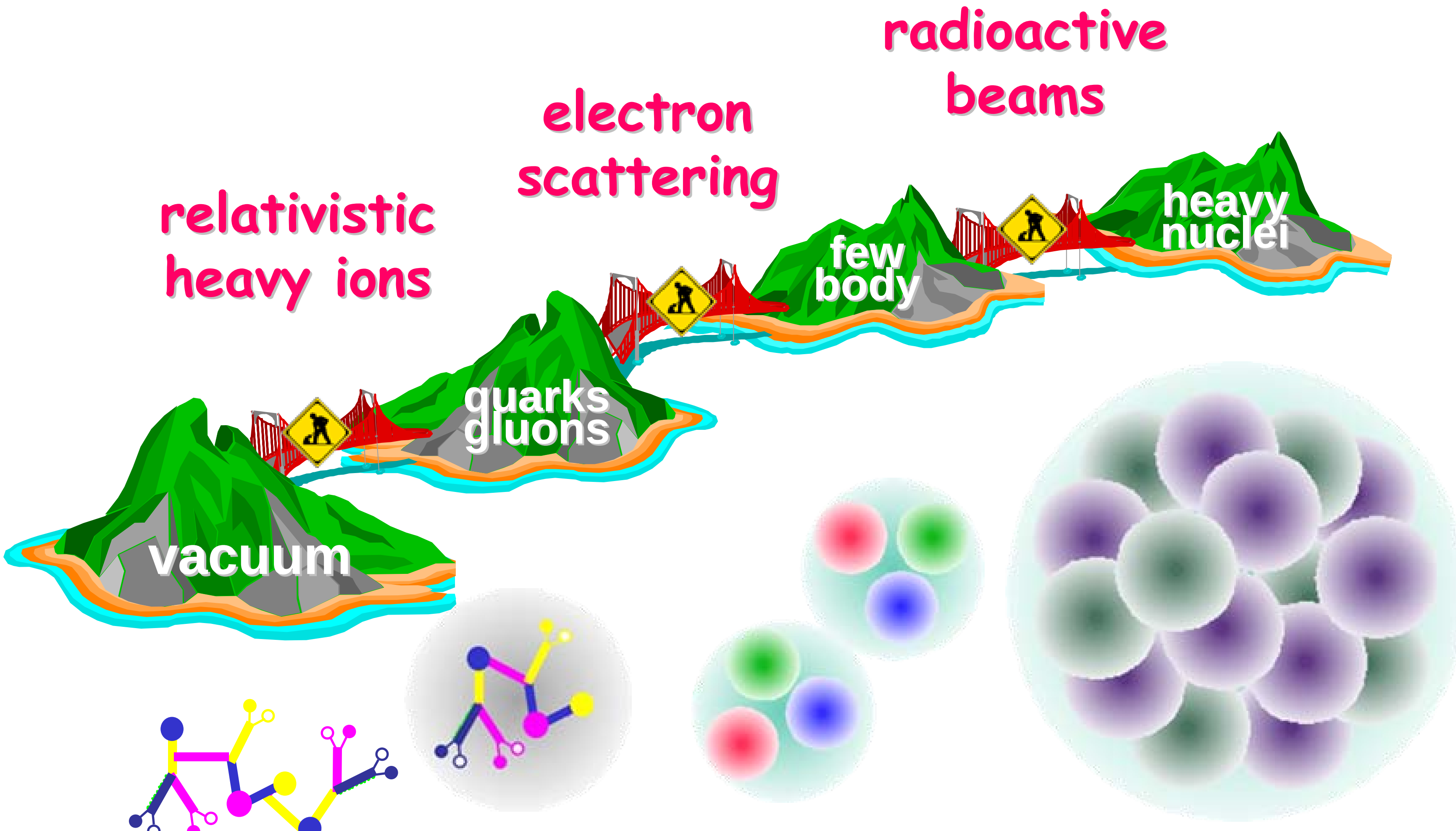
The Nuclear Many-Body Problem



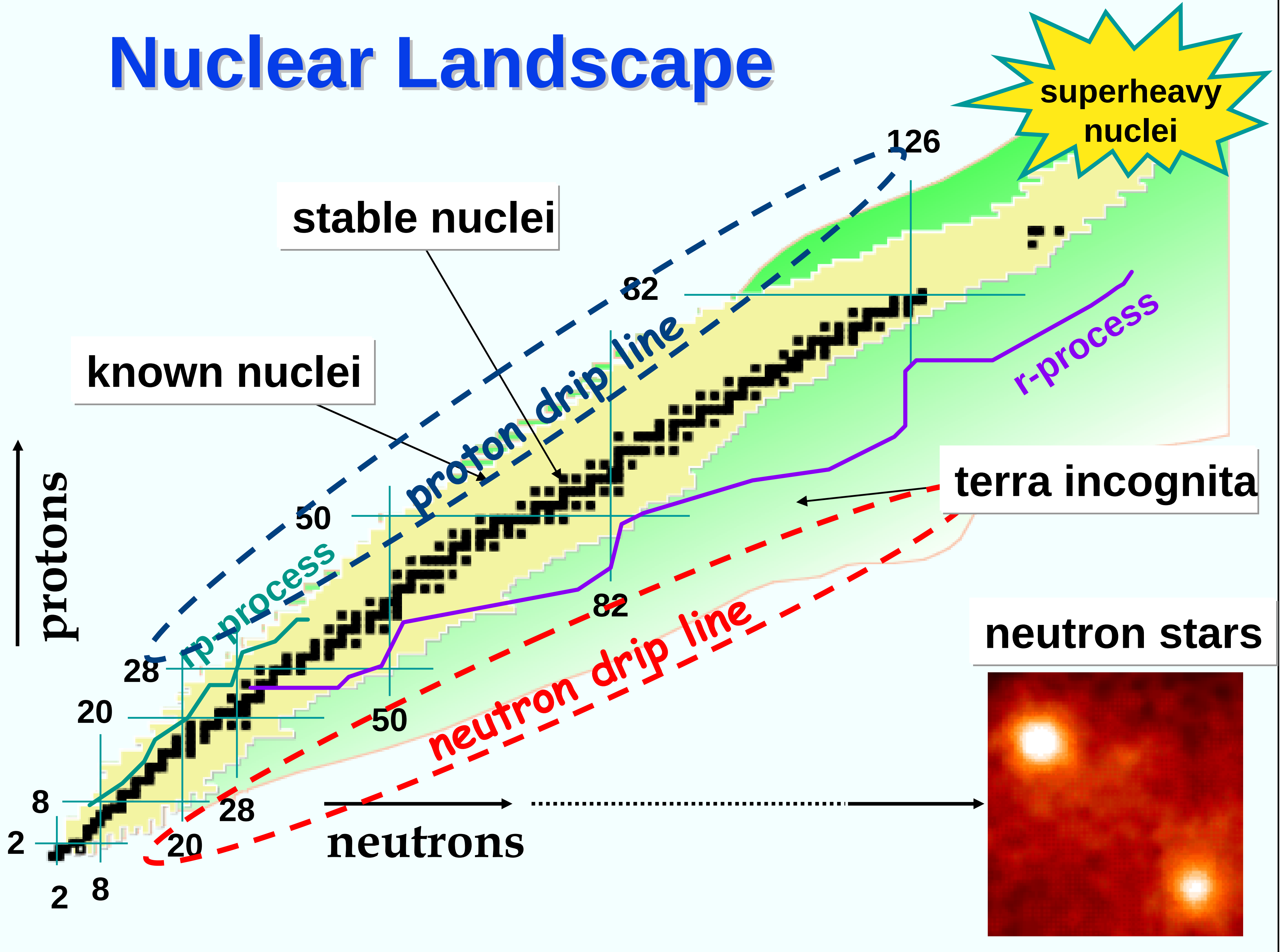
quark-gluon soup
QCD

nucleon
QCD

few body systems
bare NN force

many body systems
effective NN force

Nuclear Landscape

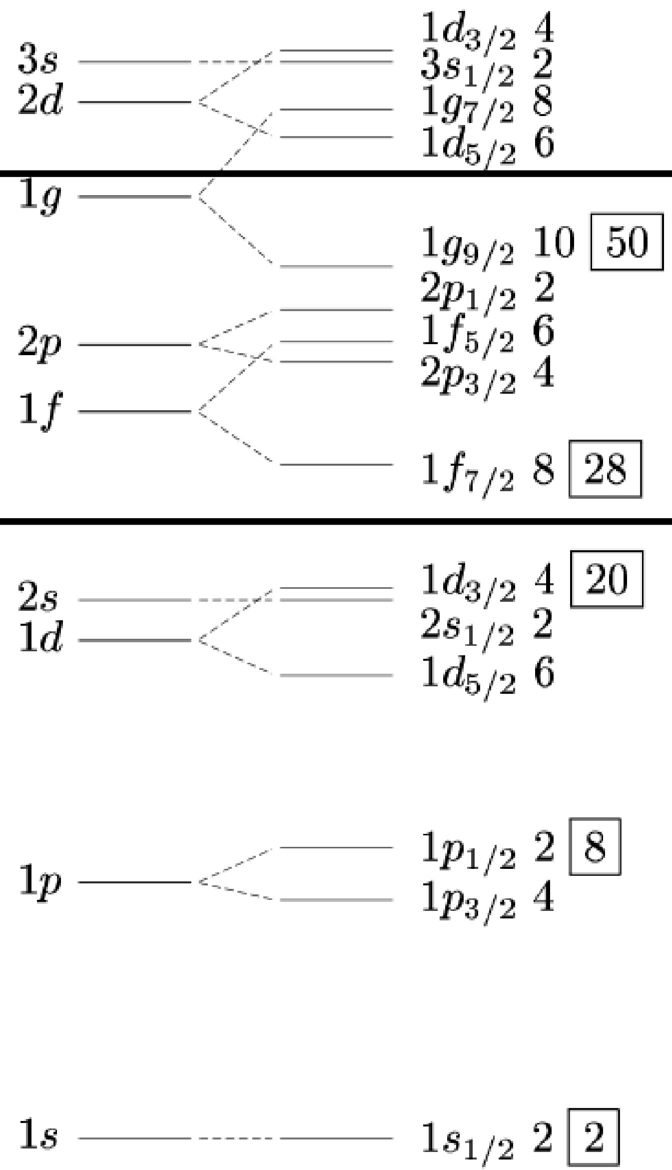


The shell model: a schematic view

All upper shells neglected

Active space:

