From Elementary Particles to Nuclei and Their Interactions

Part I: Sub-Nuclear Physics

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In This Presentation:

We discuss experiment-and-theory research strategies
in Studying the Universe

Our motto for today:

- All human knowledge comes from Observations
- All human understanding comes from Theories

What do you think?

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Part I

Elementary Constituents of Matter - Early Evolution

Classical Elements in Ancient Philosophies

*The most frequently occurring theory of classical elements, held by the Hindu, Japanese, and Greek systems of thought, is that there are five elements, namely Earth, Water, Air, Fire, and a fifth element which is called "quintessence" or Aether**

Four Classical Elements according to Aristotle:

** The term quintessence derives from "quint" meaning "fifth"*

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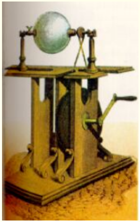
Beginning of New Science: Electromagnetism



Francis Hauksbee (The Elder), English physicist, wrote a famous book "Physico-Mechanical Experiments on Various Subjects" in 1709; it describes his studies of, in todays language, light and electricity.

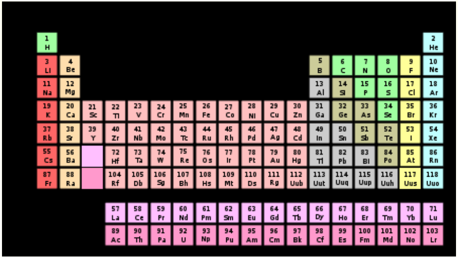
Published his book on his own costs, selling from his haus in London. To the right: an Italian translation.

ESPERIENZE
FISICO-MECCANICHE
SOPRA VARJ SOGGETTI
CONTENENTI
Un racconto di diverſi ſtupendi fenomeni
INTORNO
LA LUCE E L' ELETTRICITA'
Producibile dallo ſtrofinamento de' corpi.
con molte altre notabili apparenze
non mai prima oſſervate.
Colle ſpiegazioni di tutte le macchine.
O P E R A
DI F. HAUKSBEE
DELLA SOCIETA' REGIA
Tradotta dall' Idioma Ingleſe.

IN FIRENZE. M.DCC.XVI.
NELLA STAMPERIA DI SUA ALTEZZA REALE.
Per Jacopo Guiducci e Santi Franchi.
Con Licenza de' Superiori.

Classical Elements in Modern Times

Here we have several new elements = constituents of the surrounding Universe - according to principle proposed in 1869 by D. Mendeleiev. They are more e l e m e n t a r y right?



Dmitri Mendeleiev

The First Elementary Particle - Discovered

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The Discovery of the Electron

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The Nobel Foundation (NOBELSTIFTELSEN)

The Nobel Foundation is a private institution established in 1900 based on the will of Alfred Nobel. The Foundation manages the assets made available through the will for the awarding of the Nobel Prize in Physics, Chemistry, Physiology or Medicine, Literature and Peace.

It represents the Nobel Institutions externally and administers informational activities and arrangements surrounding the presentation of the Nobel Prize.

... and the Contributing Nobel-Prize Winners:

• In 1906: Joseph John Thomson, Nobel Prize "in recognition of the great merits of his theoretical and experimental investigations on the conduction of electricity by gases" (discovery of the electron)

• In 1923: Robert Andrews Millikan, Nobel Prize "for his work on the elementary charge of electricity and on the photoelectric effect"

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Atomic Scale Seen Today

The Search for Atoms from Today's Perspective

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B. The discovery of nucleons implies that the Atomic Nucleus is not an elementary object either

Historical Achievements from Today's Perspective

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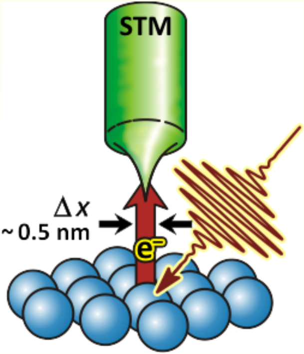
Today's Techniques Allow to 'See' Atoms

A. We will be able to see spectacular images of atoms in molecules

B. We will have to have a closer look at the microscopic particles - much smaller than atoms

Watching Atoms and Molecules Using STM Technique

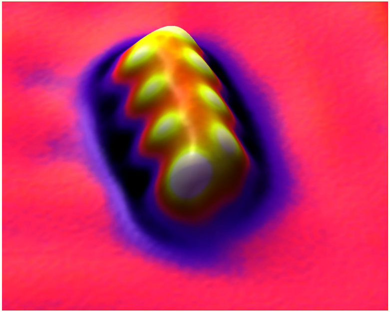
- Schematic Illustration: Principles of Scanning Tunneling Microscopy



The STM is a non-optical microscope. An atomically sharp probe ('the tip') is moved over the surface of the material under study, and a voltage is applied between probe and the surface. Depending on the voltage electrons will tunnel or jump from the tip to the surface (or vice-versa), resulting in a weak electric current. The size of this current is exponentially dependent on the distance between probe and the surface.

