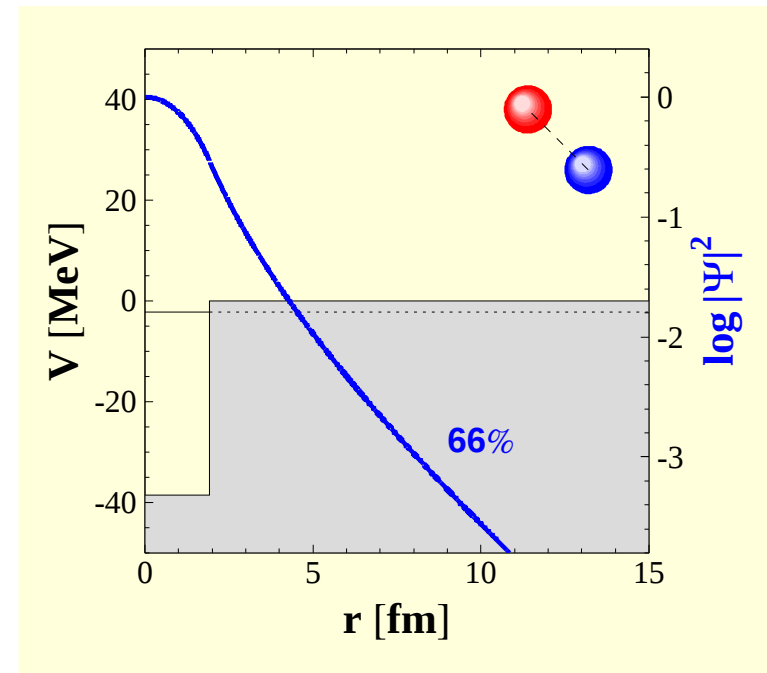
► the simplest case : $\ell=0$ neutron

▷ no Coulomb barrier $[Z/r]$

▷ no centrifugal barrier $[\ell(\ell+1)/r^2]$

▷ “only” the nuclear potential ...

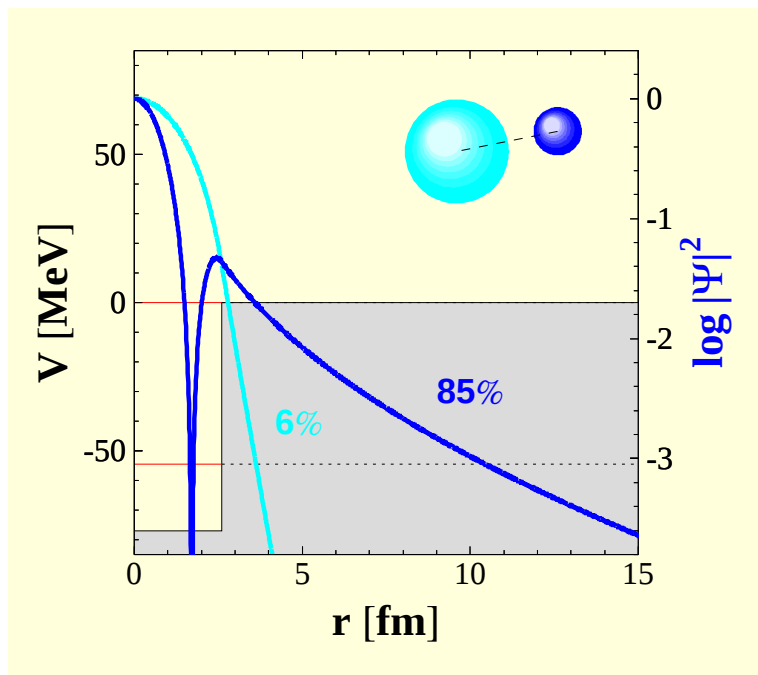
► the deuteron $[B(n) = 2.2 \text{ MeV}]$:



► very n-rich nuclei ? $[B(n) \rightarrow 0]$

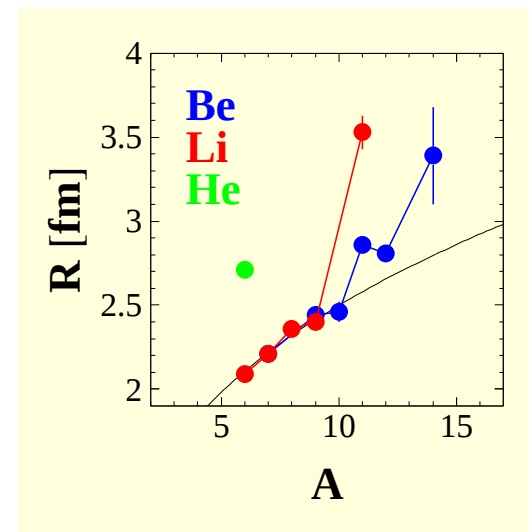
The neutron(s) halo

- ▶ neutron ($\ell=0$) in $A=10$ nucleus :



- ▷ very **big** nuclei !!!
- ▷ two phases : (**core**)+**halo** ...

- ▶ interaction cross-sections [$\propto \pi R^2$] :

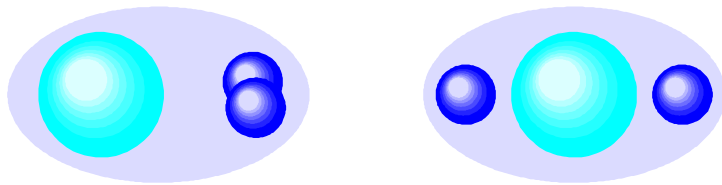


- ▷ $^{11}\text{Li} \equiv ^{48}\text{Ca} \dots$ **but** : $2n \equiv ^{208}\text{Pb}$!!!

- ▶ when **two** neutrons in the halo :
 - ▷ first seen as core-dineutron
 - ▷ but core- n - n  correlations ...

The structure of the neutron cloud

- ▶ nucleons at $\rho \neq \rho_0$!!!
 - ▷ a laboratory for $N-N$ interaction
- ▶ how do they “move” [] ?



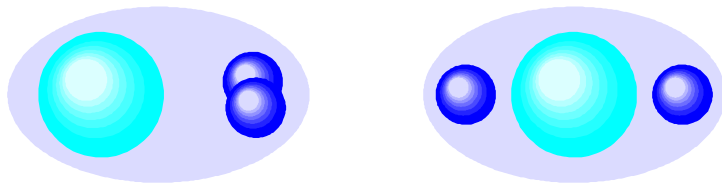
- ▷ a step inside the halo ...
- ▶ more subtle experiments ($\sigma \downarrow$) :
 - ▷ halo **transfer** $[^6\text{He} + \alpha]$
 - ▷ proton **capture** $[^6\text{He} + p]$
 - ▷ or more subtle **techniques** ...

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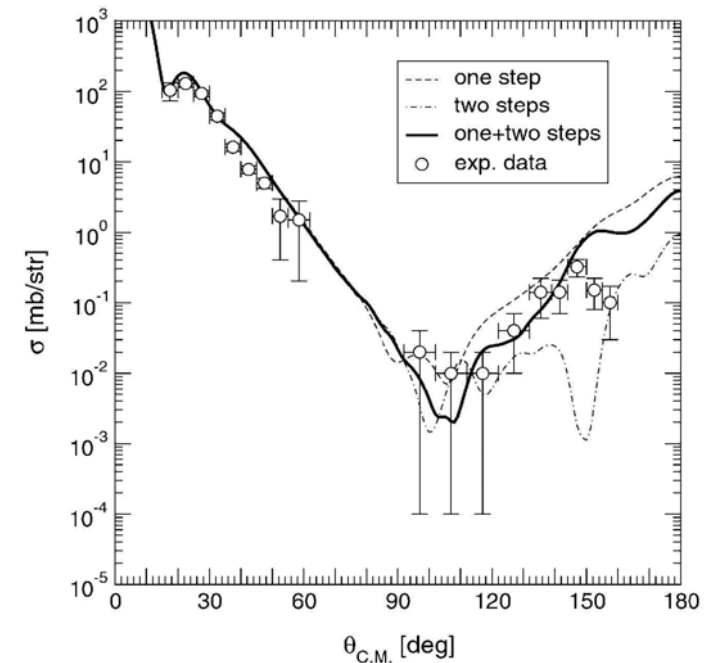
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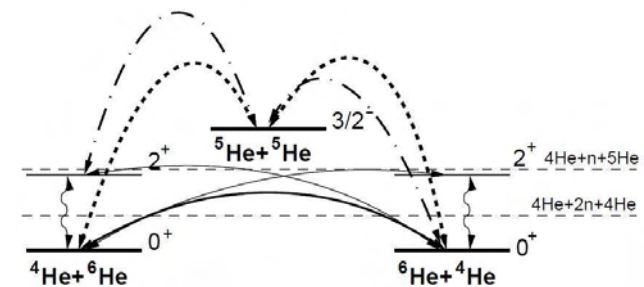
▷ or more subtle **techniques** ...

► halo transfer : $\alpha (^6\text{He}, \alpha) ^6\text{He}$



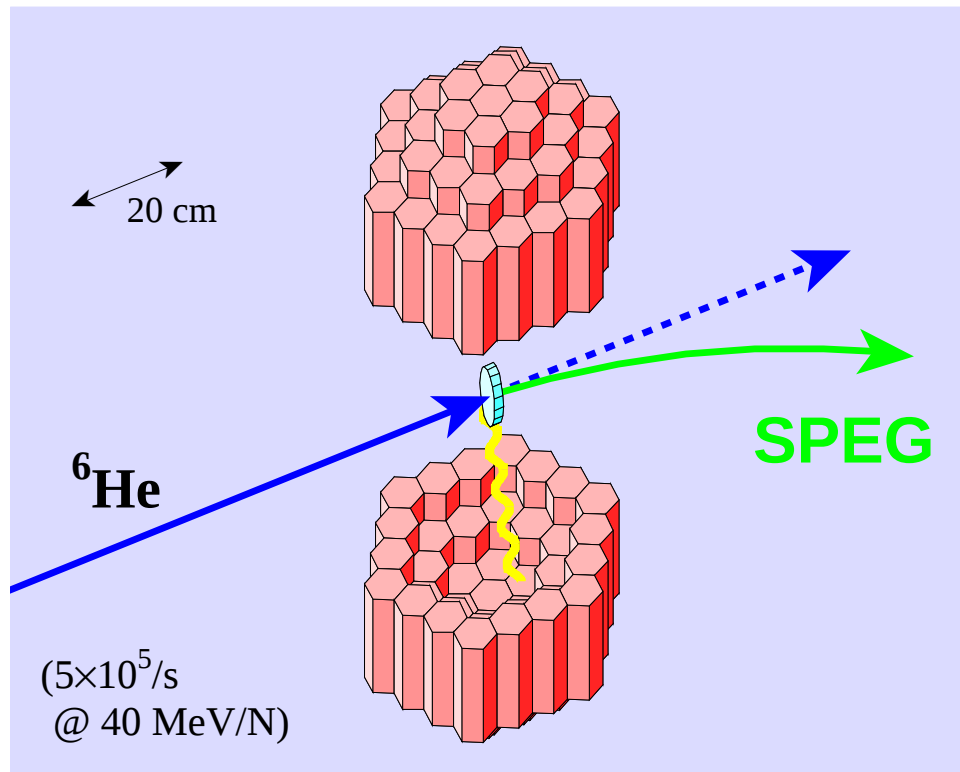
▷ dineutron configuration ! (?)

▷ assumption : **one-step** process...

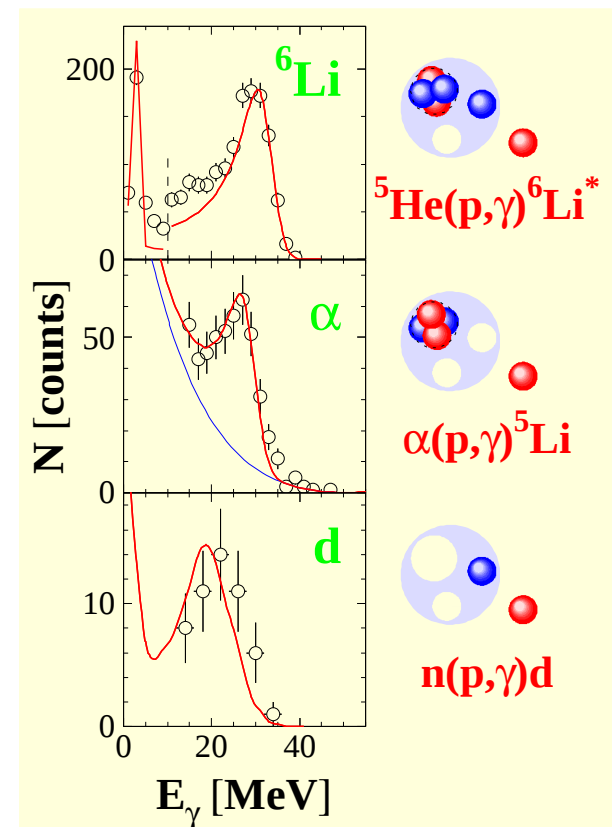
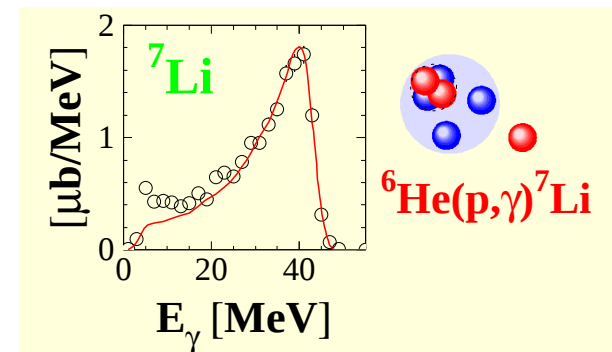


Radiative capture

► proton capture on clusters: ${}^6\text{He} (p, \gamma) X$



- a reference: capture on ${}^6\text{He}$
- several **quasi-free** captures !!!
- no $[{}^2n (p, \gamma) t]$ and $[t (p, \gamma) \alpha]$ events...
- dominant α - n - n configuration !



The n-n interaction

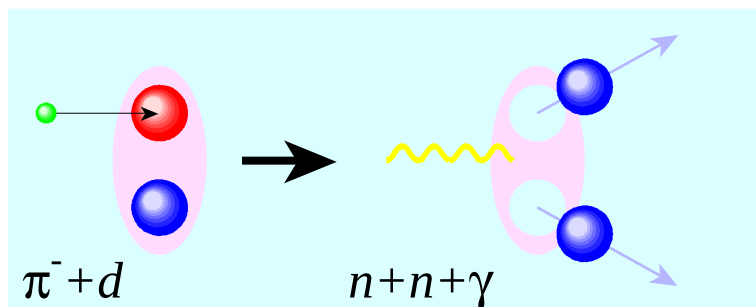
► low energy N - N interaction :

$$f_s(k) = \frac{e^{i\delta(k)}}{k} \sin \delta(k)$$

$$\sigma_s(E) = \frac{4\pi}{k^2} \sin^2 \delta \stackrel{k \rightarrow 0}{=} 4\pi a_0^2$$

$$k \cot \delta = \frac{-1}{a_0} + 1/2 d_0 k^2 + \dots \left[\frac{-1}{a(k)} \right]$$

► neutron-neutron “collisions” ?



The n-n interaction

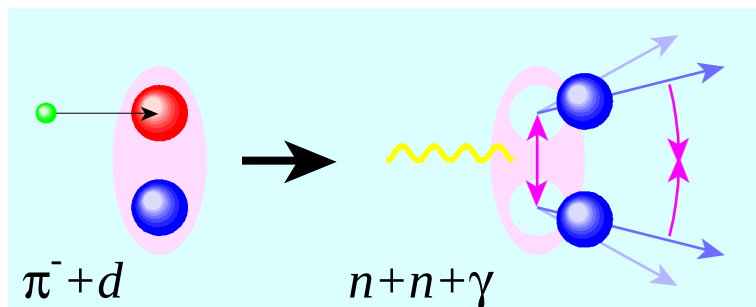
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► neutron-neutron “collisions” ?



► final state **modified** by V_{nn} !

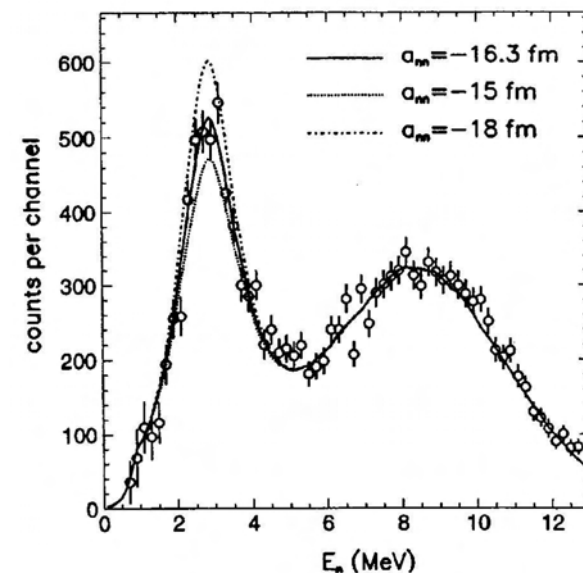
► how is it modified ?