Inactive core ^{40}Ca



Active space

$1g_{9/2}$ 10 $\boxed{50}$
$2p_{1/2}$ 2
$1f_{5/2}$ 6
$2p_{3/2}$ 4

$1f_{7/2}$ 8 $\boxed{28}$

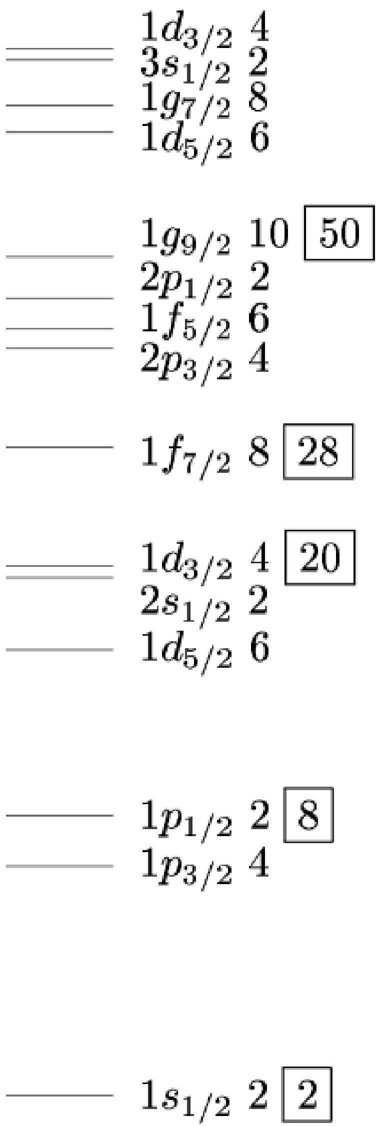
2-body matrix elements between all pairs of states

Diagonalization of a big matrix in the active space

Starting point of the model: nucleons in a potential well.
Problem of the model: active space grows very quickly.

Respect always symmetries of the 2-body hamiltonian:
work in the laboratory frame of reference

Mean-field methods



Construct all the orbitals from the mean

2-body interaction of 1 particle with the others

All orbitals are active!

Caution: not true single particle levels

Break symmetries and work in a frame of

reference intrinsic to the nucleus!