Watching Atoms and Molecules Using STM Technique

- Cesium-Iodine Molecule (8 Cesium and 8 Iodine atoms) using STM



Color (a function of light) is added after images are rendered by the STM. The data gathered by the STM are manipulated with custom graphics software to make the gathered information about a sub-visual object into something that the observer can process visually. Color and light effects are added to delinate different objects within the image, and to show curvature and other surface properties.*

* Credits: IBM - STM Image Gallery

Part II

Quantum Relativistic Wave Equation, Related Symmetries

Dirac's Search for the Relativistic Wave Equation

- Dirac constructs his equation that describes relativistic $s = \frac{1}{2}$ particles and admits probabilistic interpretation:

$$(i\hbar\gamma^{\mu}\hat{p}_{\mu} - m_0 c)\psi(x) = 0; \;\; \{\gamma^{\mu}, \gamma^{\nu}\} = 2 \cdot \mathbb{I}\, g^{\mu\nu}, \;\; 4 \times 4 \text{ matrices}$$

- Solutions ψ, Dirac spinors also called bi-spinors, have the structure

$$\psi = \binom{\xi}{\eta}, \;\; \xi = \binom{\xi_1}{\xi_2}, \;\; \eta = \binom{\eta_1}{\eta_2}; \;\; \gamma^0 = \begin{bmatrix} \mathbf{I} & 0 \\ 0 & -\mathbf{I} \end{bmatrix}, \;\; \gamma^j = \begin{bmatrix} 0 & \sigma^j \\ \sigma^j & 0 \end{bmatrix}$$

Notation: $x \equiv \{x^{\mu}\} = \{ct, \vec{r}\}, \;\; p \equiv \{p^{\mu}\} = \{\frac{E}{c}, \vec{p}\}, \;\; \bar{\psi} \equiv \psi^{\dagger}\gamma^0$

- Dirac demonstrated the searched conservation of probability

$$j^{\mu} \stackrel{df}{=} \bar{\psi}\gamma^{\mu}\psi, \;\; \partial_{\mu} j^{\mu} = 0 \qquad j^0 = \bar{\psi}\gamma^0\psi = \psi\gamma^0\gamma^0\psi = \psi^{\dagger}\psi \geq 0$$

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Dirac's Problem with the Relativistic Wave Equation

- The problem: Dirac equation admits negative energy solutions

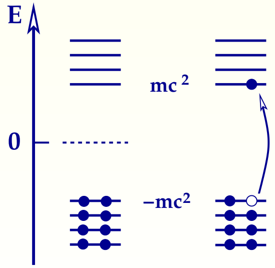
$$\psi^{(\pm)} = \binom{\xi}{\eta}_o \exp\left\{ \mp i\, p\, x/\hbar \right\} \;\text{with}\; E = \pm\sqrt{(c\vec{p})^2 + (m_0 c^2)^2}$$

- These solutions forced a bit artificial Dirac sea interpretation: infinitely many particles that form permanent 'vaccum'

- On the other hand it allowed for the pair-creation mechanism:

- The hole of a given-charge appears as one opposite charge particle;

- The corresponding excitation energy is always positive, masses remain equal



Discovery of Anti-Electrons: Matter and Anti-Matter

- In 1932: Paul Adrien Maurice Dirac, after a series of difficulties with the negative energy solutions to the Dirac equation, postulates the existence of a positron, anti-particle associated with electron
- In 1932: Carl David Anderson, Swedish American finds positrons, electron-positron pairs, using gamma rays produced by the natural radioactive nuclides: particle-anti-particle production
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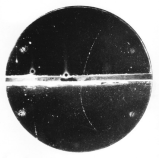
Discovery of Anti-Electrons: Historical Documents



The discovery of the positron in 1932 by Carl Anderson studying cosmic rays. The particle was deflected by a magnetic field in the opposite direction to the electron, but was too light to be a proton

*This bubble chamber photograph shows an electron and a positron (anti-electron) that are spiralling in opposite directions**

** Credits: CERN*

Particles and Anti-Particles: a New Symmetry

As a physicist - Whenever you see such an image - recall: particle-antiparticle symmetry and doubling the universe. In physics: charge conjugation

Some Historical Steps in Retrospection

• The free Dirac equation generalises simply, within the so-called minimal coupling scheme, for electromagnetic interactions

$$p_\mu \to \left(p_\mu - \tfrac{e}{c}A_\mu\right) : \;\Rightarrow\; \left[\gamma^\mu(\hat{p}_\mu - \tfrac{e}{c}A_\mu) - m_0 c\right]\psi(x) = 0$$

• We can introduce a charge conjugation operator, $\hat{\mathcal{C}}$, transforming a given solution ψ into opposite-charge same-mass solutions ψ_c:

$$\hat{\mathcal{C}} = \hat{\mathcal{C}}(\{\gamma^\mu\}) \;\to\; \hat{H}_c = \hat{\mathcal{C}}\,\hat{H}\,\hat{\mathcal{C}}^{-1} \text{ and } \psi_c = \hat{\mathcal{C}}\,\psi$$

• With the help of the charge conjugation operation one can show

$$\langle\psi_c|\hat{H}|\psi_c\rangle = -\langle\psi|\hat{H}|\psi\rangle \text{ and } \langle\psi_c|\hat{p}|\psi_c\rangle = -\langle\psi|\hat{p}|\psi\rangle$$

i.e. the negative-energy particles seen as positive-energy anti-particles moving in the opposite sense of time

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Some Historical Steps in Retrospection (2)

• Similarly to the concept of charge conjugation $\hat{\mathcal{C}}$, the other discrete symmetries such as inversion $\hat{\mathcal{P}}$ and time-reversal $\hat{\mathcal{T}}$ have been introduced

• This led to the discoveries of the $\hat{\mathcal{C}}\hat{\mathcal{P}}$ as well as $\hat{\mathcal{C}}\hat{\mathcal{P}}\hat{\mathcal{T}}$ symmetries and consecutively to the discovery of partial $\hat{\mathcal{C}}\hat{\mathcal{P}}$ symmetry breaking

• All these concepts were developped further by Richard Feynman while constructing quantum electrodynamics (QED); Nobel Prize in 1965, together with Julian Schwinger and Sin-Itiro Tomonaga

• Using field theory formalism the concept of charge-conjugation has been generalized to other charges such as baryonic, leptonic ... etc.