▷ by the n-n **distance**

▷ by the n-n **interaction**

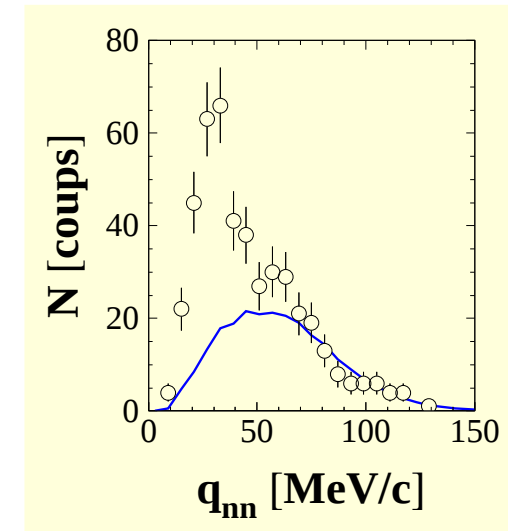
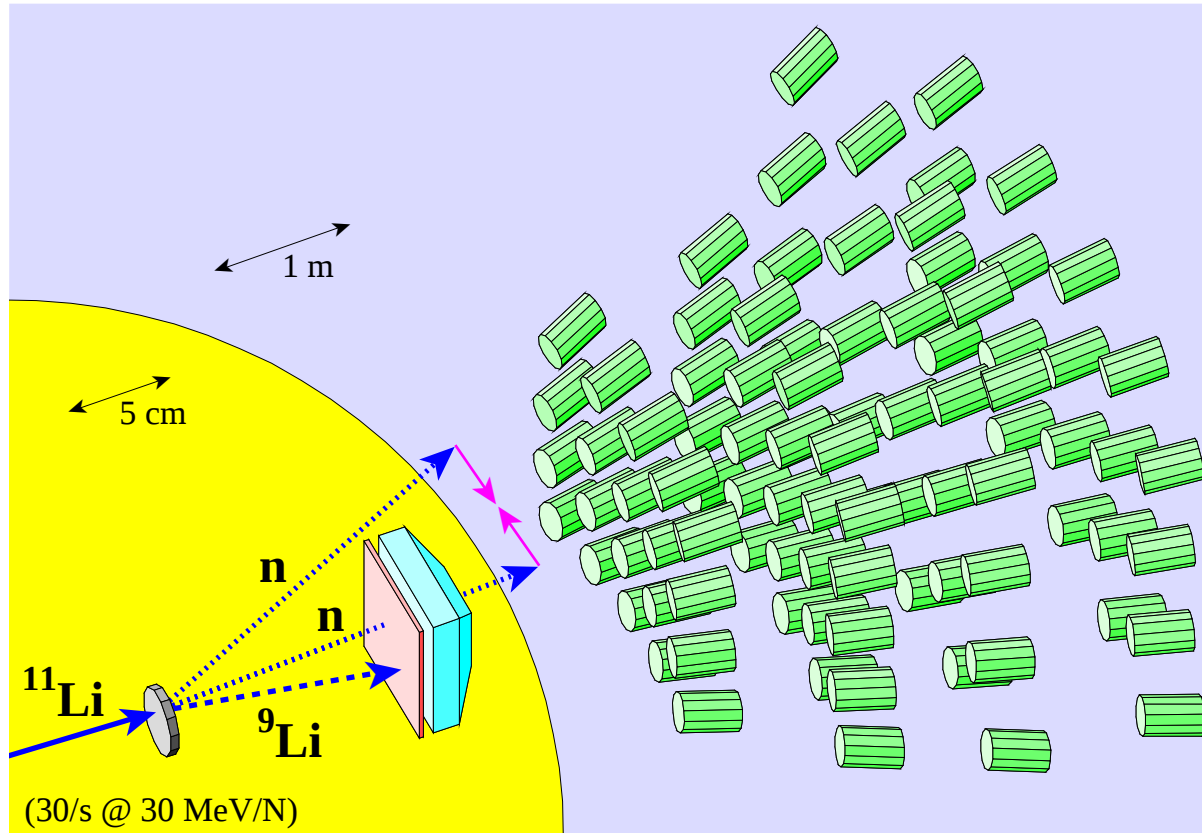
$$\sigma(q) \approx \Omega(q) \times \left| \int \psi_d \psi_s^*(a_{nn}) d^3r \right|^2$$

$$\approx \Omega(q) \times \frac{1}{1 + q^2 a_{nn}^2}$$



The neutron femtoscope

► the halo of ^{11}Li : $\bigcirc \leftrightarrow \bigcirc \bigcirc$?



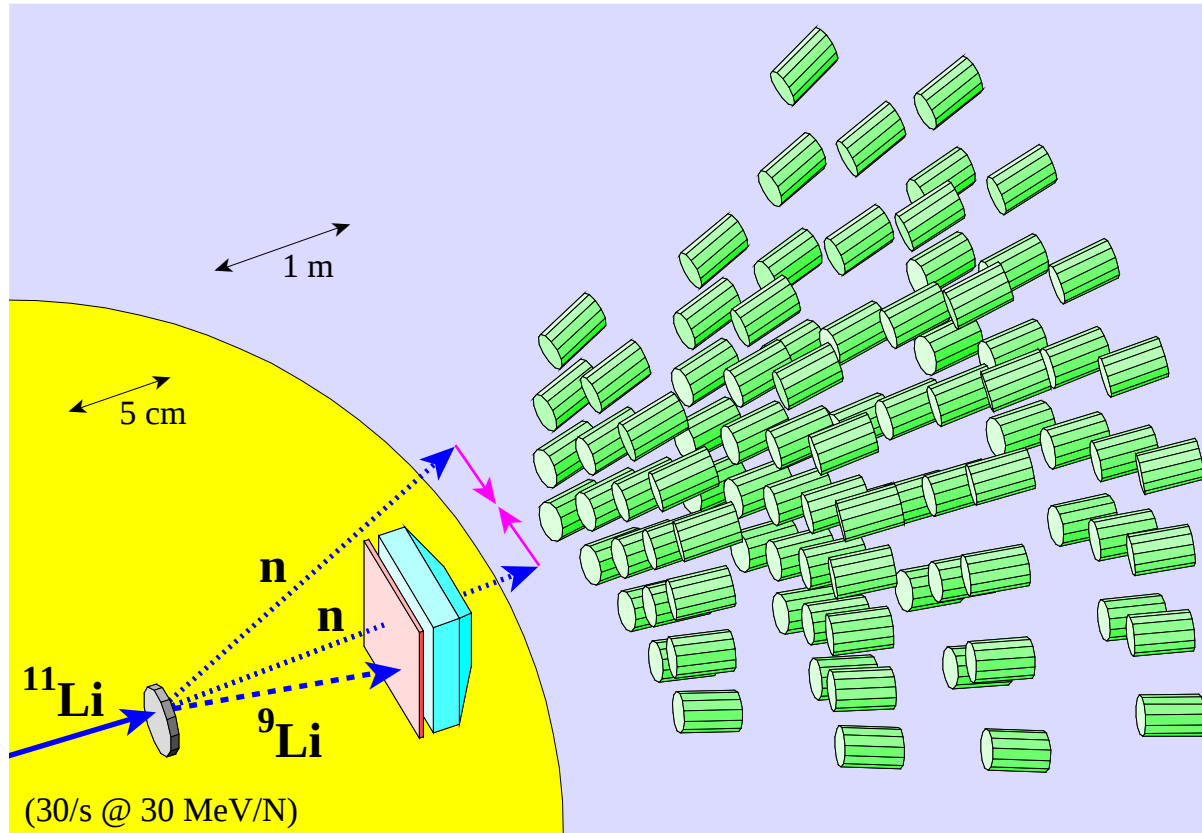
▷ $\sigma(q) \equiv \Omega(q) \times C_{nn} \{ \psi(r_{nn}), a_{nn} \} :$

↔ $\sigma(q)$ is measured

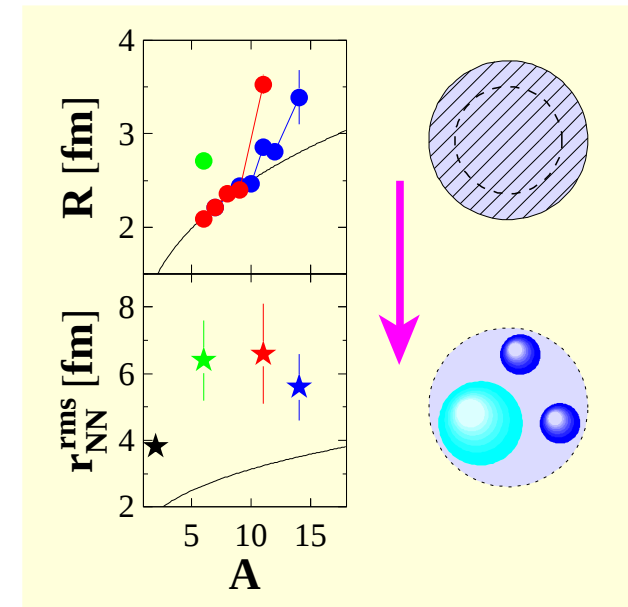
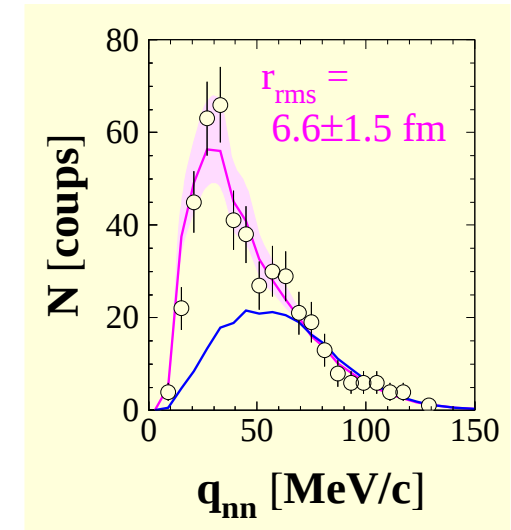
↔ event mixing provides $\Omega(q)$...

The neutron femtoscope

► the halo of ^{11}Li : $\text{O} \leftrightarrow \text{O} \text{ with halo} ?$



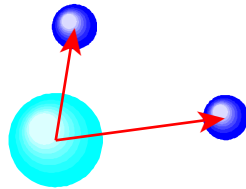
▷ $\sigma(q) \equiv \Omega(q) \times C_{nn} \{ \psi(r_{nn}), a_{nn} \} :$
 $\rightsquigarrow \sigma(q)$ is measured
 $\rightsquigarrow$ event mixing provides $\Omega(q) \dots$



Three-body models

► few-nucleon dilute halo :

▷ **correlations** overwhelm mean field...



$$\Psi \equiv \Psi(2n)$$

$$V(2n) = V_{cn}(1) + V_{cn}(2) + V_{nn}$$

$$H_{2n}\Psi(2n) = E_0 \Psi(2n)$$

V_{nn} : zero range...

V_{cn} : “Woods-Saxon”...

▷ expand on core- n basis :

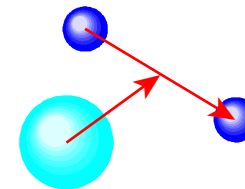
$$\Psi^{JM\pi} = \sum_{ij} c_{ij} |\phi_{cn}^i \otimes \phi_{cn}^j\rangle_{JM\pi}$$

▷ Lagrange mesh :

$$\phi_{cn} = \sum_i^N f_i(\mathbf{r}/h) Y_l(\Omega) \left[f_i(x_j) \propto \delta_{ij} \right]$$

► more complex formalisms :

▷ Jacobi coordinates ($\mathbf{x}, \mathbf{y}$)



▷ Hyperspherical Harmonics :

$$\Psi^{JM\pi} = \rho^{-5/2} \sum \chi_{Kl_x l_y}^{LS}(\rho) |\mathcal{Y}_{KL}^{l_x l_y}(\Omega_5) \otimes X_S\rangle_{JM\pi}$$

$$\rho = \sqrt{x^2 + y^2} \quad \text{hyperradius}$$

$$K = l_x + l_y + 2n \quad \text{hypermoment}$$

$$\left[-\frac{d^2}{d\rho^2} + \underbrace{\frac{(K+3/2)(K+5/2)}{\rho^2}}_{\text{effective HH barrier}} - \frac{2m}{\hbar^2} E \right] f(\rho) = 0$$

▷ **asymptotics** from “3 → 3” scattering

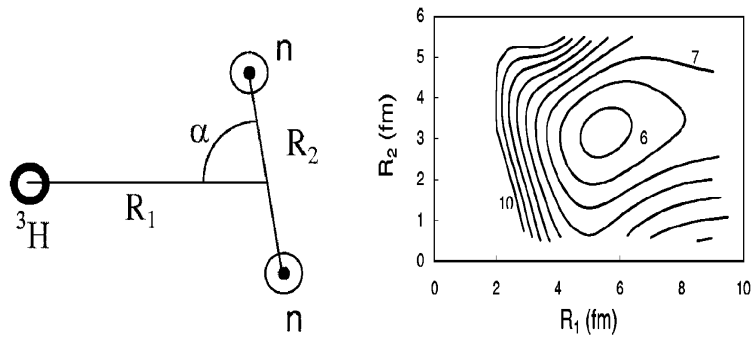
▷ still **inert** core and V_{cn} ...

More-body models

► Generator Coordinate Method [⁵H] :

$$H = \sum_i^5 T_i + \sum_{i<j} V_{ij}$$

$$\Phi_{\nu_1\nu_2\nu_3}(R_1, R_2, \alpha) = \mathcal{A}\{\phi_t^{\nu_1}\phi_n^{\nu_2}\phi_n^{\nu_3}\}$$



$$\Psi^{JM\pi} = \sum_{\substack{R_1 R_2 \alpha \\ \nu_1 \nu_2 \nu_3}} f_{\nu_1 \nu_2 \nu_3}^{J\pi} \Phi_{\nu_1 \nu_2 \nu_3}^{JM\pi}(R_1, R_2, \alpha)$$

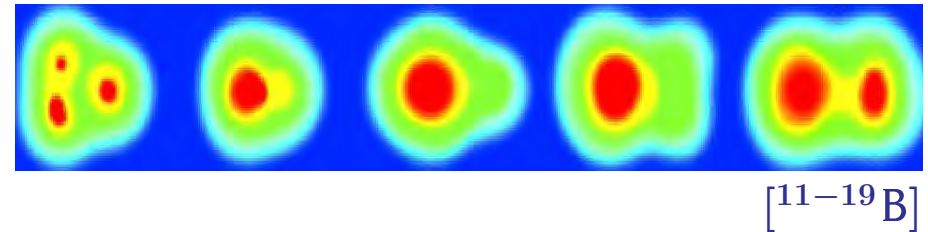
- ▷ core excited states !
- ▷ a priori clustering
- ▷ HO [same b] wave functions...
- ▷ very effective V_{NN} ...

► Antisymmetrized Molecular Dynamics :

$$H = \sum_i^A T_i + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk}$$

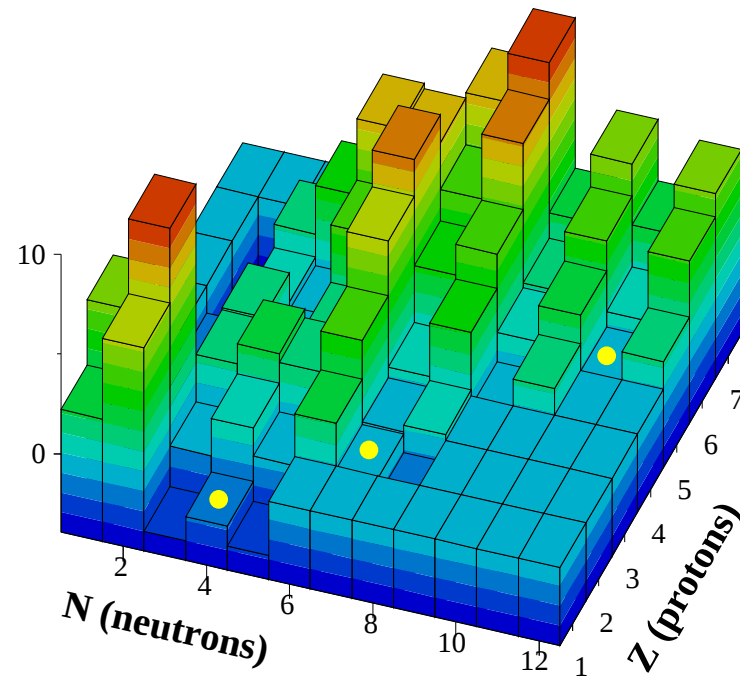
$$\Phi_{\text{AMD}}(X_i, \nu) = \mathcal{A}\{\phi_1, \phi_2, \dots, \phi_A\}$$

$$\Psi^{JM\pi} = \sum_j c_j \Phi_{\text{AMD}}^{JM\pi}(X_i^j)$$



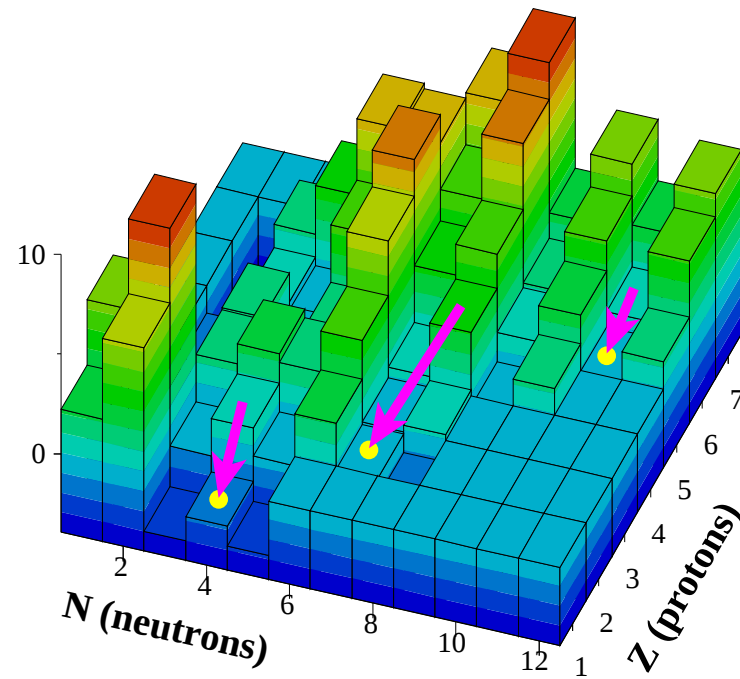
- ▷ A wave packets ! [Gaussian]
- ▷ NO assumptions... (???)
- ▷ V_{NN} as effective as in GCM
- ▷ $V_{ijk} \propto \delta(r_i - r_j) \delta(r_i - r_k) \dots$
- ▷ “valence” neutrons :
 $\{\phi_i\} \rightarrow \{\phi'_\alpha\}_{\text{o.n.}} \rightarrow h_{\alpha\beta} \rightarrow E_\alpha$

Formation of unbound nuclei



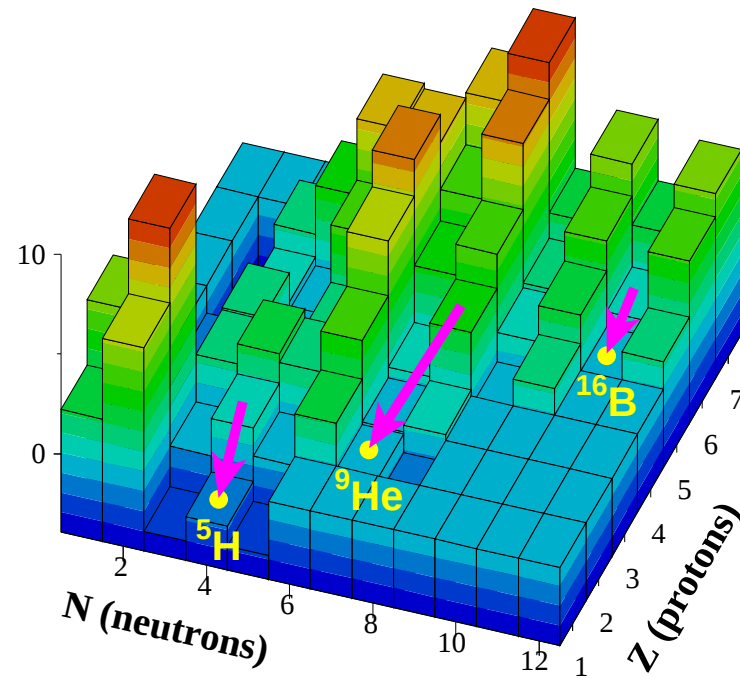
- how to dive into the sea ?
 - ▷ **strip** nucleons from a beam !
- how to find a “nucleus” ?
 - ▷ look for energy **levels** ...