• These new concepts helped in constructing modern theories of elementary particles (see below)

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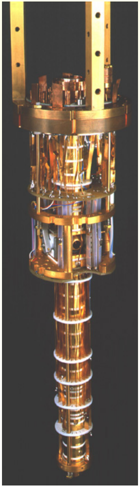
Producing Anti-Matter: Today

Producing Anti-Hydrogen: ATRAP Experiment

- Basing on his general equation governing the motion of relativistic Fermions, Dirac has formulated for the first time the prediction of the existence of antiparticles
- This prediction opened the way to the idea that each particle has an anti-particle partner, not just electrons
- In this way we arrive at the hypothesis of Doubling the Forms of Matter: *There exists Matter and Anti-Matter*



To the right: The ATRAP* apparatus combines positrons (which enter from the top) with anti-protons (which enter from below) and meet about one-third of the way up from the bottom to make neutral anti-hydrogen atoms. To do this the positrons pass through a special rotable electrode (the element with the circular hole near the bottom of the wide part of the apparatus)

***Credits: Physical Review Letters, November 2002**

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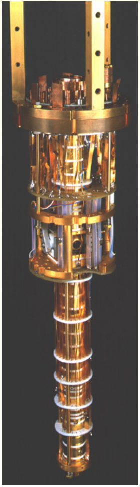

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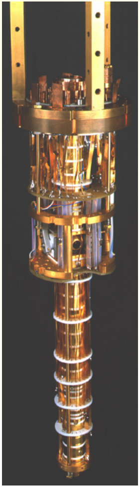

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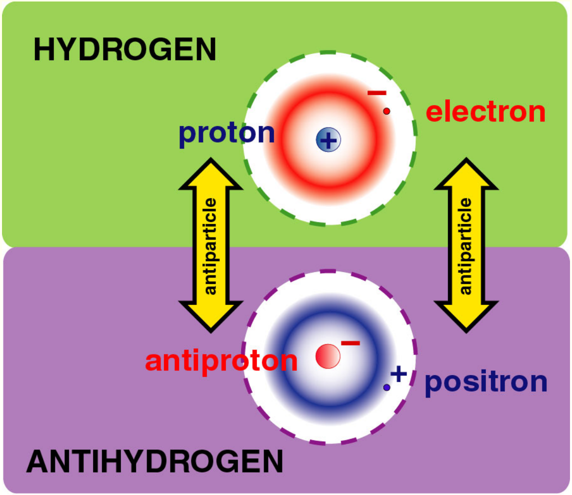
Anti-Hydrogen and Its Twin Brother: Portraits



Part III

Sub-Atomic and Sub-Nuclear Particles

Discovering New Sub-Atomic Particles: The Pion

- Following the prediction of Hideki Yukawa of 1935...

The Discovery of the Pion

- In 1947: *Charged pions* $\pi^{\pm}$ are discovered by Cecil Powell, César Lattes and Giuseppe Occhialini at the University of Bristol
- In 1949: Nobel Prize awarded to H. Yukawa, for predicting the existence of mesons
- In 1950: Nobel Prize awarded to C. Powell, for developing the technique of particle detection using photo-emulsions
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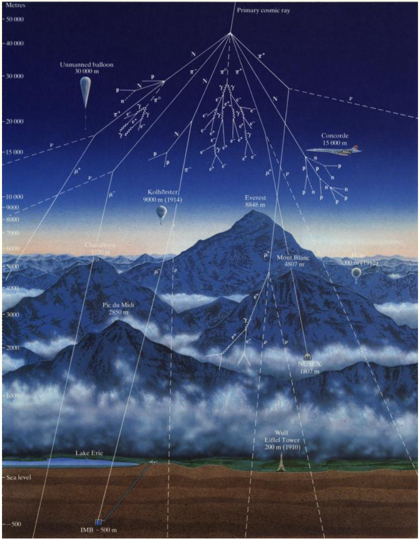
Information that Is Just Falling from Heaven



First Discovery of Particles Called Strange

- **The discovery of the neutral, strange Λ-particle, in 1951**

Here: An example of the results from liquid hydrogen bubble chamber* at Brookhaven National Laboratory. The yellow line at the bottom is an incoming high-energy proton, it collides with a proton at rest in the liquid hydrogen creating many particles. Seven positive pions, a proton, and a positive kaon (shown in red) curve off to the right, while seven negative pions (blue) move to the left. A neutral Λ is also produced which travels upwards undetected and then decays into a proton (yellow) and a negative pion (purple). NB: the green curve at the bottom is due to an electron which has been knocked out of its orbit by the passing proton.

$$\Lambda \longrightarrow \pi^- + p$$

π^- - purple, p - yellow

*Credits: Brookhaven National Laboratory

Even More Strange Particles Discovered Soon After

- *These new particles can be grouped; within the group they decay very fast, but only very slowly to the outside of the group wherefrom their name*
- *By attributing a new quantum number*, 'strangeness' S, we are able to systematize their decay and reaction properties (Table for $s = \frac{1}{2}$ particles)*