Formation of unbound nuclei

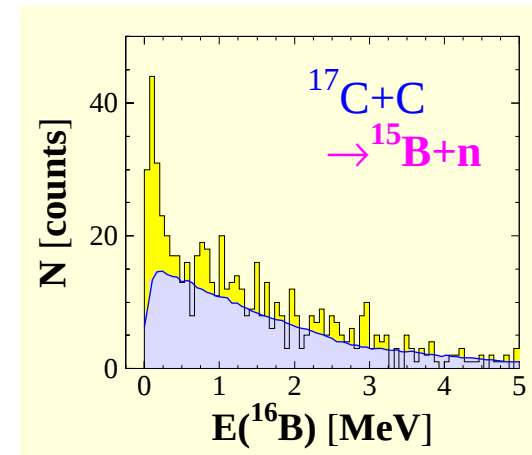
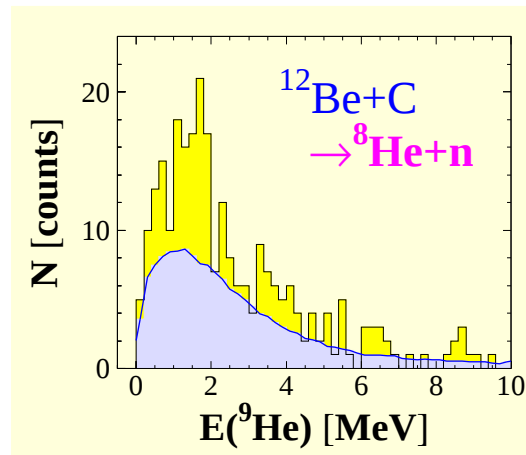
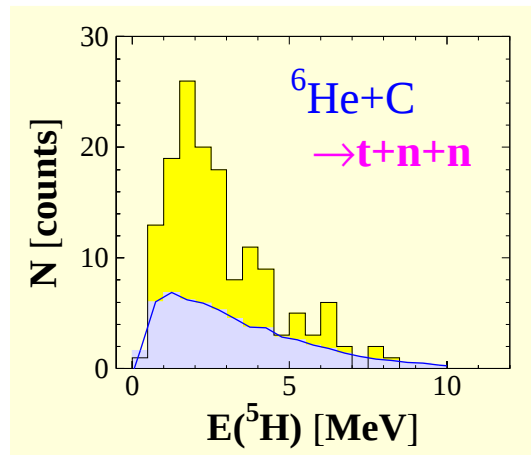


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Formation of unbound nuclei



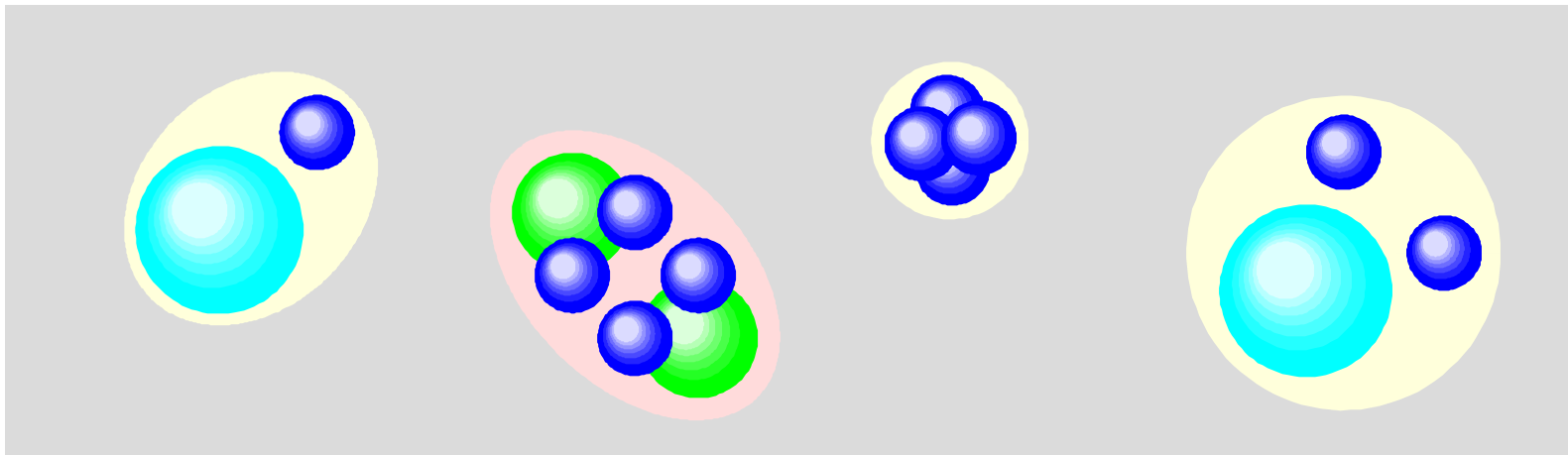
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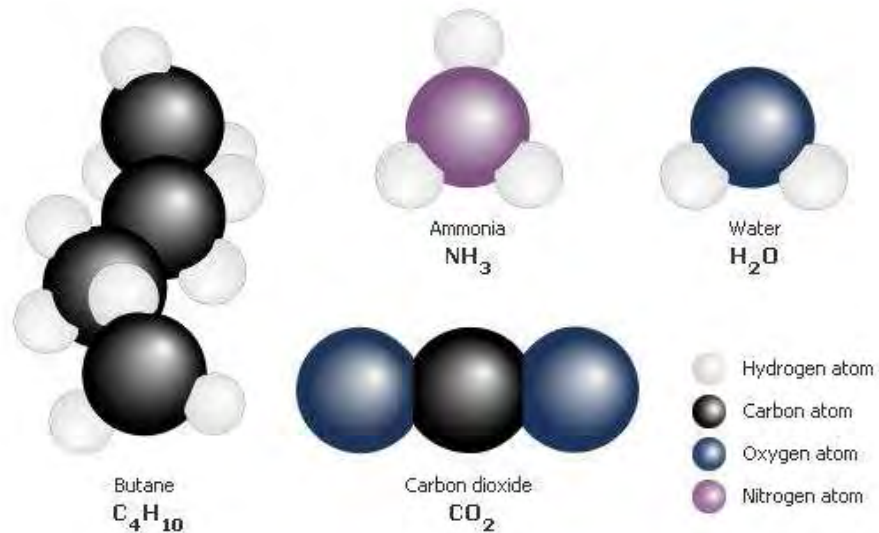
(I) The Nuclear Halo

(II) Nuclear Molecules

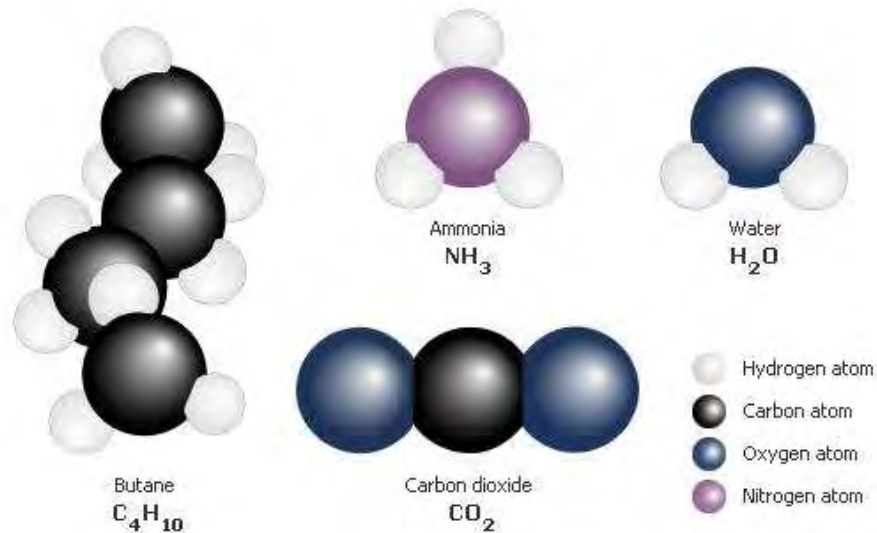
(III) Neutron Clusters ?



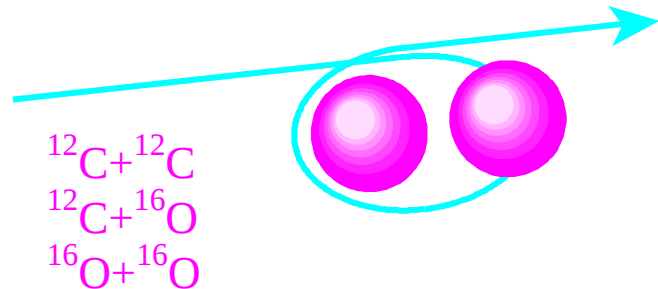
Molecules and nuclei



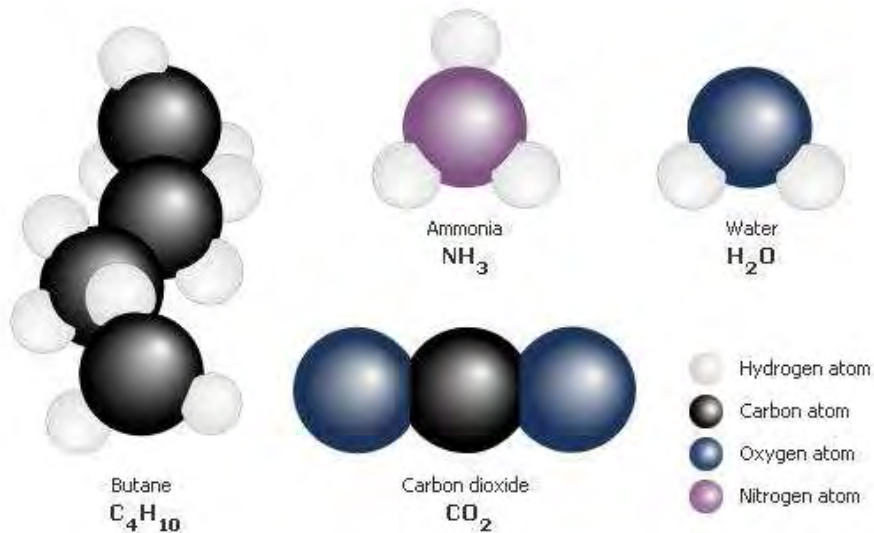
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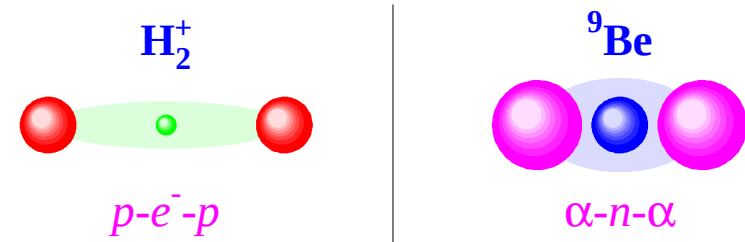
► scattering resonances :



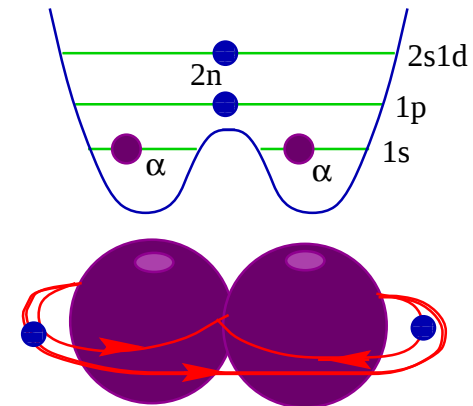
- ▷ E^* vs $J(J+1)$ systematics
- ▷ very sharp : ~ 100 keV
- ▷ orbiting : molecule-like **properties** !



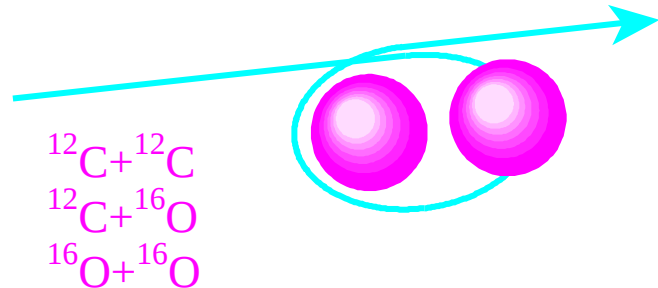
► covalent molecules ?



► two-center potential [${}^{10}Be$] :



► scattering resonances :



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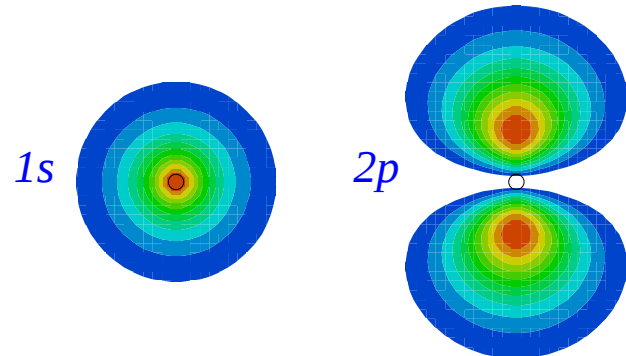
- ▷ two-center orbits !
- ▷ binding from nucleon exchange !!

Atomic orbitals

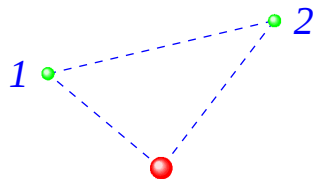
► the **hydrogen** (and -like) atom :

$$H = -\frac{1}{2}\nabla^2 - \frac{Z}{r}$$
$$E_n = -\frac{Z^2}{2n^2}$$

▷ atomic orbitals : $|\phi|^2$



► the **helium** atom :



$$H = -\frac{1}{2}[\nabla_1^2 + \nabla_2^2] - \frac{2}{r_1} - \frac{2}{r_2} + \frac{1}{r_{12}}$$

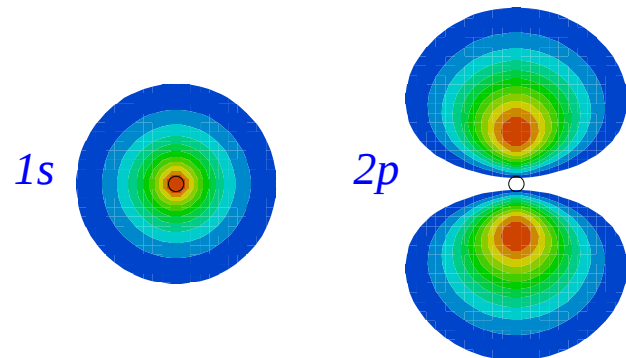
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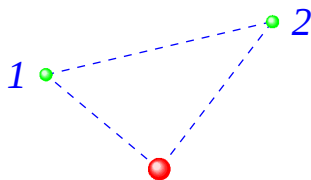
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- ▶ neglect electron “correlation” :

$$H' = -\frac{1}{2}[\nabla_1^2 + \nabla_2^2] - \frac{Z'}{r_1} - \frac{Z'}{r_2}$$

$$\Psi = \phi_H(1) \phi_H(2)$$

$$\rightsquigarrow E = E_H(1) + E_H(2) \quad +38\%$$

$$\rightsquigarrow \langle \Psi | H | \Psi \rangle \quad +5\%$$

$$\rightsquigarrow \Phi(Z') : Z' = 2 - \frac{5}{16} \quad +2\%$$

- ▷ electrons **screen** each other ...

- ▶ more complex atoms :

$$H = -\frac{1}{2} \sum_i \nabla_i^2 - \sum_i^N \frac{Z}{r_i} + \sum_{i < j} \frac{1}{r_{ij}}$$

$$\Psi = \prod_i^N \phi_H(i)$$

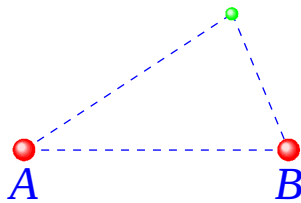
$\approx N$ one-electron [Z'] problems !

“True” molecules

Why do they form at all (and how) ?

Melvin W. Hanna (1965)

► the simplest molecule : H_2^+



$$H = -\frac{1}{2} \sum_i \nabla_i^2 - \frac{1}{r_A} - \frac{1}{r_B} + \frac{1}{R_{AB}}$$

▷ Born-Oppenheimer : **fix** nuclei...

▷ variational principle : **LCAO**

$$\Psi_{\pm} = (\phi_A \pm \phi_B)$$

$$E_{\pm} = \langle \phi_A | H_A | \phi_A \rangle + \langle \phi_B | H_B | \phi_B \rangle \\ + \left\{ \frac{1}{R_{AB}} - 2 \langle \phi_A | \frac{1}{r_B} | \phi_A \rangle \right\} \mp \langle \phi_A | H | \phi_B \rangle$$

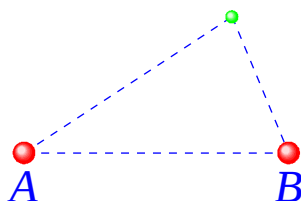
+Coulomb **-exchange** terms !

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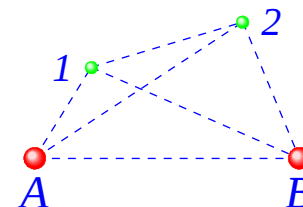
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+Coulomb –exchange terms !

► next : the H_2 molecule



$$H = H_1^+ + H_2^+ - \frac{1}{R_{AB}} - \frac{1}{r_{12}}$$

$$\Psi = \phi^+(1) \phi^+(2)$$

▷ different Ψ_{MO} approximations :

$$\rightsquigarrow \phi_{\{AB+BA\}} + \phi_{\{A+B^--A-B^+\}} \quad +6.4\%$$

$$\rightsquigarrow \phi_{\{AB+BA\}} \quad +4.9\%$$

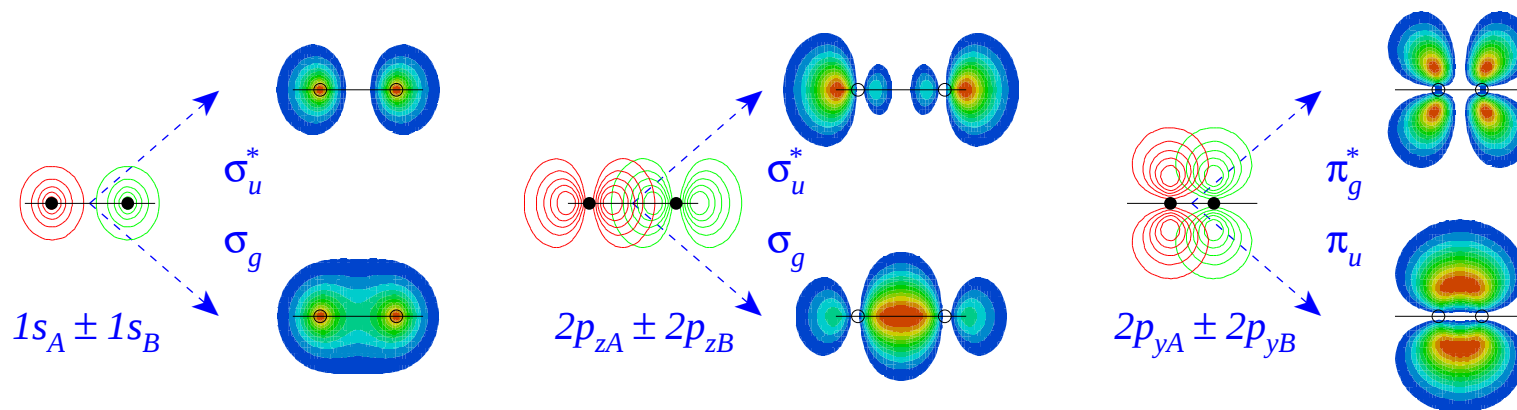
$$\rightsquigarrow \phi_{\{AB+BA\}} + \lambda \phi_{\{A+B^--A-B^+\}} \quad +4.7\%$$

$$\rightsquigarrow \{1s + \lambda 2p\}_{\{AB+BA\}} \quad +4.2\%$$

The best function has 50 variational terms. Unfortunately, all intuitive concepts disappear. It is still an open question whether accurate calculations will solve fundamental problems...