Symbol	S	⟨Life-time⟩ sec	Q	Decay
Λ^0	−1	2.6×10^{-10}	0	$\begin{cases} p+\pi^- \\ n+\pi^0 \end{cases}$
Σ^+	−1	8.0×10^{-11}	+1	$\begin{cases} p+\pi^0 \\ n+\pi^+ \end{cases}$
Σ^0	−1	7.4×10^{-20}	0	$\Lambda^0+\gamma$
Σ^-	−1	1.4×10^{-10}	−1	$n+\pi^-$
Ξ^0	−2	2.9×10^{-10}	0	$\Lambda^0+\pi^0$
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- *These new particles can be grouped; within the group they decay very fast, but only very slowly to the outside of the group wherefrom their name*

- *Moreover, strange particles are produced always in pairs in the strong interactions of the non-strange hadrons for instance* $\pi^+ + p \to K^+ + \Sigma^+$

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Summarizing the Results of the First Discoveries

• There exist 'heavy' particles such as nucleons (fermions), 'medium heavy' mesons (bosons) and 'light' particles (fermions)

• Names: for nucleons, mesons and other heavy particles

Baryons - from Greek: $\beta\alpha\rho\upsilon\zeta =$ heavy

and for the light particles:

Leptons - from Greek: $\lambda\epsilon\pi\tau\upsilon\varsigma =$ delicate

• Experiments show that baryons and mesons interact, create other particles and decay in very short times comparable with 10^{-24} secs

• We identify the short-time processes with the strong interactions; nucleons and mesons belong to this cathegory

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Elementary Sub-Nuclear Particles

Baryons, Mesons and Partons

- **We may observe that baryons are, on average, heavier than mesons thus they may contain more really elementary constituents (partons)**
- If we wish to keep simplicity: the smallest number of elementary constituents must be 2 (mesons) and one bigger must be 3 (baryons)
- If we attribute the baryonic 'charge' to all the baryons $B = +1$, then anti-baryons must have $B = -1$ and all other particles $B = 0$
- It follows that elementary constituents must have $B = \frac{1}{3}$ so that baryons may have $B = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$
- ... while mesons must be composed of pairs: parton - anti-parton and thus $B = \frac{1}{3} + \frac{\bar{1}}{3} = \frac{1}{3} - \frac{1}{3} = 0$
- Elementary charges of partons must be a multiple of $Q_{el.} = \frac{1}{3}$ e
- Partons must be Fermions so that both half-integer and integer spins can be constructed

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From Baryons, Mesons and Partons → Quarks

- **The smallest number of partons is 2; the simplest interaction law assures that the interactions do not depend on the type of parton**

$$\begin{bmatrix} u' \\ d' \end{bmatrix} = \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} \times \begin{bmatrix} u \\ d \end{bmatrix} \leftrightarrow \text{SU(2)-symmetry}$$

- We must introduce the electric charges by convention:

Quark	Symbol	Spin	B	Q
up	u	1/2	1/3	+2/3
down	d	1/2	1/3	−1/3

- Partons with these properties are called quarks (see below)
- Test for the nucleons and pions

$$p = uud, \quad n = udd, \quad \pi^{+} = u\bar{d}, \quad \pi^{-} = d\bar{u}, \quad \pi^{0} = u\bar{u} \;\text{ and/or }\; d\bar{d}$$

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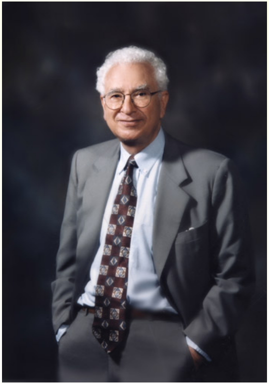
Historical Remarks: Parton and Quark Models

- **Quark Model, 1964 by Murray Gell-Mann and George Zweig**

The quark model uses the concept of quarks with several properties just as introduced above (see also below).

The initial reaction of the physics community to the proposal was mixed. There was particular contention about whether the quark was a physical entity, or an abstraction used to explain certain new concepts that were not well understood at the time.



Murray Gell-Man

Historical Remarks: Parton and Quark Models

- **Parton Model formulated in 1969 by Richard P. Feynman**

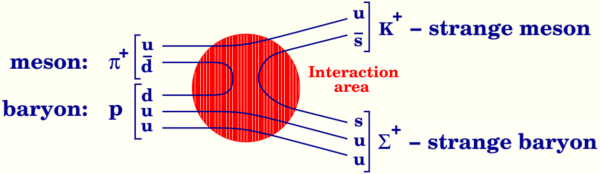
In this model, a hadron is composed of a number of point-like constituents, called "partons". Additionally, the hadron is in a reference frame where it has infinite momentum - a valid approximation at high energies.

Quark model can be seen as a particular realisation of the parton model.