The chemical bond

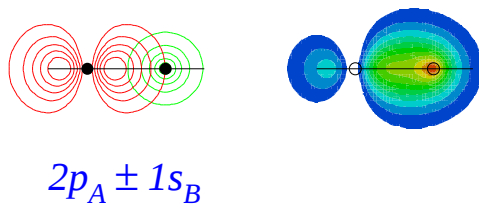
► MO theory and LCAO :



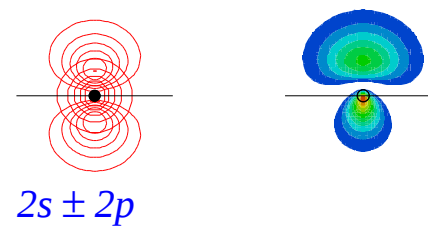
▷ $\Psi_{\pm m} : \sigma, \pi, \delta \dots$

▷ $(2N)_{AO} = (N + N^*)_{MO} \rightarrow \text{stability} !$

► proportions [H₂O] :



► geometries [$sp^3 \rightarrow 109^\circ \dots$] :

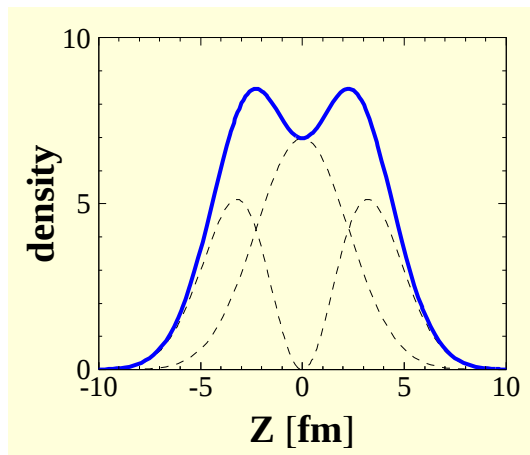


► deformed harmonic oscillator :

$$V = m \omega_0^2 r^2 / 2 \rightarrow (\omega_t, \omega_z)$$

$$E = \hbar \omega_0 (N + 3/2) \rightarrow (n_t, n_z)$$

► ${}^8\text{Be}$ [$\alpha + \alpha$] : $\omega_t / \omega_z = 2$

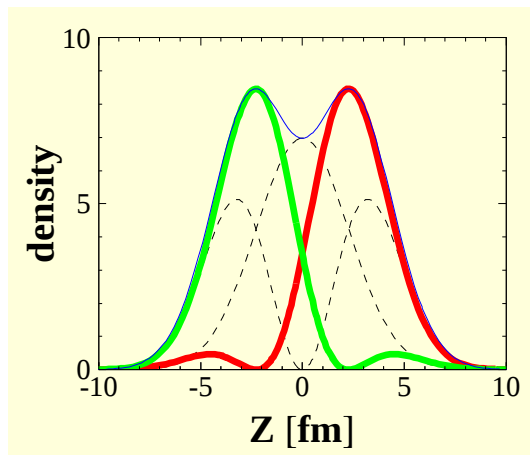


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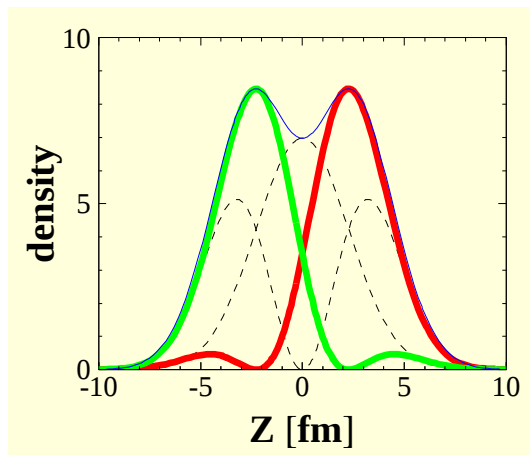
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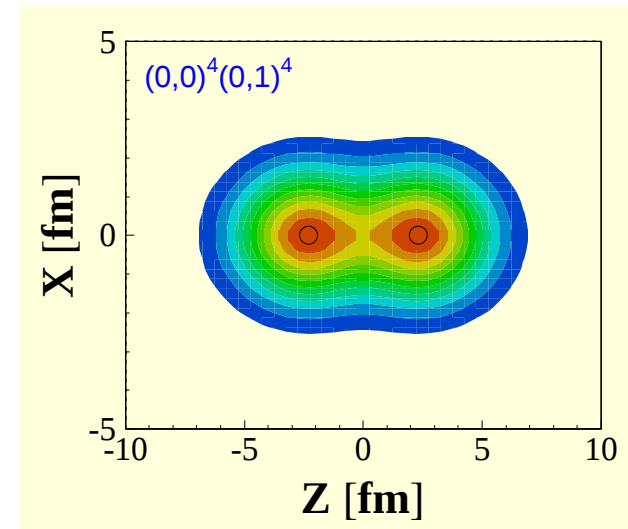


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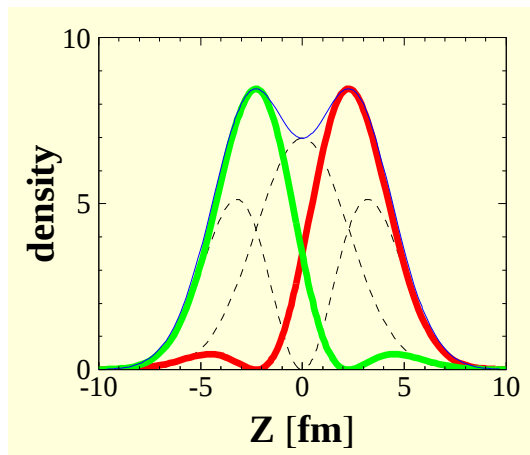
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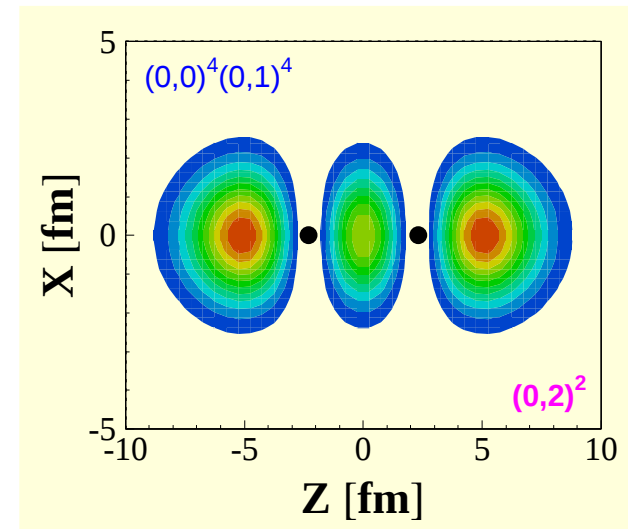


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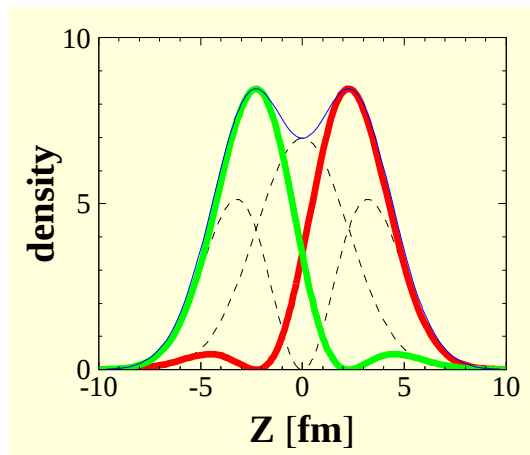
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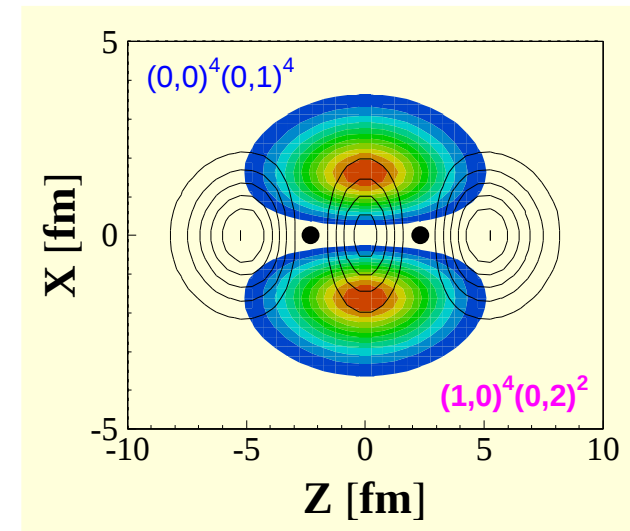


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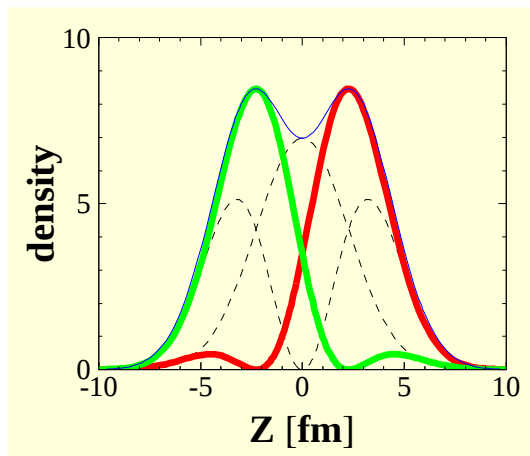
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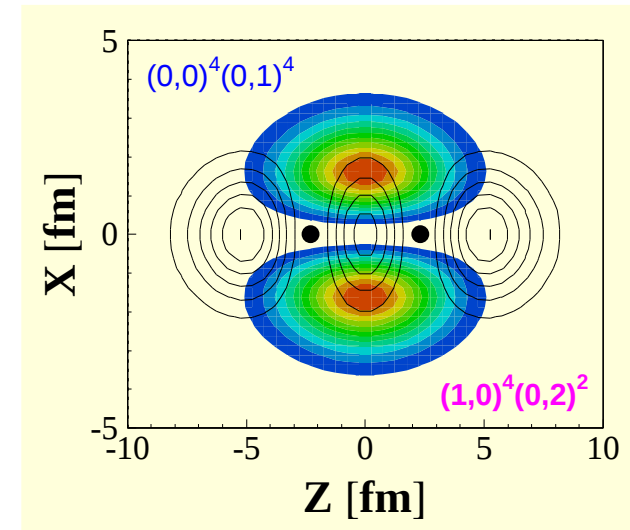


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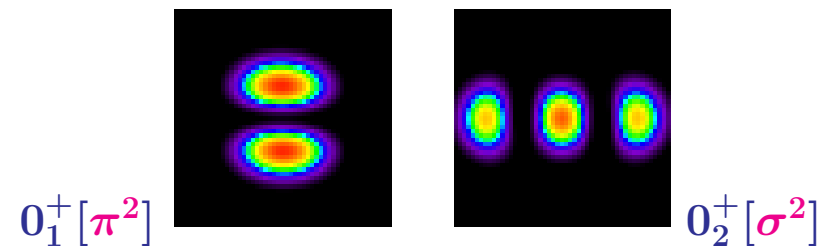
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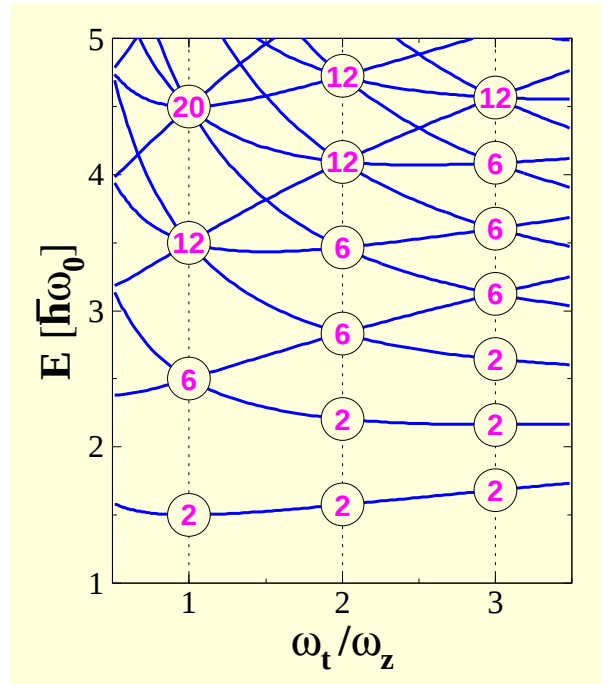
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► AMD calculations [^{10}Be] :



Deformation and clustering

► deformed HO level scheme :



$$E = \hbar \omega_0 ([n_x + n_y + n_z] + 3/2)$$

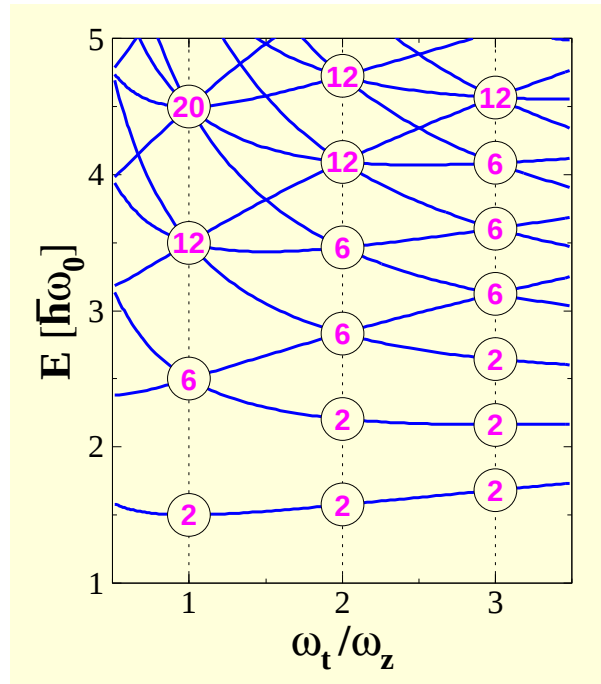
$$\rightarrow \hbar \omega_t (n_t + 1) + \hbar \omega_z (n_z + 1/2)$$

▷ new magic numbers !

▷ ^8Be , ^{20}Ne , ^{24}Mg , $^{12}\text{C}^{*/\text{gs}}$...

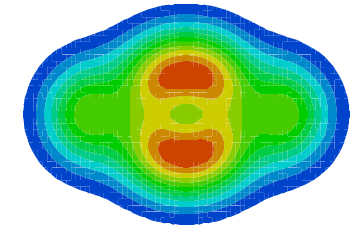
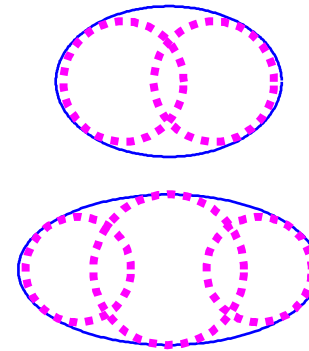
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► not only $|2p2n\rangle_{L=0} [\alpha]$:

▷ **spherical** degeneracies repeated !



$^{24}\text{Mg} \equiv \alpha^{16}\text{O} \alpha$

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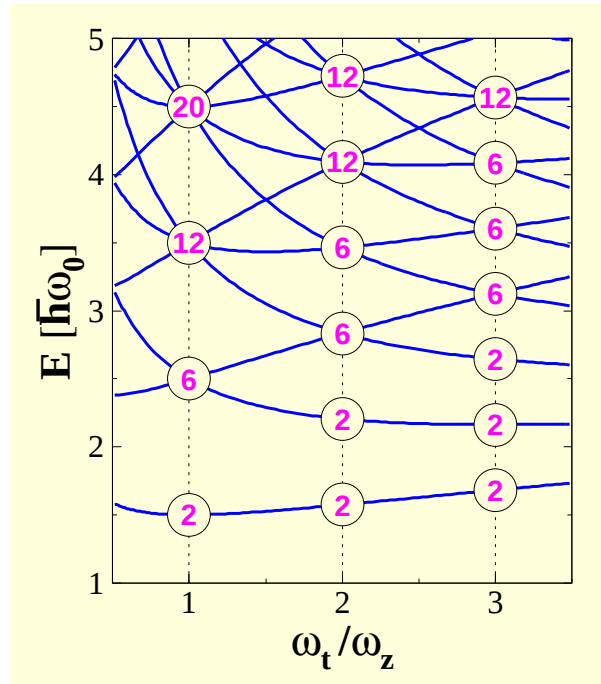
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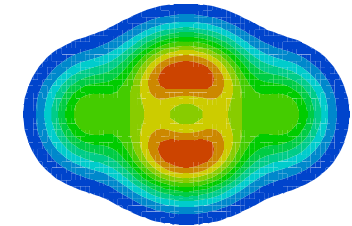
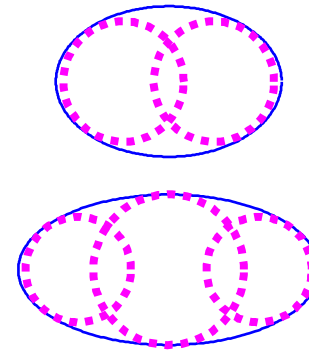
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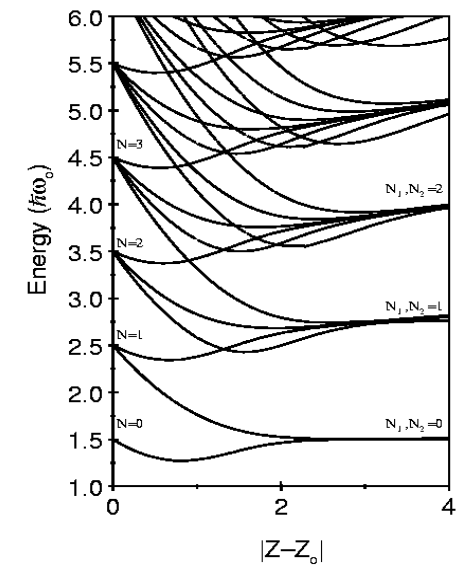
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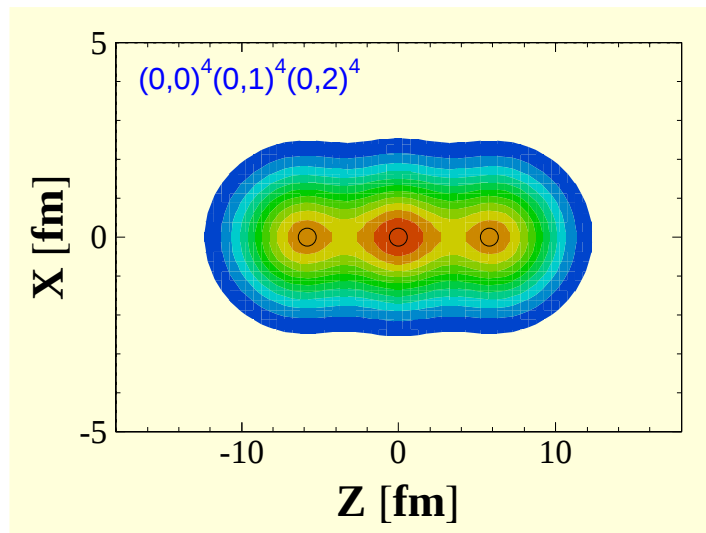
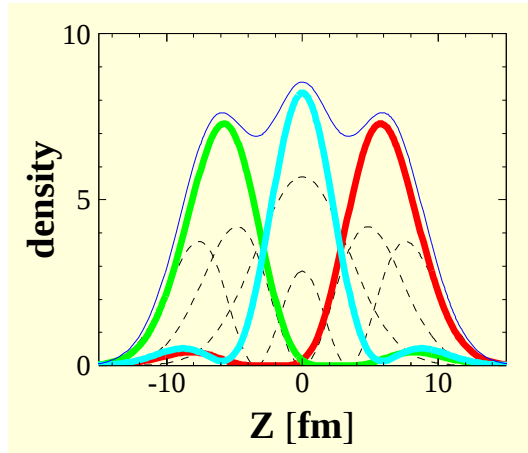
- two-center HO :

- fission
- fusion



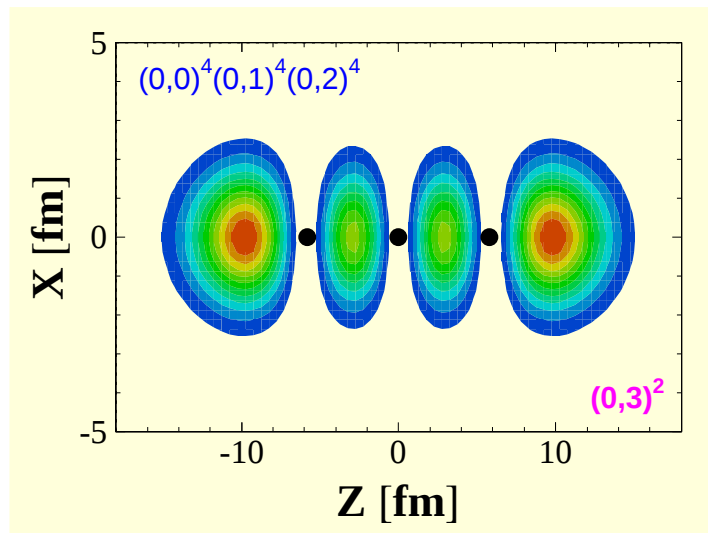
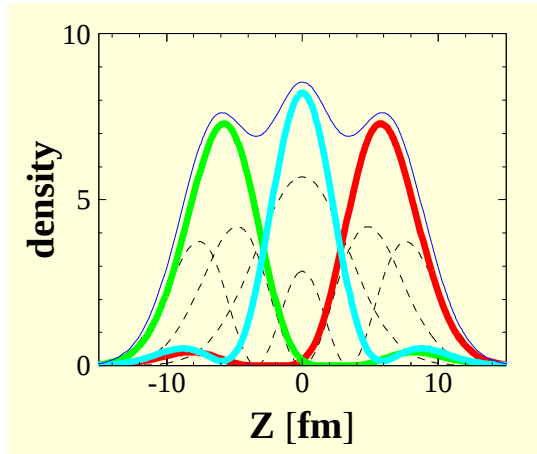
Binding more complex molecules

► three centers [$\omega_t/\omega_z = 3$] : ^{12}C



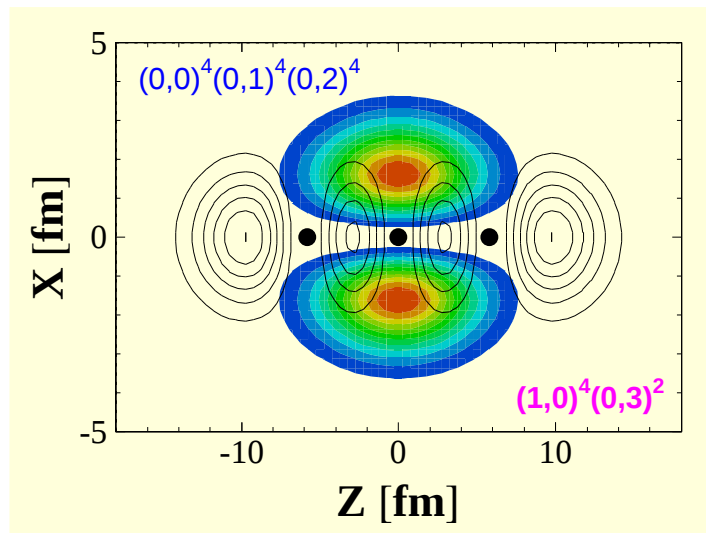
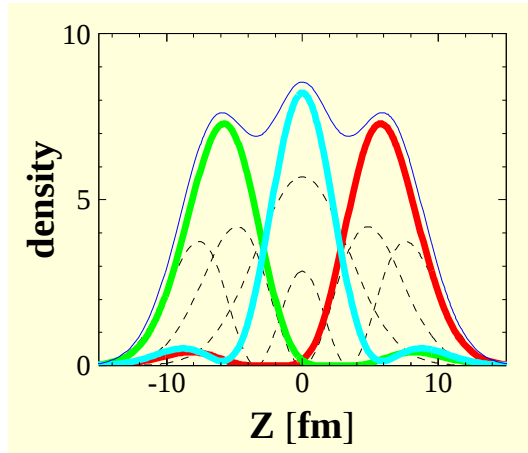
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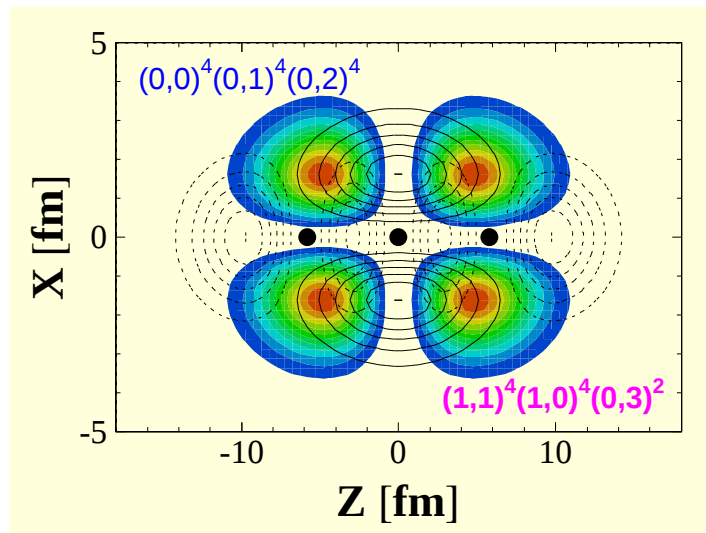
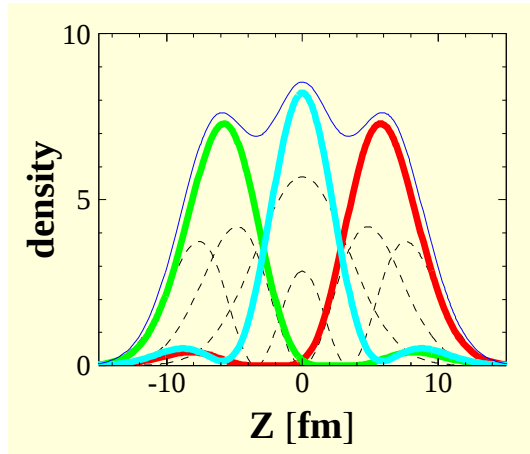
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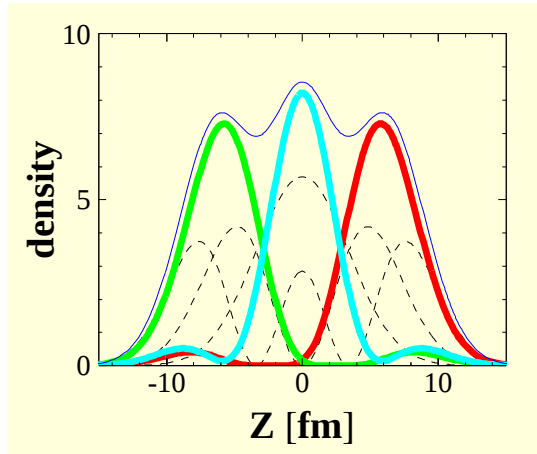
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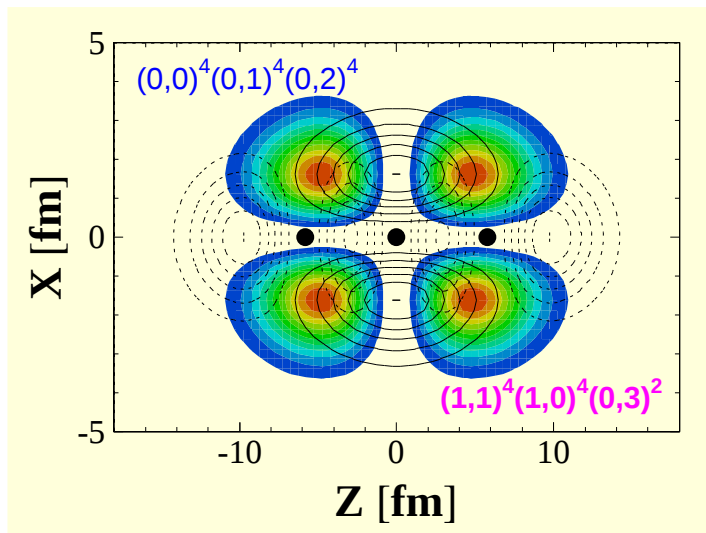
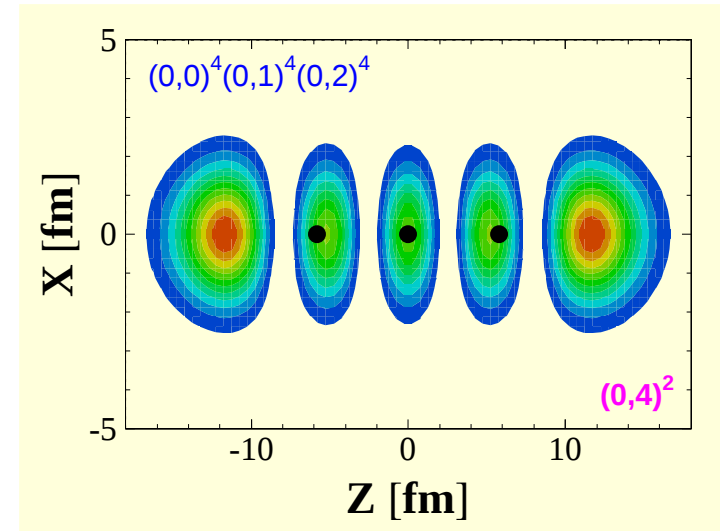


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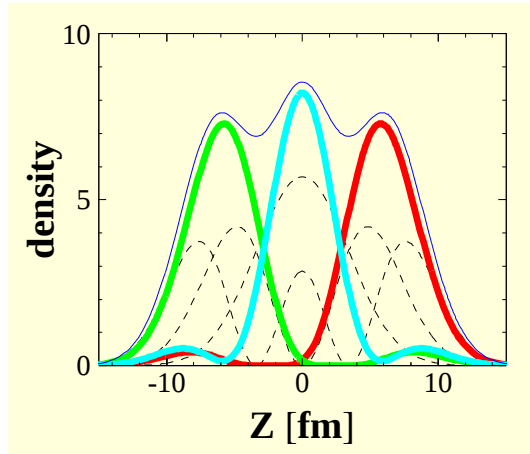


► beyond ^{22}C : σ^* , π^* ...

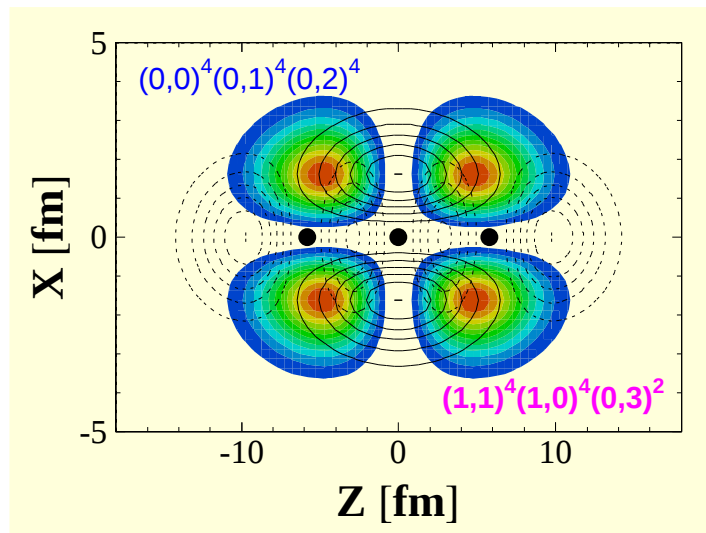
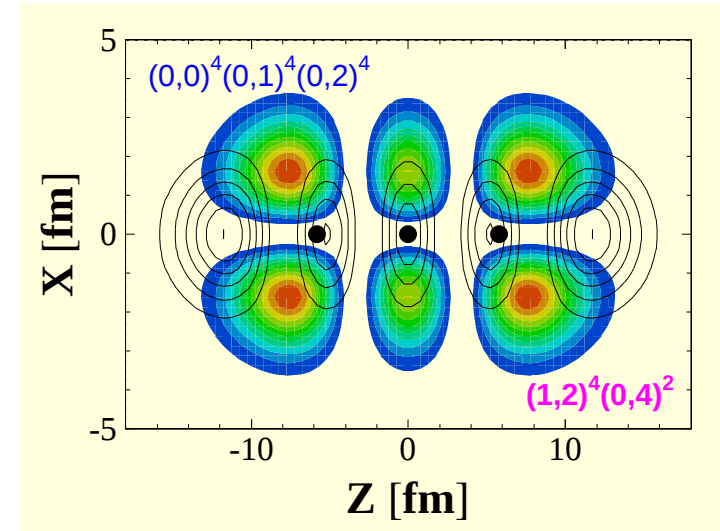


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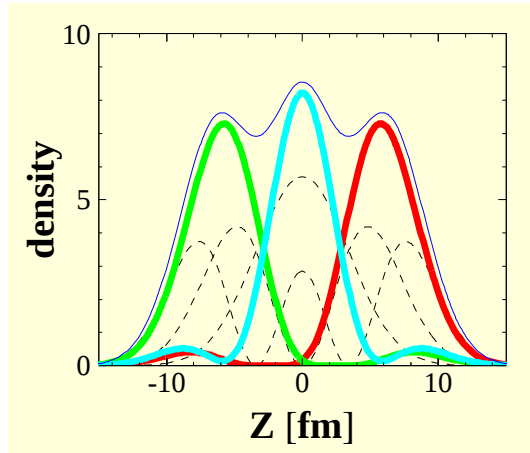


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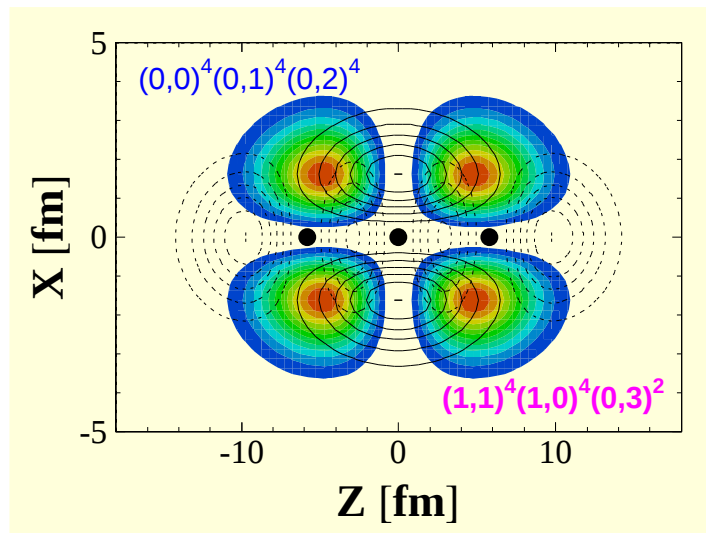
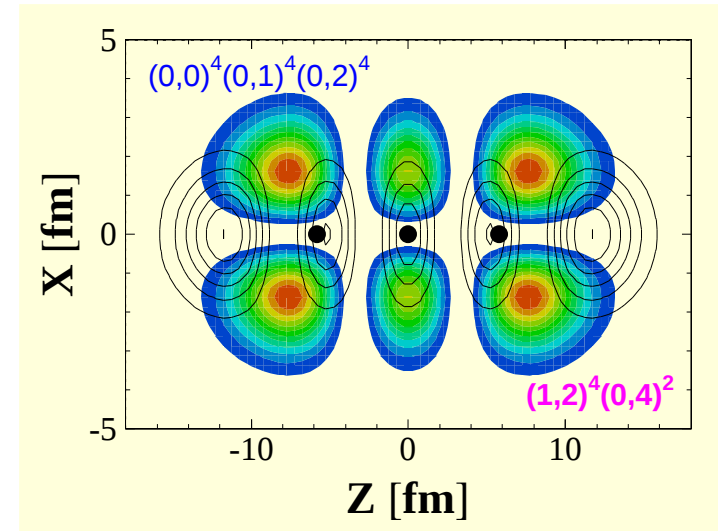