Richard P. Feynmann

Strange Particles: Extension of the Quark Model

• The conservation of strangeness could not be accounted for with the presence of two quarks only, wherefrom the new hypothesis of the existence of the third ('strange') quark s

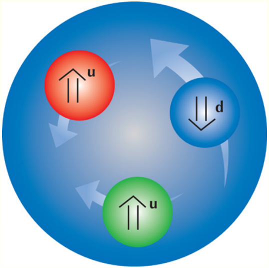


• Illustration of the process of quark - anti-quark annihilation in a central 'interaction area'. It can be viewed in analogy to the other annihilation processes such as $e^+ + e^- \to 2\gamma$ and many others

Moreover: Nucleon Spin & Orbital Motion of Quarks

Jefferson National Accelerator Facility, Virginia, USA. Report of a discovery that the spins of the proton's two up quarks (u) are aligned parallel to the overall spin of the proton, but the same is not true for the proton's down quark (d)

In order to make the experimental data on quark spin agree with theory, the authors had to take into account the once-neglected orbital motion of quarks inside the proton



Credits: Jefferson Lab. and Zheng et al., Phys. Rev. Lett. 2003

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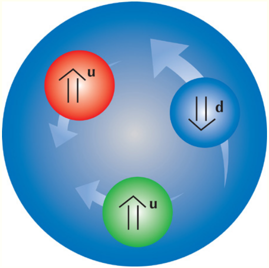

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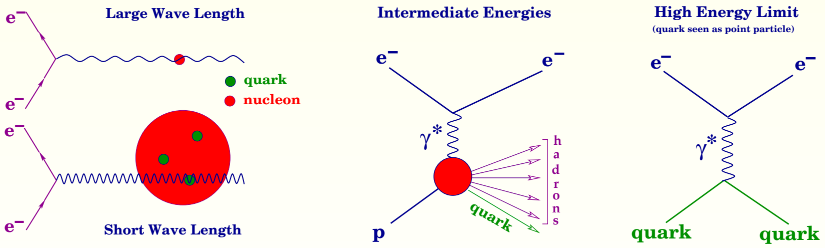
Part IV

Today's Truly Elementary Particles: Quarks

Principle of Experimental Tests of the Quark Model

- Probing the quark structure of protons through deep inelastic scattering of high-energy electrons; the quark structure is resolved through the virtual photons when $\lambda \ll 1$ Fm



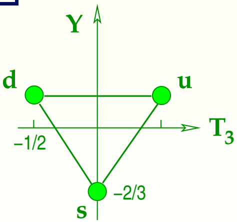
- Experiments on high energy $e + p$ scattering fully confirmed these qualitative considerations providing the basis for the quark model

'Convenient' Representation: Hypercharge, Y, Isospin T

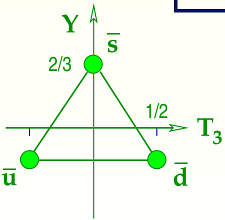
Quark	Spin	B	Q	T_3	S	Y
u	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{2}$	0	$\frac{1}{3}$
d	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$\frac{1}{3}$
s	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{3}$	0	-1	$-\frac{2}{3}$

$$Y \stackrel{df}{=} B + S$$

$$Q = T_3 + \frac{1}{2}Y$$



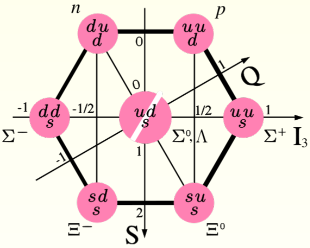
Quarks

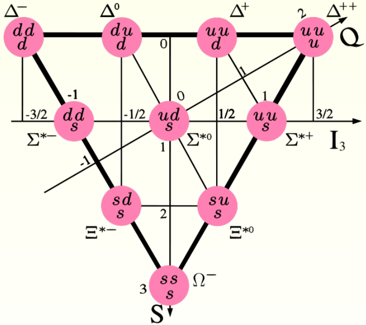


Anti–Quarks

Three Quarks and Resulting Baryon Periodic Tables

• Combining u, d and s quarks and using Y (alternatively* S) vs. isospin T_3 representation we obtain octet and decuplet structures





• The predictions of the existence of all these particles have been confirmed experimentally supporting the idea of quark constituents

*Since $Y \stackrel{df}{=} B + S$, one can use alternatively S; indeed $B = const.$ implies contant shift in this case

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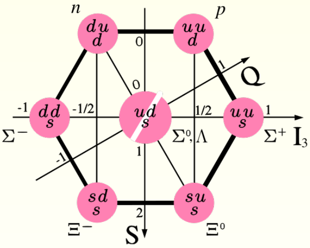

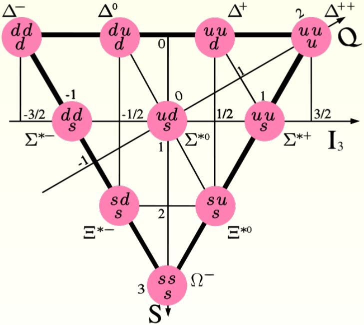

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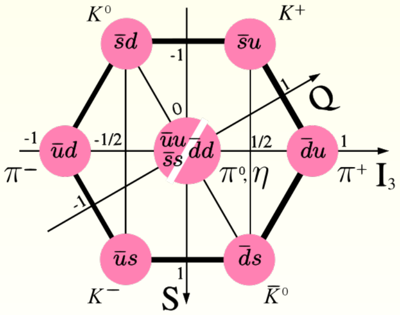

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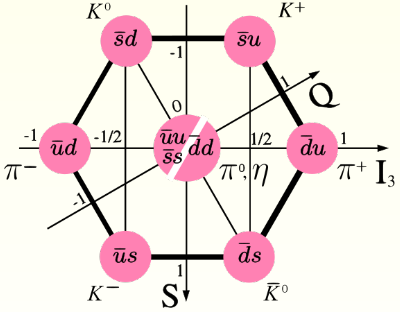

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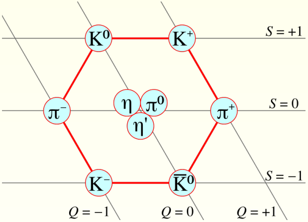
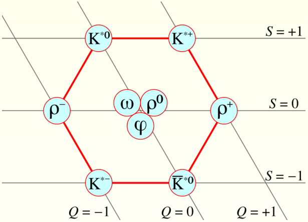
- Quark - anti-quark pairs implies $s = 0$ and $s = 1$ meson nonets