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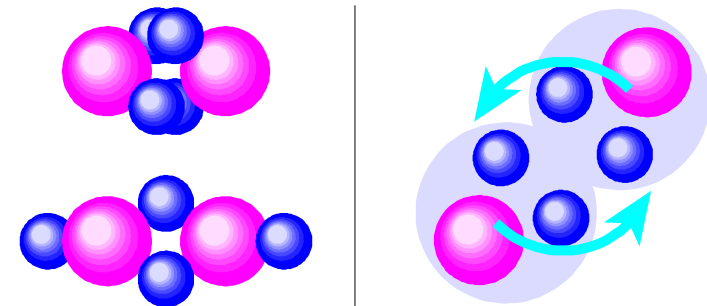
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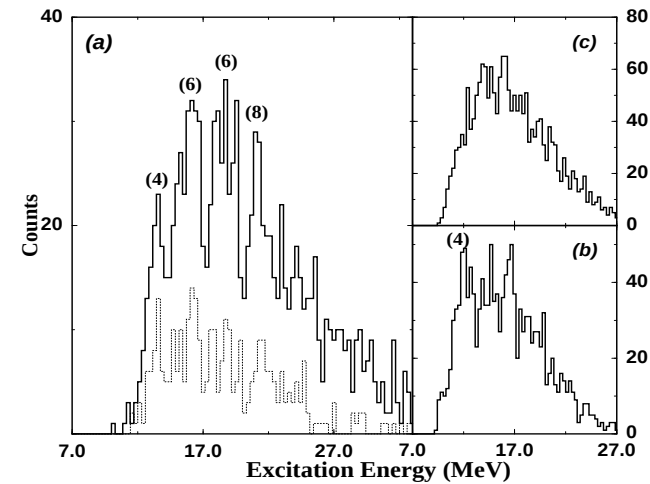
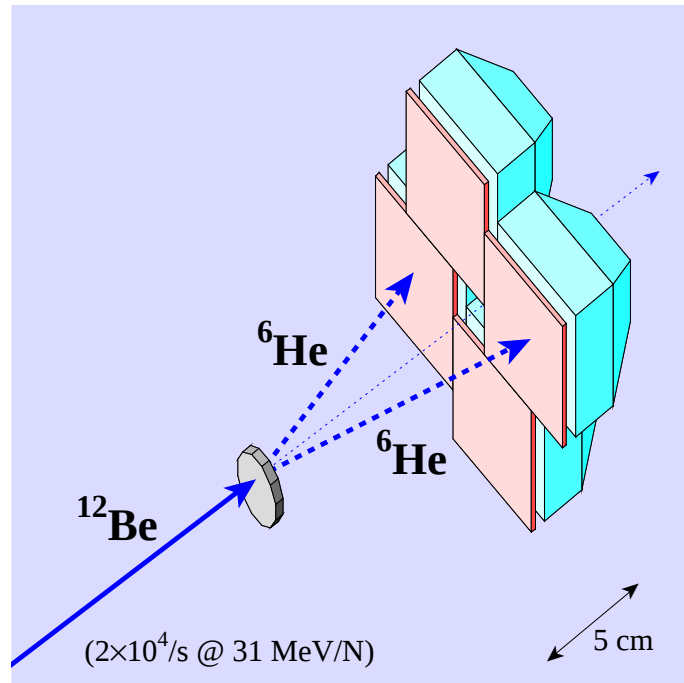


► heavier molecules ? [^{12}Be]



A halo around another halo

► inelastic excitation of ^{12}Be :

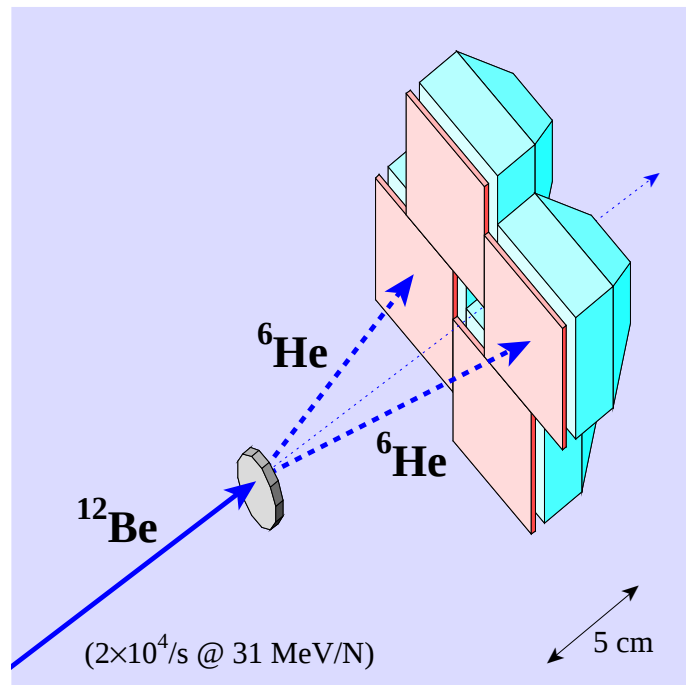


▷ just breakup or $^{12}\text{Be}^* \rightarrow {}^6\text{He} + {}^6\text{He}$?

▷ are they associated to α -4n- α ?

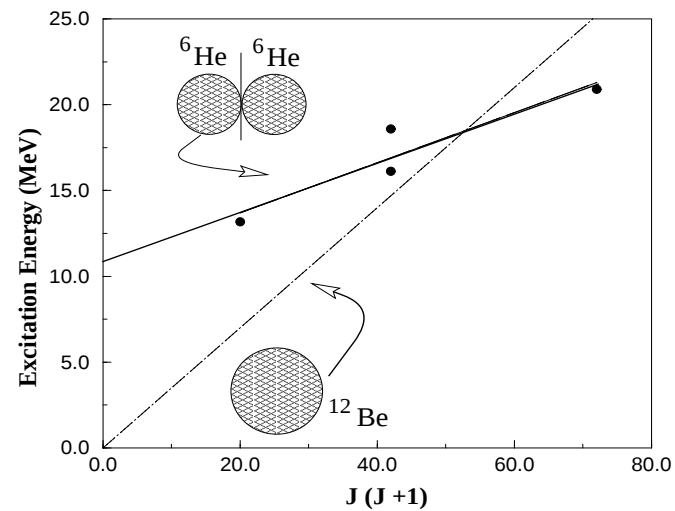
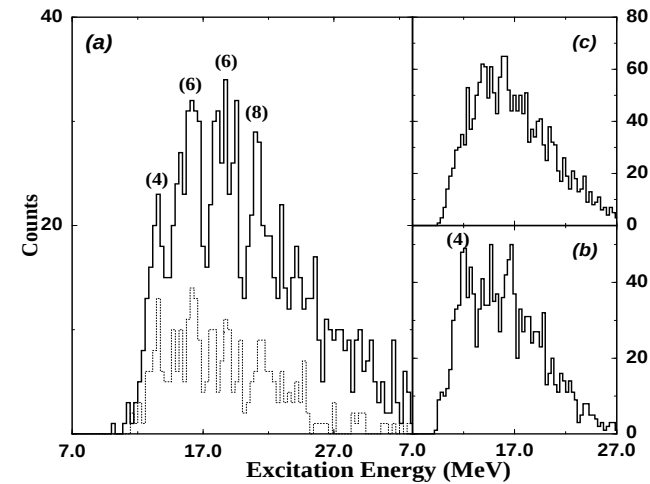
A halo around another halo

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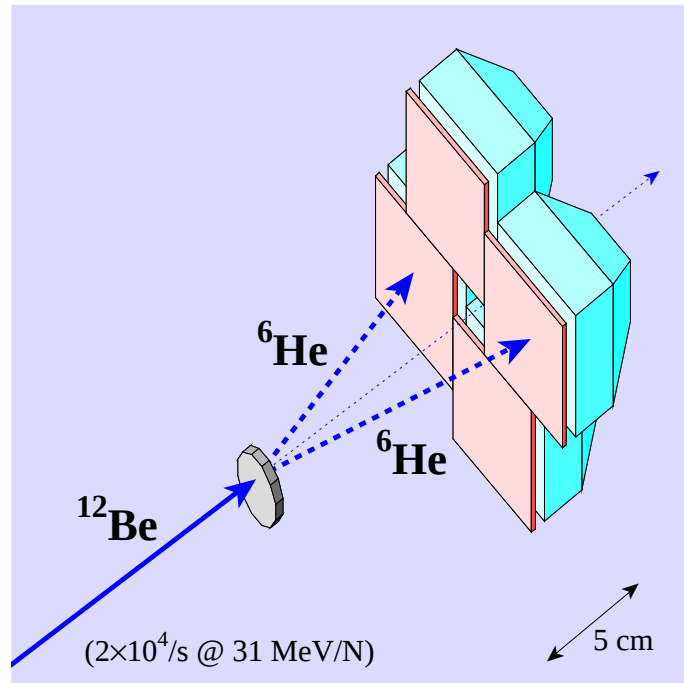
▷ just breakup or $^{12}\text{Be}^* \rightarrow {}^6\text{He} + {}^6\text{He}$?

▷ are they associated to α -4n- α ?



A halo around another halo

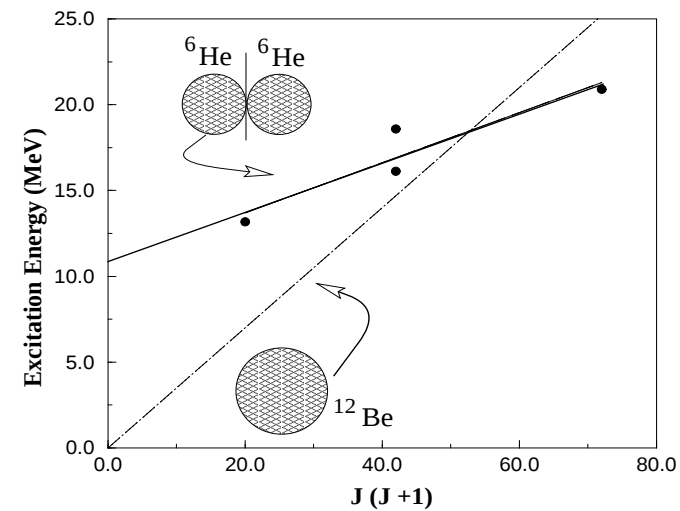
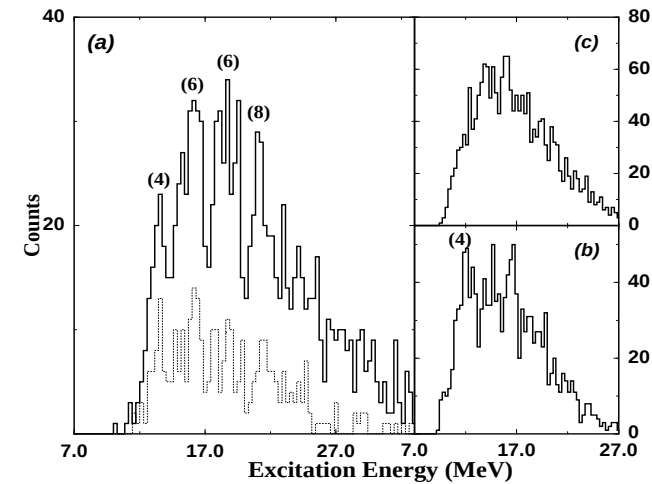
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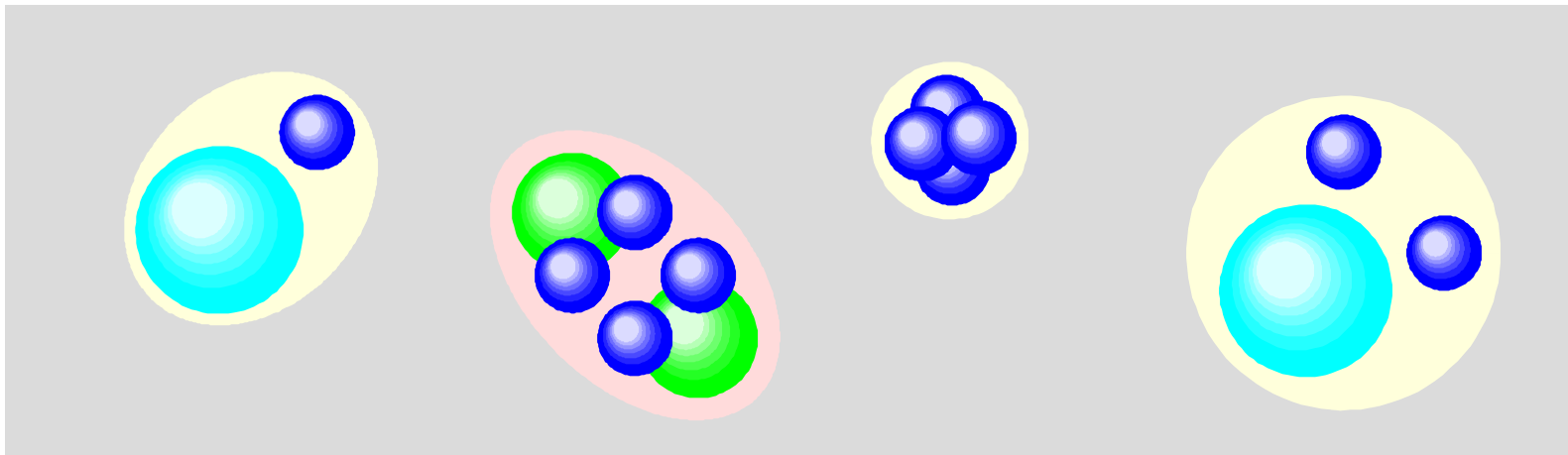
► missing mass $[{}^9\text{Be}({}^{13}\text{C}, {}^{11}\text{C}){}^{11}\text{Be}^*]$...



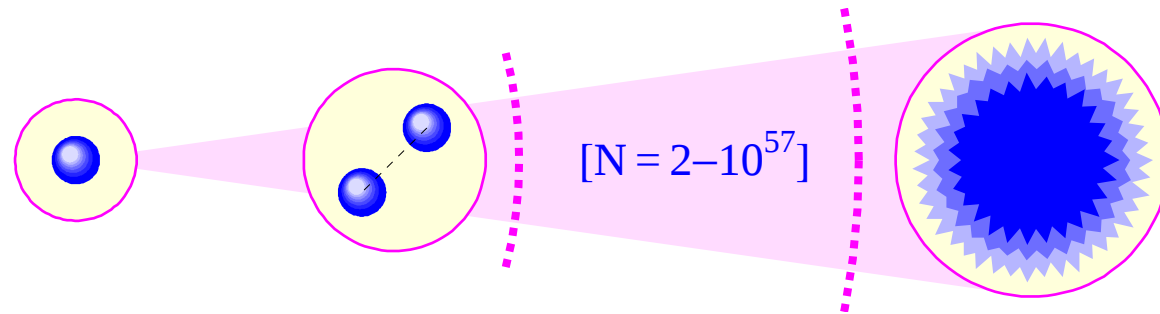
(I) The Nuclear Halo

(II) Nuclear Molecules

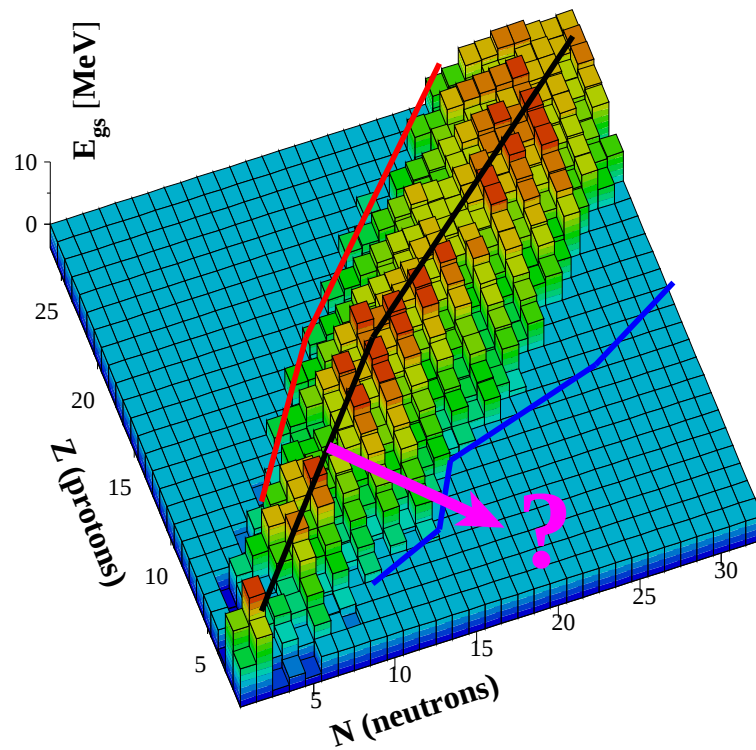
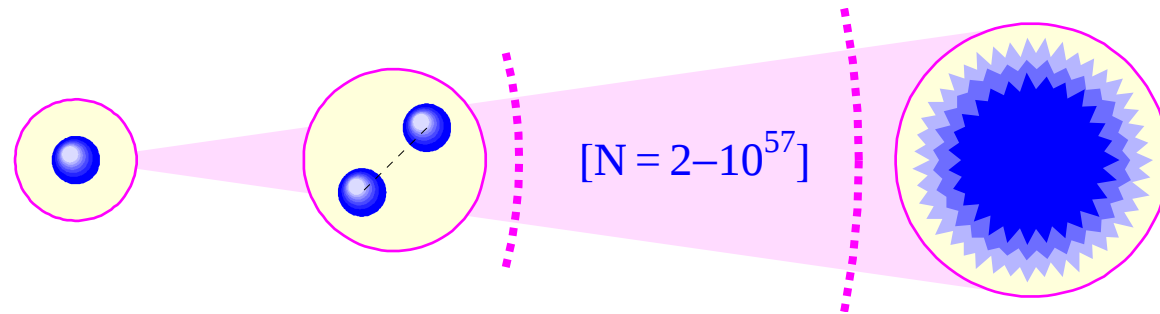
(III) Neutron Clusters ?



More than 2 neutrons



More than 2 neutrons



► neutron-rich beams :

$$\rightsquigarrow N \gtrsim 2 ???$$

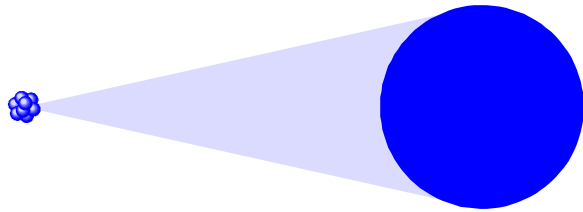
Neutron stars : from few to 10^{57}

► a huge liquid, neutral drop :

$$B/A \approx (a_v - a_a) + \frac{3}{5} G \frac{m_n^2}{r_0} A^{2/3}$$

▷ $(A_{min})_{B>0}$ depends on a_a !!!

a_a [MeV]	A_{min}	R [km]	$M/M_\odot$
23	$4 \cdot 10^{55}$	3.4	0.04
$28 - \frac{33}{A^{1/3}}$	10^{57}	10	1



▷ roughly a neutron star...

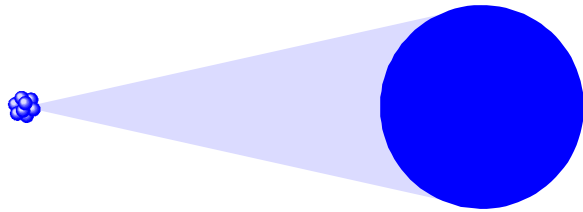
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▷ roughly a neutron star...

► Fermi gas model :



▷ Fermi pressure vs gravity :

$$\langle E_{kin}/N \rangle \propto (N/R^3)^{2/3}$$

$$\langle E_{pot}/N \rangle = -\frac{3}{5} G \frac{N m_n^2}{R}$$

$$\frac{d}{dR} \langle E/N \rangle = 0$$

▷ for $M = 1.5 M_\odot$:

$$\rightsquigarrow R = 12 \text{ km}$$

$$\rightsquigarrow \rho = 1.5 \rho_{nuc}$$

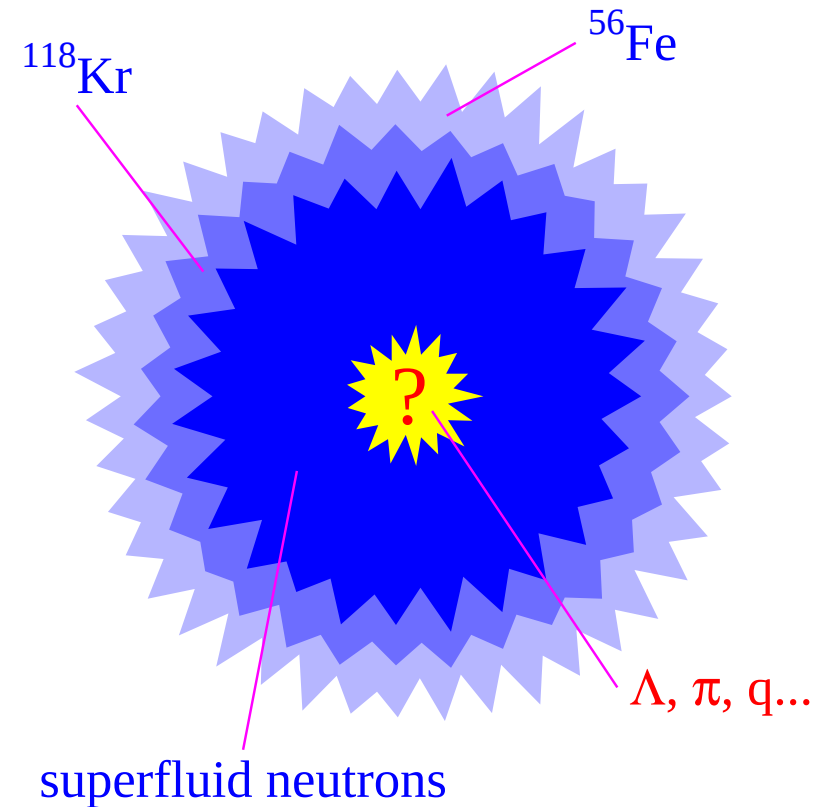
► ρ can be higher : Λ , quarks ...

Neutron stars : from 10^{57} to few ?

► the $^{56}\text{Fe} \rightarrow 56n$ collapse :

ρ [g/cm ³]	stage
10^9	p capture e^-
10^9 – 10^{11}	n-rich nuclei
10^{11}	n drip
10^{12}	superfluid n pairs
$2 \cdot 10^{14}$	nuclei dissolve
$> 4 \cdot 10^{14}$	superfluid n + ?

- ▷ rotation period $\sim$ ms !!!
- ▷ magnetic field $\sim 10^{14}$ G !!!
- ▷ $^A n$ in inner crust ?

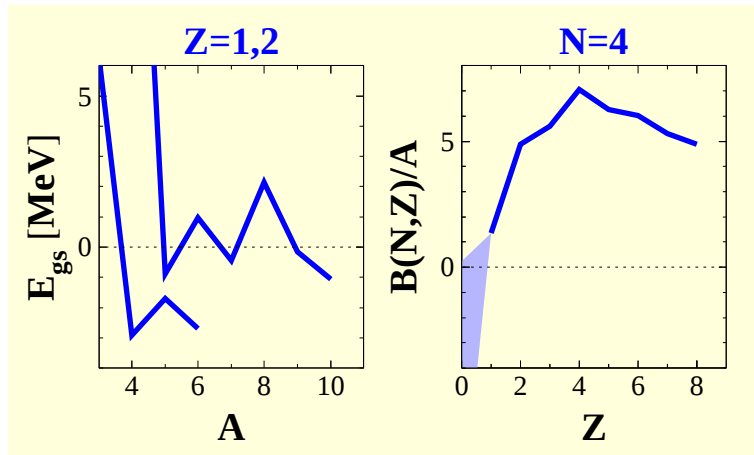


► models explore $V_{NN}(N > Z)$:

- ▷ $R_n(^{208}\text{Pb}) \leftrightarrow R_{star} \dots$
- ▷ better explore $V_{nn}(Z=0)$!!!

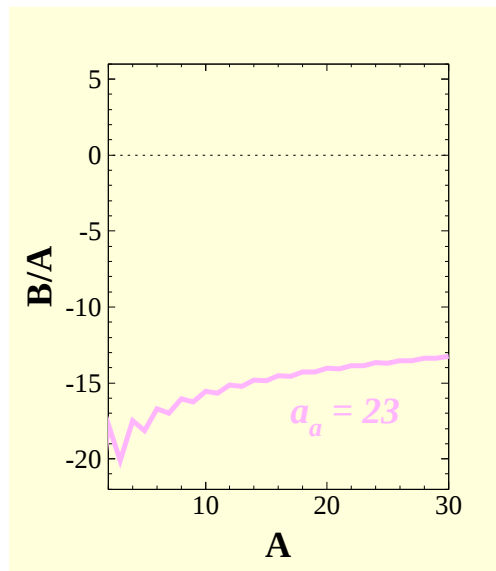
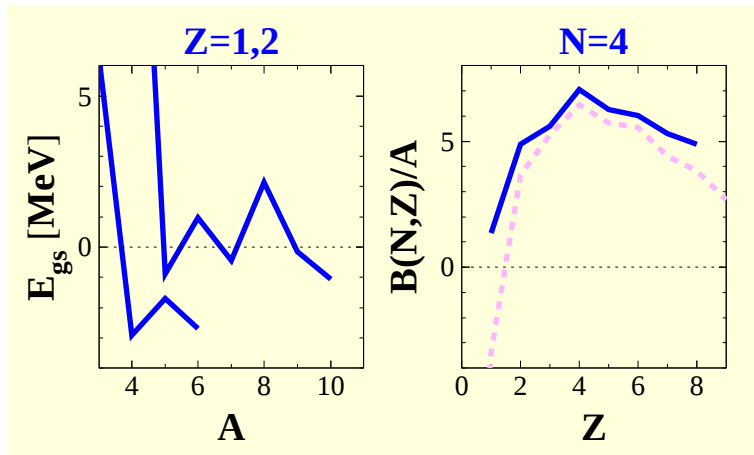
The landscape in 2001

► known masses & asymmetry :



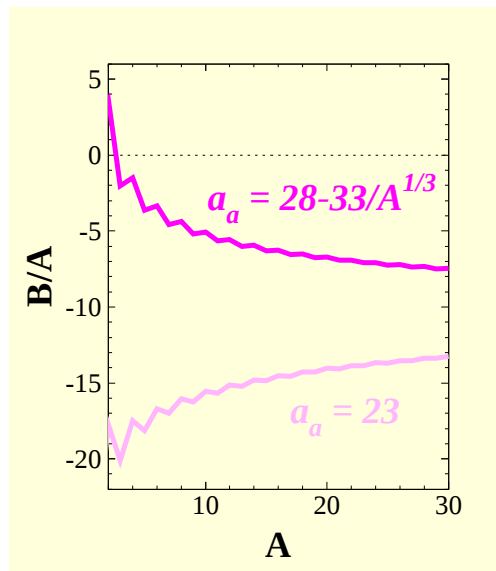
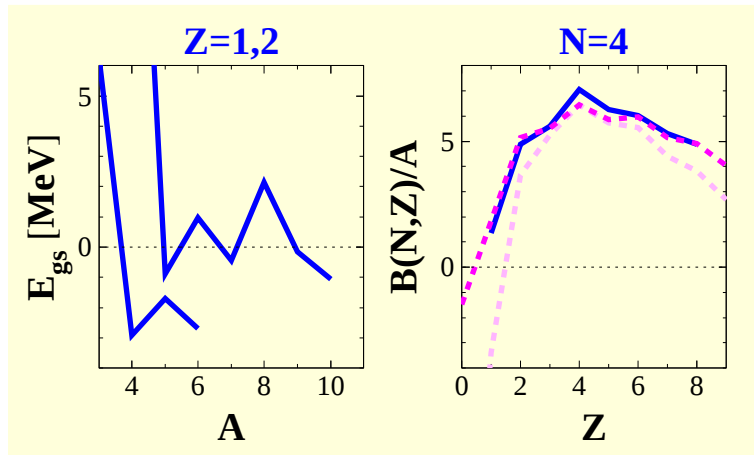
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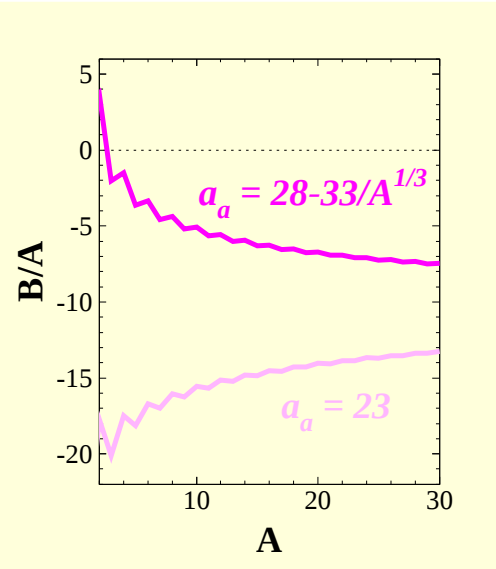
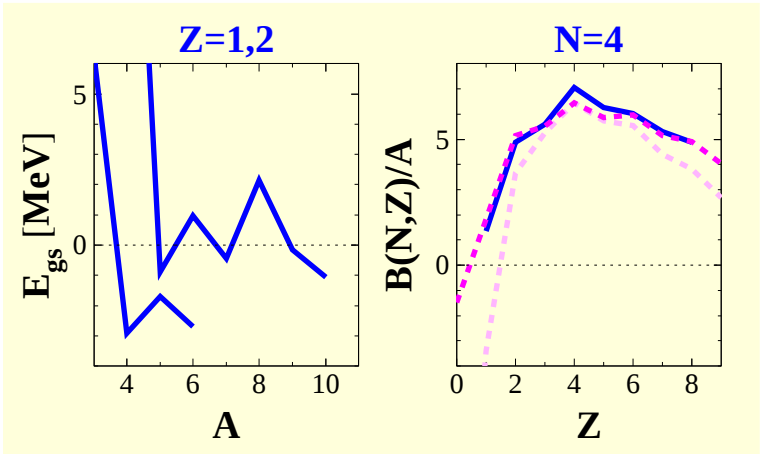
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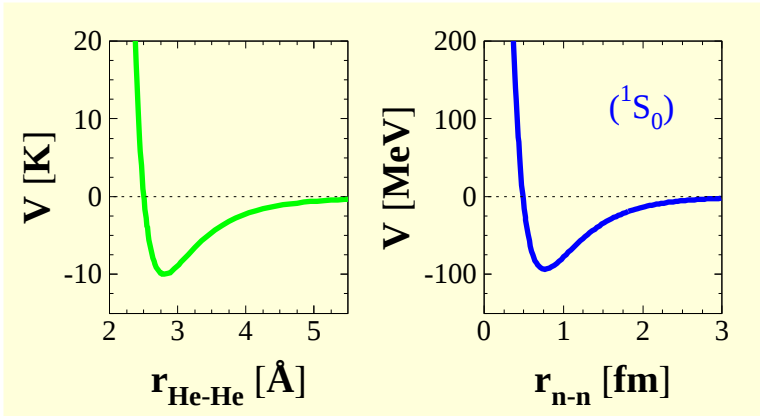


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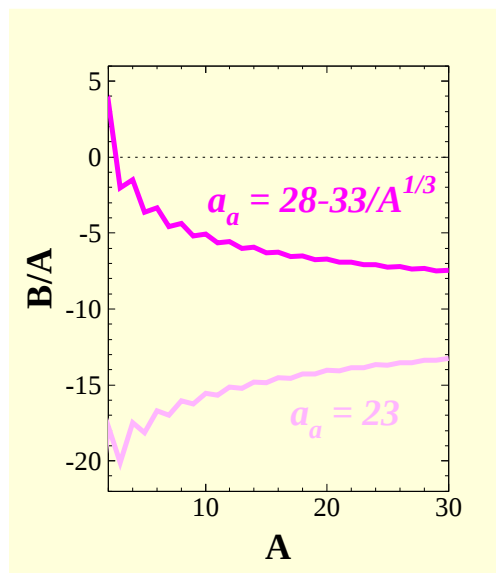
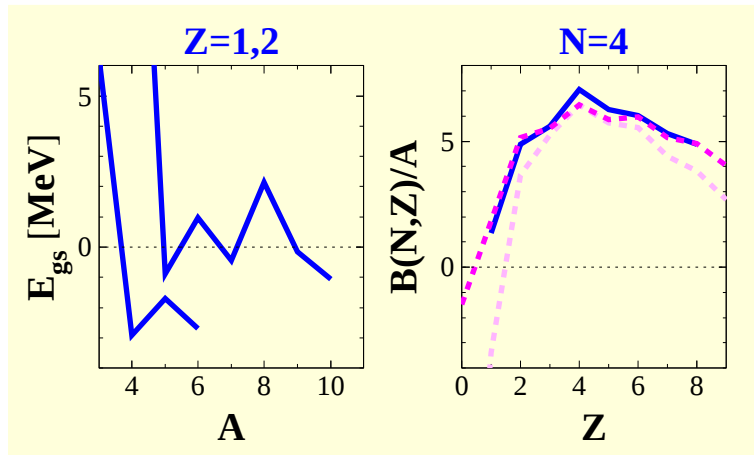


► few fermions bound ?

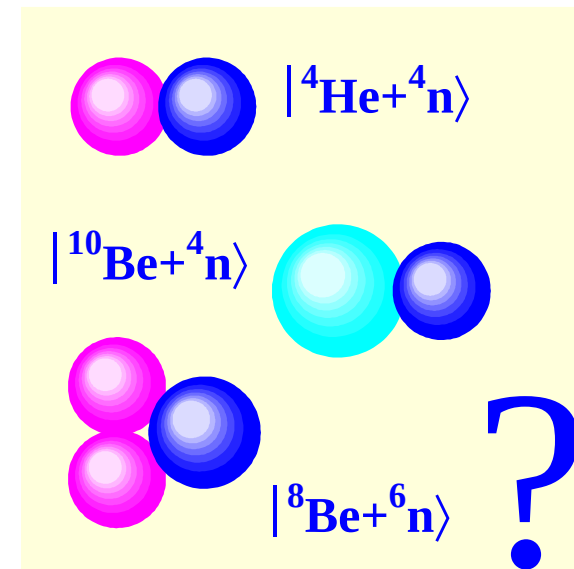
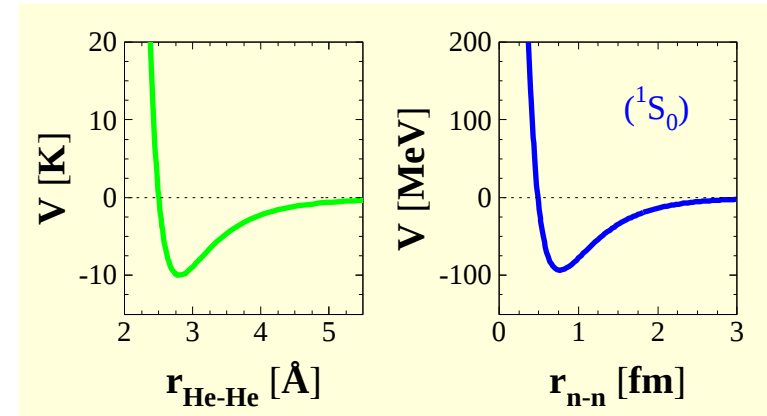


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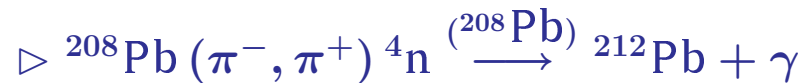
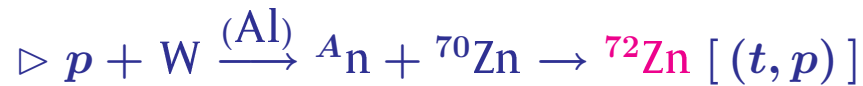


► few fermions bound ?



1960s-2000s : a long, unsuccessful quest

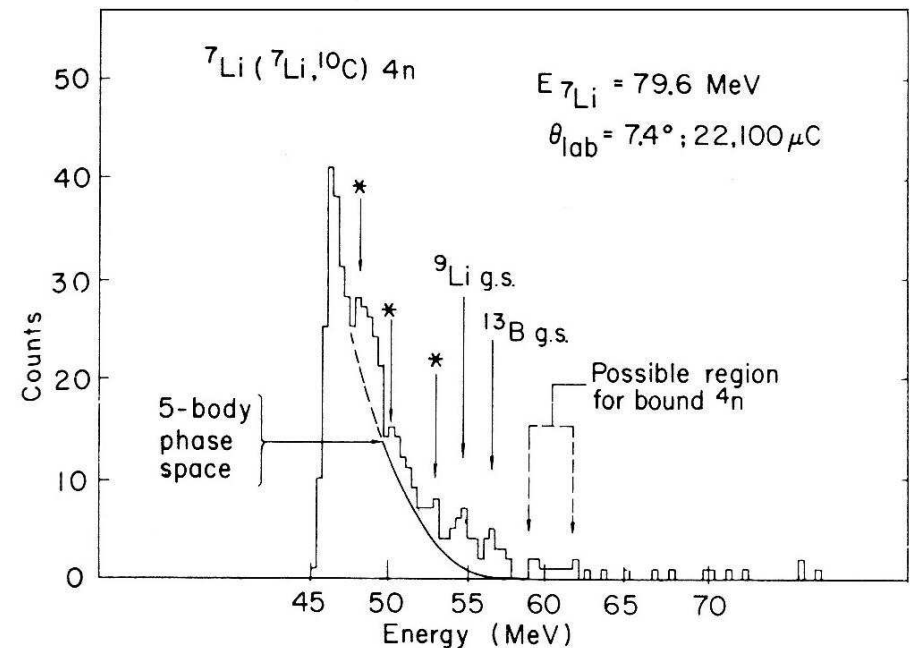
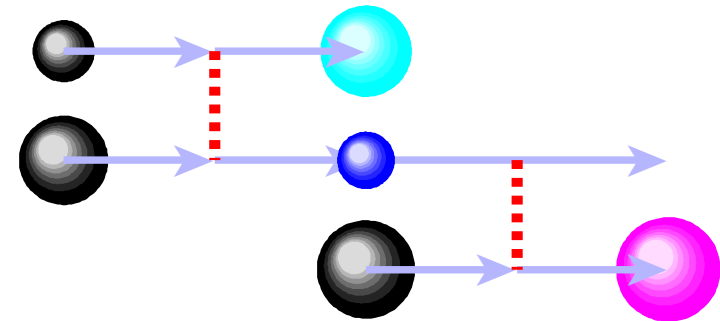
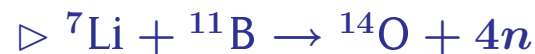
► two-step reactions :



► pion charge exchange :



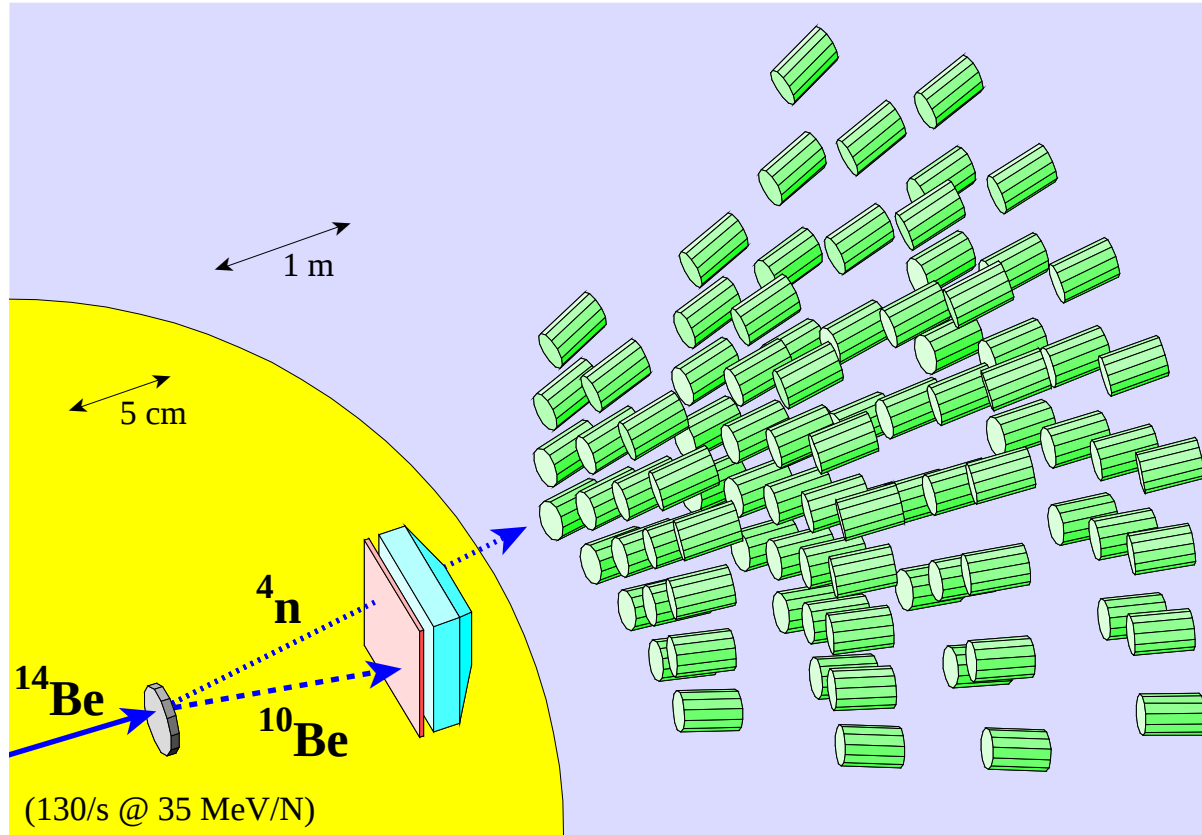
► multinucleon transfer :



↪ bcks + cross-sections ...