Spin s=0 Pseudo-Scalar Mesons



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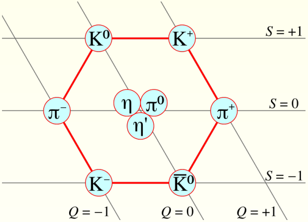

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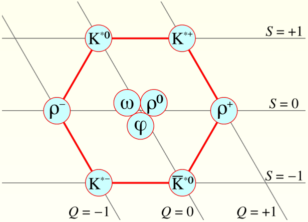

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Part V

Fermions, Bosons and Problem of Symmetrisation

About Identical Particles

• Consider a many-body system composed of n identical particles.

We use position, linear momentum and spin, $\hat{r}$, $\hat{p}$, $\hat{s}$, to describe a particle $\hat{x} \equiv \{\hat{r}, \hat{p}, \hat{s}\}$. The Hamiltonian $\hat{H} = \hat{H}(\hat{x}_1, \hat{x}_2, \ldots \hat{x}_n)$ must be symmetric under any permutation

$$\hat{\mathcal{P}}_{ij}\,\hat{H}(\hat{x}_1 \ldots \hat{x}_i \ldots \hat{x}_j \ldots \hat{x}_n)\,\hat{\mathcal{P}}_{ij}^{-1} \stackrel{df}{=} \hat{H}(\hat{x}_1 \ldots \hat{x}_j \ldots \hat{x}_i \ldots \hat{x}_n) = \hat{H}(\hat{x}_1 \ldots \hat{x}_i \ldots \hat{x}_j \ldots \hat{x}_n)$$

and it follows: $\boxed{\hat{\mathcal{P}}_{ij}\,\hat{H}\,\hat{\mathcal{P}}_{ij}^{-1} = \hat{H} \quad \rightarrow \quad [\hat{\mathcal{P}}_{ij}, \hat{H}] = 0, \quad \forall\, i \neq j \leq n\,.}$

• Conclusions: 1. Both observables $\hat{\mathcal{P}}_{ij}$ and $\hat{H}$ can be diagonalized simultaneously; 2. Eigenvalues of $\hat{\mathcal{P}}_{ij}$ are constants of motion.

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$$\begin{aligned} \hat{\mathcal{P}}_{\mathbf{ij}} \, \hat{\mathbf{H}}(\hat{\mathbf{x}}_1 \ \ldots \ \hat{\mathbf{x}}_{\mathbf{i}} \ \ldots \ \hat{\mathbf{x}}_{\mathbf{j}} \ \ldots \ \hat{\mathbf{x}}_{\mathbf{n}}) \, \hat{\mathcal{P}}_{\mathbf{ij}}^{-1} &\overset{\mathbf{df}}{=} \hat{\mathbf{H}}(\hat{\mathbf{x}}_1 \ \ldots \ \hat{\mathbf{x}}_{\mathbf{j}} \ \ldots \ \hat{\mathbf{x}}_{\mathbf{i}} \ \ldots \ \hat{\mathbf{x}}_{\mathbf{n}}) \\ &= \hat{\mathbf{H}}(\hat{\mathbf{x}}_1 \ \ldots \ \hat{\mathbf{x}}_{\mathbf{i}} \ \ldots \ \hat{\mathbf{x}}_{\mathbf{j}} \ \ldots \ \hat{\mathbf{x}}_{\mathbf{n}}) \end{aligned}$$

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$$\mathbf{p_{ij}^2 = 1 \quad \longrightarrow \quad p_{ij} = \pm 1}$$

This implies that identical particles are either

$$\boxed{\textbf{Fermions :} \quad \hat{\mathcal{P}}_{\mathbf{ij}}\, \mathbf{\Psi}_{\mathbf{n_1, \dots\, n_i, \dots n_j, \dots\, n_n}} = -\mathbf{\Psi}_{\mathbf{n_1, \dots\, n_i, \dots n_j, \dots\, n_n}}, \quad \forall\ \mathbf{i, j}}$$

or

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About Identical Particles: Pauli Principle

- We say that the wave-functions for identical Fermions are totally anti-symmetric and those for Bosons are totally symmetric
- By setting for Fermions $i = j$ (two identical states) we have

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Anti-Symmetrising Fermion Wave-Functions

Fermion Wave-Functions: Anti-Symmetrisation

• Let us begin by posing a certain elementary problem that some of you know already how to tackle:

What is the structure of $s = \frac{1}{2}$ two-particle wave functions at $\vec{r}_1$ and $\vec{r}_2$?

• The wave functions depend on the spatial parts $\varphi_\alpha(\vec{r}\,)$ and $\varphi_\beta(\vec{r}\,)$ and the spin part χ_{s,s_z}: the total wave functions must be antisymmetric:

$$\boldsymbol{\Psi_{\alpha\beta} \sim \textbf{Anti-symm.}[\varphi_\alpha(\vec{r}_1\,), \varphi_\beta(\vec{r}_2\,)] \times \textbf{Symm.}[\chi_{s,s_{z,1}}, \chi_{s,s_{z,2}}]}$$

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Fermion Wave-Functions: Anti-Symmetrisation

• Let us begin by posing a certain elementary problem that some of you know already how to tackle:

What is the structure of $s = \frac{1}{2}$ two-particle wave functions at $\vec{r}_1$ and $\vec{r}_2$?

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Fermion Space-Symmetrisation

Symmetrisation and anti-symmetrisation in space can be done in a simple, unique manner: for spatially anti-symmetric functions

$$\hat{\mathcal{A}}\Psi_{\alpha\beta}(\vec{r}_1,\vec{r}_2) = \frac{1}{\sqrt{2}}[\varphi_\alpha(\vec{r}_1),\varphi_\beta(\vec{r}_2) - \varphi_\alpha(\vec{r}_2),\varphi_\beta(\vec{r}_1)]$$
$$\leftrightarrow \frac{1}{\sqrt{2}}[\varphi_\alpha(\vec{r}_1),\varphi_\beta(\vec{r}_2) - \varphi_\beta(\vec{r}_1),\varphi_\alpha(\vec{r}_2)]$$

and for spatially-symmetric functions

$$\hat{\mathcal{S}}\Psi_{\alpha\beta}(\vec{r}_1,\vec{r}_2) = \frac{1}{\sqrt{2}}[\varphi_\alpha(\vec{r}_1),\varphi_\beta(\vec{r}_2) + \varphi_\alpha(\vec{r}_2),\varphi_\beta(\vec{r}_1)]$$
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and only one spin-anti-symmetric function

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Isospin - The Younger Brother of Spin

Isospin - a Younger Brother of Spin

- The concept of isospin (isobaric spin) has been introduced by Werner Heisenberg in 1932; the name proposed by Eugene Wigner in 1937

- Spin $s = \frac{1}{2}$ is a dichotomic variable associated with one and the same particle: we say that masses of $s_z = +\frac{1}{2}$ and $s_z = -\frac{1}{2}$ particles are equal

- Compare: The nucleons (protons and neutrons) have nearly the same mass and the dichotomic variable is here the electric charge $q = 0$ or 1 e

- We say that the nucleon has isospin $t = \frac{1}{2}$ and $t_z = +\frac{1}{2}$ if the charge is $q = +1$e (proton) while $t_z = -\frac{1}{2}$ if the charge is $q = 0$ (neutron)

- Analogy:

$$\chi_{\mathbf{s}=\frac{1}{2},\mathbf{s_z}=\pm\frac{1}{2}} \leftrightarrow \text{spin-up vs. spin-down}$$

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Pauli Principle within 3D-Space, Spin- and Isospin-Spaces

Pauli Principle Generalized for the Nucleons

- Generalized Pauli principle implies that the two-nucleon wave function must be totally anti-symmetric

$$\boxed{\Psi_{\alpha\beta} = \psi_{\alpha\beta}(\vec{r}_1, \vec{r}_2)\chi_{S,S_z}\chi_{T,T_z} \;\leftrightarrow\; \text{anti-symmetric}}$$

- The physically acceptable two-body wave functions are

$$\psi^A_{12}\,\chi^S_{S,S_z}\,\chi^S_{T,T_z} \;\longrightarrow\; \psi^A_{12}\,\chi^S_{S,S_z}\,[\underbrace{\chi^S_{T=1,-1}}_{pp} \;\text{or}\; \underbrace{\chi^S_{T=1,0}}_{pn} \;\text{or}\; \underbrace{\chi^S_{T=1,+1}}_{nn}]$$

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Proton and Neutron Quark-Symmtrised Wave Functions

Symmetrisation Principles on the Quark Level

- The total wave-functions of the final objects must be totally anti-symmetric
- We will denote as usually the two signs of the spin-projections with the spin-up ($\psi_\uparrow$) and spin-down ($\psi_\downarrow$) symbols
- Example of the results: spin-up proton structure

$$
\begin{aligned}
p_\uparrow \;\; = \;\; \mathcal{N}\big[& 2\cdot u_\uparrow d_\downarrow u_\uparrow + 2\cdot u_\uparrow u_\uparrow d_\downarrow + 2\cdot d_\downarrow u_\uparrow u_\uparrow \\
& - u_\uparrow u_\downarrow d_\uparrow \quad - u_\uparrow d_\uparrow u_\downarrow \quad - u_\downarrow d_\uparrow u_\uparrow \\
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Part VI

Elementary Constituents of Matter - Today

November Revolution: J and ψ that Became J/ψ

• The existence of the fourth quark has been predicted, among others, by Glashow, Iliopoulos and Maiani, in 1970

• On the 14^{th} of November 1974 a discovery of a new particle has been announced simultaneously by Stanford Linear Accelerator Center (SLAC) and Brookhaven National Laboratory (BNL) groups

• The SLAC group called the new particle ψ and the BNL called it J - both discoveries concerned the same particle

• The particle (the only one named with two letters) was called J/ψ - and the leaders* of the teams obtained Nobel prize in 1976

• The new particle is interpreted today as a pair new-quark new-anti-quark, the former called 'charm', c: thus $J/\psi \leftrightarrow c\bar{c}$

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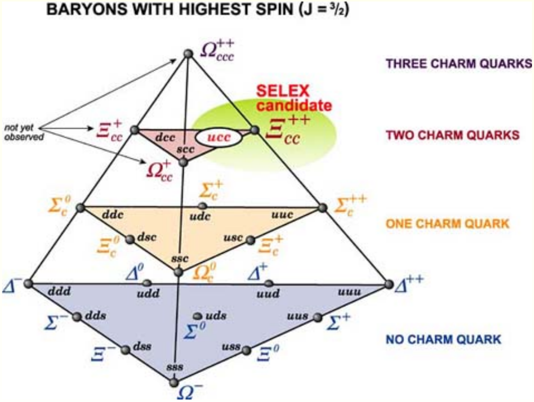
A New Version of the 'Particle Periodic Table'