The principle ...

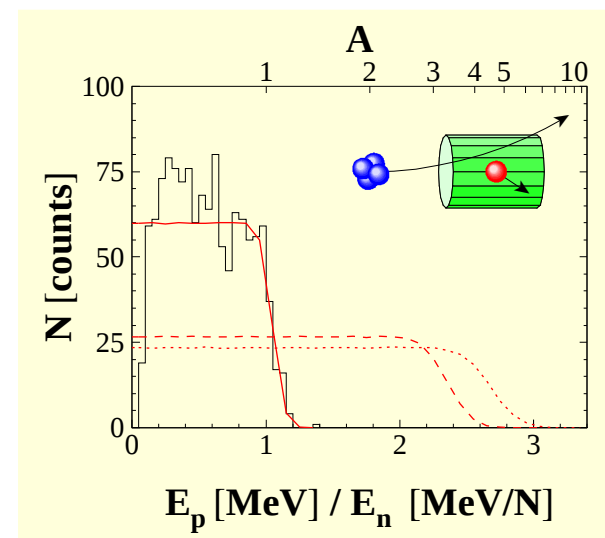
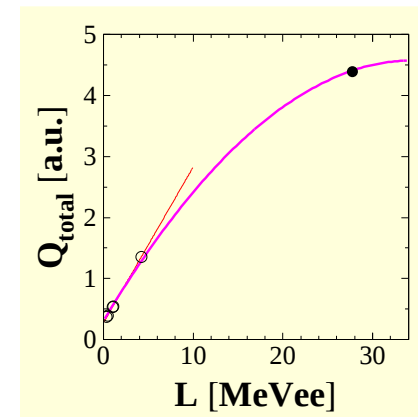
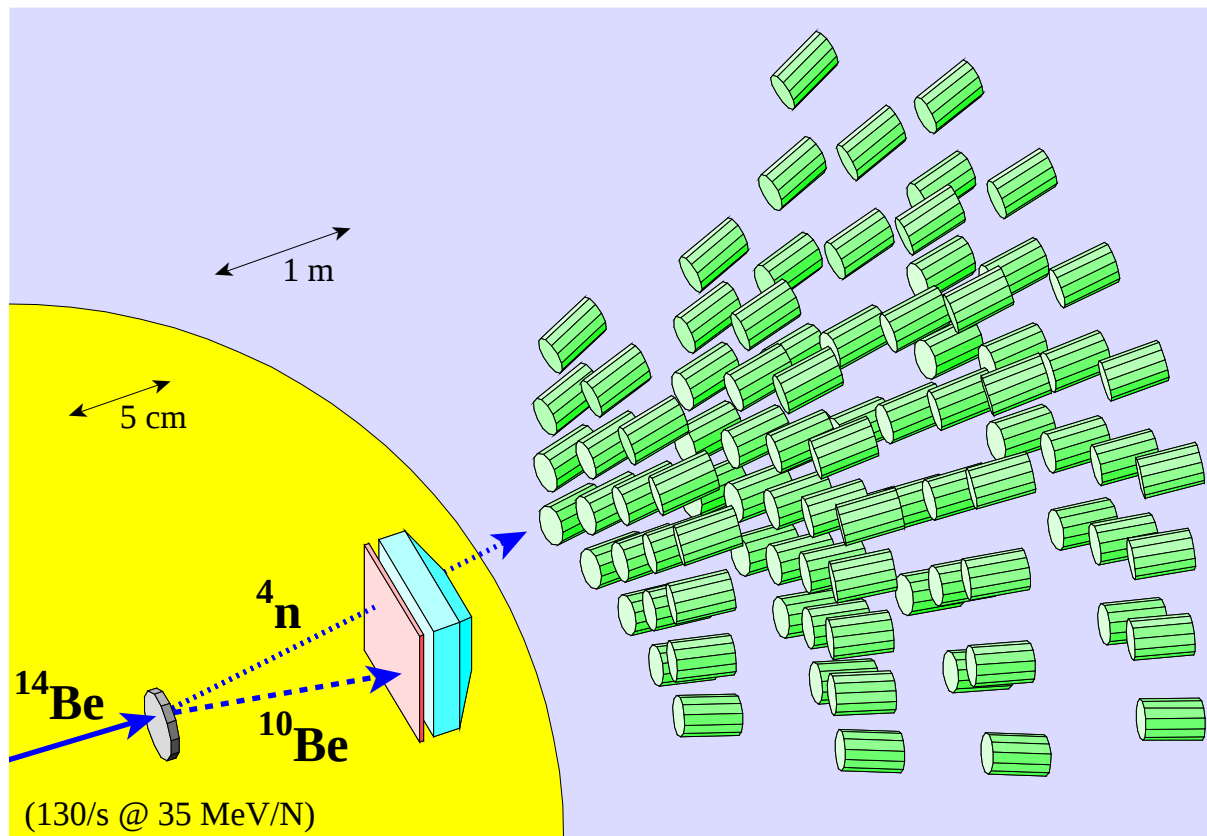
$$\triangleright |^{14}\text{Be}\rangle \equiv a |^{10}\text{Be} + {}^4\text{n}\rangle + \dots$$



▷ effective + clean

The principle ...

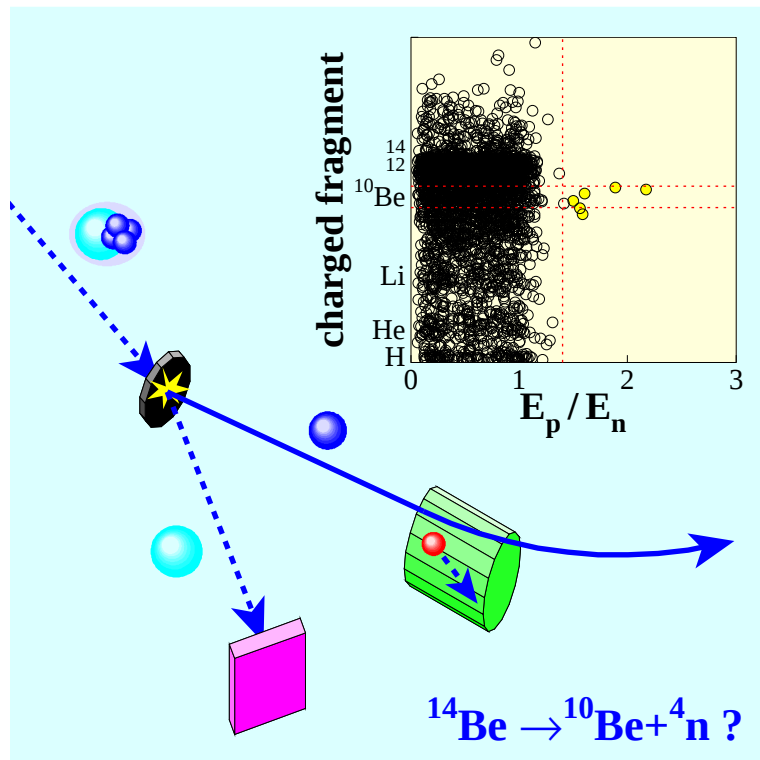
► $|^{14}\text{Be}\rangle \equiv a |^{10}\text{Be} + ^4\text{n}\rangle + \dots$



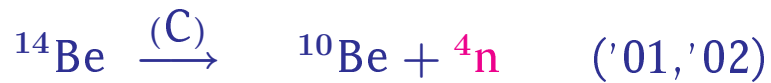
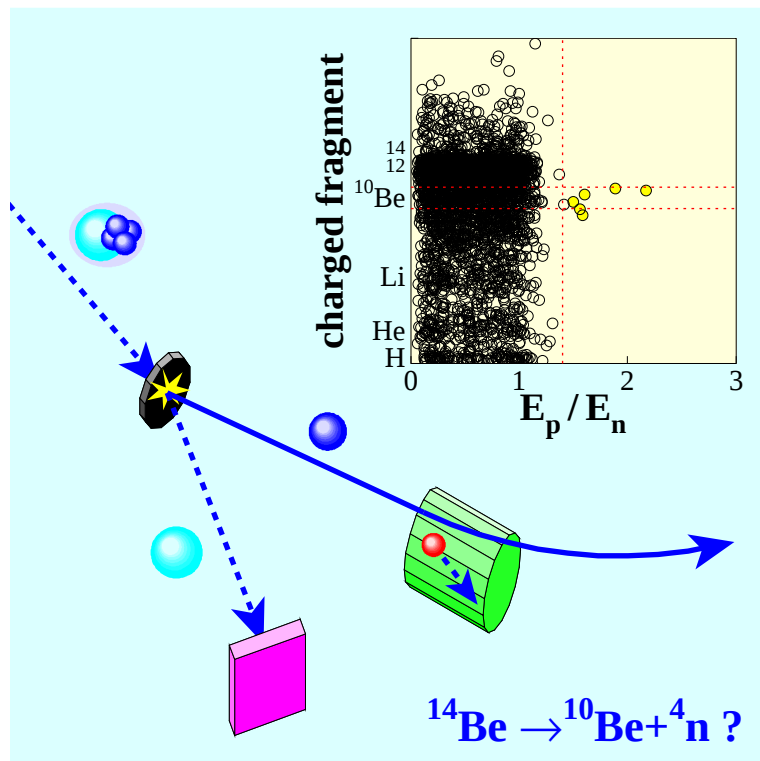
▷ effective + clean + sensitive !!!

▷ saturation (sensitive to low E_p) ...

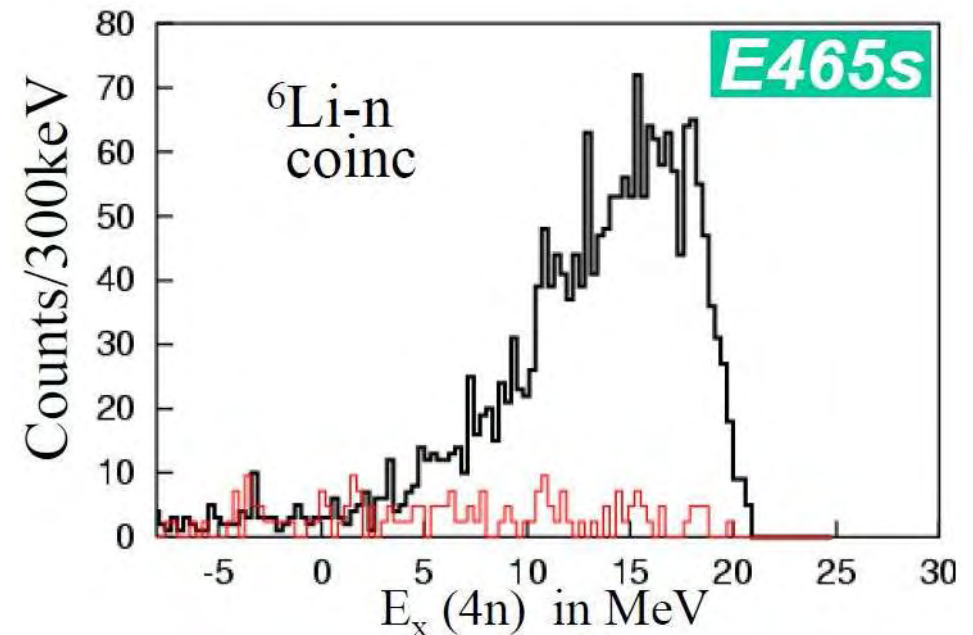
... and the results



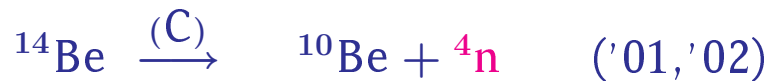
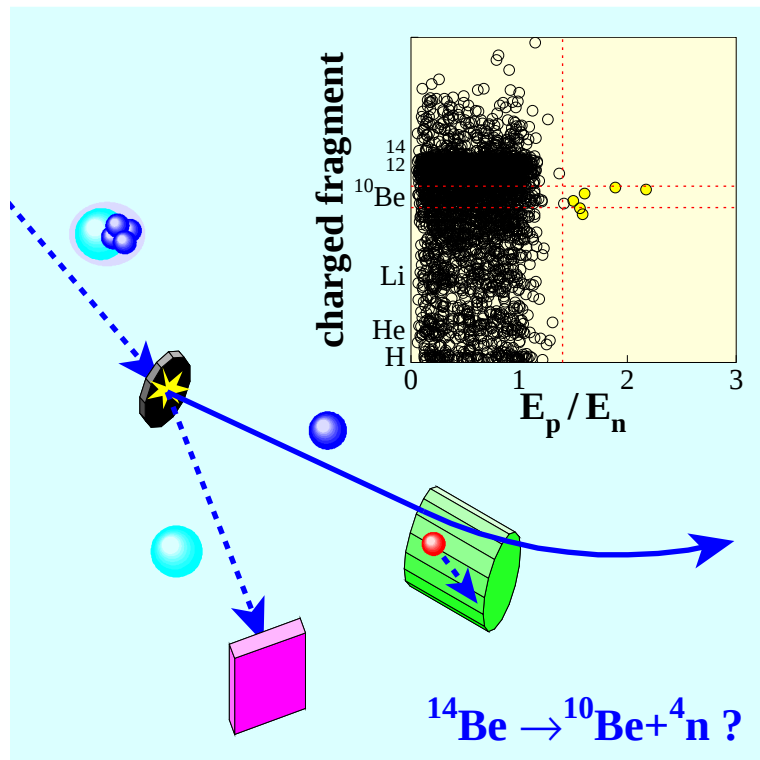
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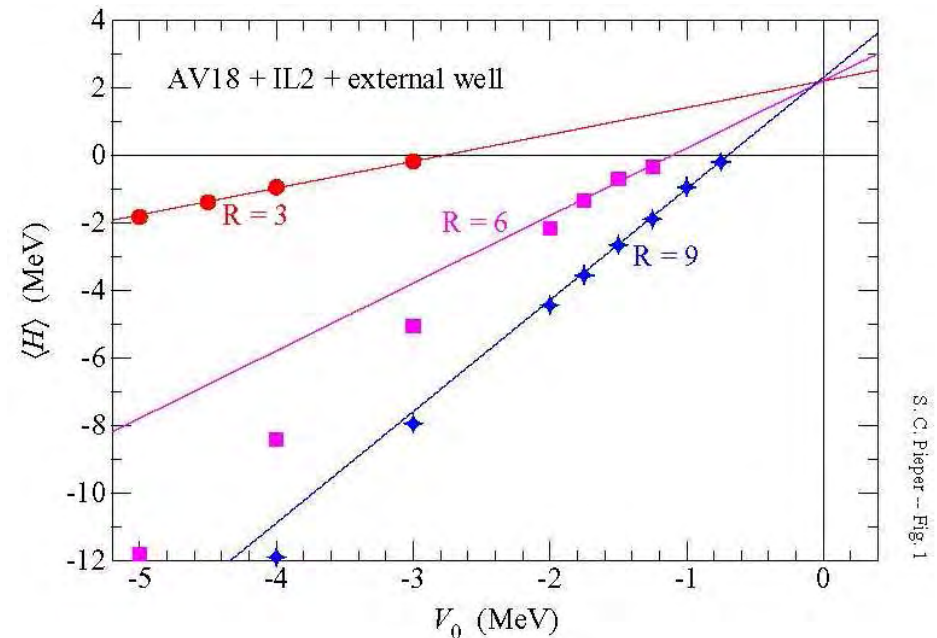
► transfer [Beaume] :



... and the results



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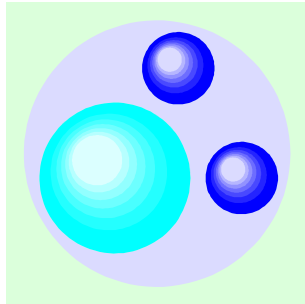


► “modern” calculations :

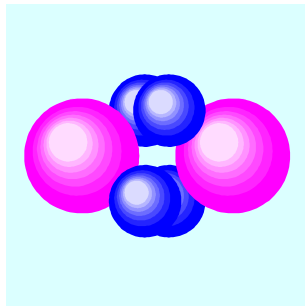
▷ $(^4\text{n}, p)$ scattering [Bertulani]

▷ bound/resonance ? [Pieper, Carbonell]

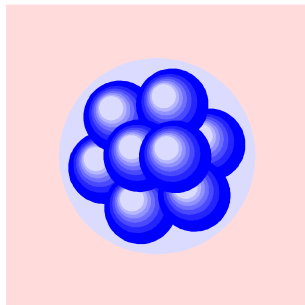
Summary & Conclusion



- ▶ understanding of cluster correlations [3α]
- ▶ very dilute matter: laboratory for NN interaction
- ▶ experimental input to three-body forces [V_{ijk}]
- ▶ where are the **limits** of the halo?



- ▶ completely **different** scales, concepts...
- ▶ completely analog **molecular orbitals** !
- ▶ deformation $\rightarrow$ WF nodes $\rightarrow$ clustering...
- ▶ **how far** can we push the analogy?



- ▶ many **arguments** are against...
- ▶ many **hopes** are for !
- ▶ **neutron-rich** beams provide new opportunities
- ▶ first "**hints**": new experiments and calculations...