• Baryons with increasing number of charmed quarks (counting from the bottom to the top of the figure)



The Heaviest Elementary: Top and Bottom Quarks

• Precision calculations of Gerardus 't Hooft and Martinus Veltman predict the existence of yet another quark, called top, t

• After these predictions the top quark anti-quark pair was discovered in 1995 at Fermilab (Tevatron) by CDF and D0 collaborations

• Nobel Prize for Gerardus 't Hooft and Martinus Veltman in 1999

• Single quark production via weak interactions: in March 2009, both CDF and D0 announced discovery of a single-top production

• According to Standard Model t-lifetime is $\sim 1 \times 10^{-25}$ sec, about 20 times shorter than the timescale for strong interactions - therefore quark t does not hadronize

• **Top t offers a unique opportunity to study a "bare" quark**

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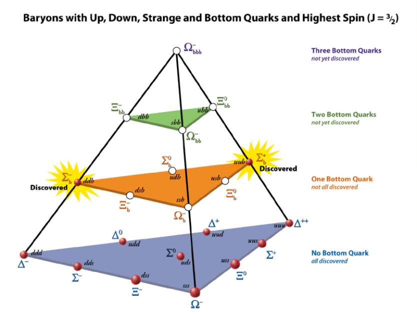
From Mendeleiev's to Baryon Periodic Table

Baryons are particles made of three quarks. The particles can exist in a ground state (J=1/2) and an excited state (J=3/2). This figure shows the various three-quark combinations with J=3/2 that are possible using the three lightest quarks – up, down and strange – and the bottom quark. Past experiments discovered all of the baryons made of light quarks. The CDF discovery is the first observation of baryons with one bottom quark and spin J=3/2.



Collider Detector at Fermilab (CDF). The discovery of the positively charged Σ_b^+ and the negatively charged Σ_b^- in both spin configurations.

Credits: Fermi Lab. Press Release

Summary of Quark Flavour Properties

- **The meaning of some symbols: J-spin, B baryon-number, Q-charge, T_z-isospin, C-charmness, S-strangeness, T-topness, B′-botomness**

Name	Symb	M MeV/c^2	J	B	Q	T_z	C	S	T	B′
Up	u	1.5 to 3.3	$\frac{1}{2}$	$\frac{1}{3}$	$+\frac{2}{3}$	$+\frac{1}{2}$	0	0	0	0
Down	d	3.5 to 6.0	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	0	0	0	0
Charm	c	1 270	$\frac{1}{2}$	$\frac{1}{3}$	$+\frac{2}{3}$	0	+1	0	0	0
Strange	s	104	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{3}$	0	0	−1	0	0
Top	t	171 200	$\frac{1}{2}$	$\frac{1}{3}$	$+\frac{2}{3}$	0	0	0	+1	0
Bottom	b	4 200	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{3}$	0	0	0	0	−1

• Quarks are considered point particles; • A quark of one flavor can transform into a quark of another flavor only through the weak interaction; • These transitions occur through emission of virtual W bosons

Quarks, Leptons and Force-Transmitting Bosons

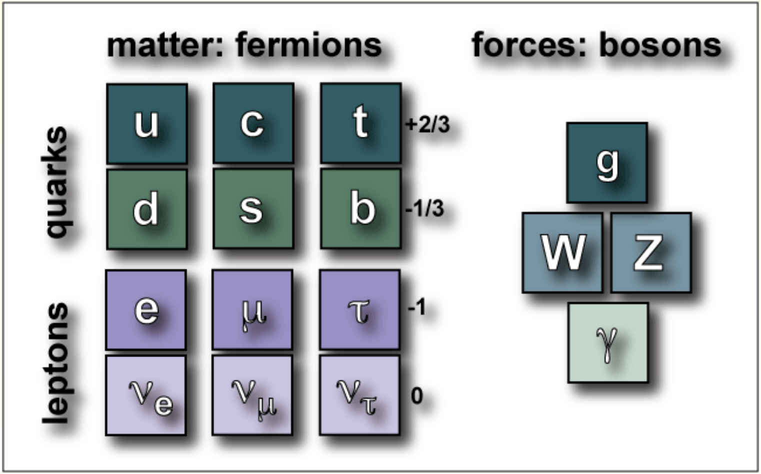
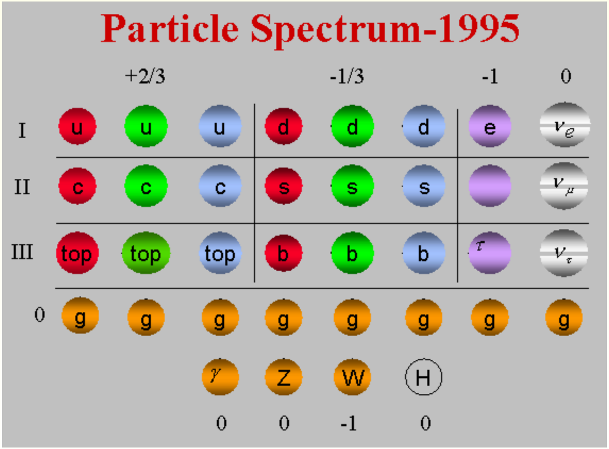


Table of Quarks with Colors



Key Issues in the Standard Model