The Shrödinger equation is equivalent to a wave function variational principle: minimize <Ψ | H | Ψ> under the constraint < Ψ | Ψ> =1:

$$\delta\{<\Psi|H|\Psi> / <\Psi|\Psi>\} = 0$$

$$H = \sum_i (T_i + U_i) + \sum_{i>j} V(\vec{r}_i - \vec{r}_j)$$

One-body

Two-body

Hartree-Fock method:

the ground state wave function is a Slater determinant.

The Hartree-Fock method

Wave-function of a many particle system= Slater determinant

$$\Psi = \frac{1}{N!}\sum_P (-)^p \phi_\alpha(\vec{r_1})\phi_\beta(\vec{r_2})...\phi_\nu(\vec{r_A})$$

The particles interact trough a 2-body interaction v($\mathbf{r_1}$-$\mathbf{r_2}$)

They are also confined by a central potential v_{ext}(r).
The total energy is:

$$\begin{aligned} E &= \sum_{i,\sigma}\int d\vec{r}\phi_i^*(\vec{r},\sigma)(-\frac{\hbar^2}{2m}\Delta + v_{ext}(\vec{r}))\phi_i(\vec{r},\sigma) \\ &+ \frac{1}{2}\sum_{i,\sigma}\sum_{j,\sigma'}\int\int d\vec{r}d\vec{r'}\phi_i^*(\vec{r},\sigma)\phi_j^*(\vec{r'},\sigma')v(\vec{r},\vec{r'})(\phi_i(\vec{r},\sigma)\phi_j(\vec{r'},\sigma') - \phi_i(\vec{r'},\sigma')\phi_j(\vec{r},\sigma)) \end{aligned}$$

direct exchange

Minimize the energy with a constraint on norm conservation:

$$
\begin{aligned}
(& -\frac{\hbar^2}{2m}\Delta + v_{ext}(\vec{r}))\phi_i(\vec{r},\sigma) + \sum_{j,\sigma'} \int d\vec{r'} \phi_j^*(\vec{r'},\sigma') v(\vec{r},\vec{r'}) \\
(& \phi_i(\vec{r},\sigma)\phi_j(\vec{r'},\sigma') - \phi_i(\vec{r'},\sigma')\phi_j(\vec{r},\sigma)) = \epsilon_i \phi_i(\vec{r},\sigma)
\end{aligned}
$$

One defines the one-body diagonal and non diagonal densities:

$$
\begin{aligned}
\rho(\vec{r'}) &= \sum_{j,\sigma'} |\phi_j(\vec{r'},\sigma')|^2 \\
\rho^{(1)}(\vec{r},\sigma,\vec{r'},\sigma') &= \sum_j \phi_j(\vec{r'},\sigma')\phi_j^*(\vec{r},\sigma)
\end{aligned}
$$

One rewrites the HF equations as a function of these densities:

$$
\begin{aligned}
&\left(-\frac{\hbar^2}{2m}\Delta + v_{ext}(\vec{r})\right)\phi_i(\vec{r},\sigma) + U(\vec{r})\phi_i(\vec{r},\sigma) \\
&- \sum_{\sigma}\int d\vec{r'}\rho^{(1)}(\vec{r},\sigma,\vec{r'},\sigma)v(\vec{r},\vec{r'})\phi_i(\vec{r'},\sigma) = \epsilon_i\phi_i(\vec{r},\sigma)
\end{aligned}
$$

where $$U(\vec{r}) = \int v(\vec{r}-\vec{r'})\rho(\vec{r'})d\vec{r'}$$

The first line is easy: problem in a potential.
The second line is complicate: non local exchange term.

HF single particle energy: $$\epsilon_i = t_{ii} + \sum_{j=1}^{A}(\bar{v}_{ijij})$$

2-body matrix element between i and all other j

Total energy: $$E^{HF} = \sum_{i=1}^{A}\epsilon_i - \frac{1}{2}\sum_{i,j=1}^{A}\bar{v}_{ij,ij}$$

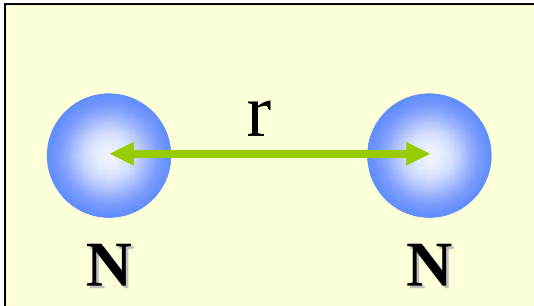
no double counting!

Mean-field Methods

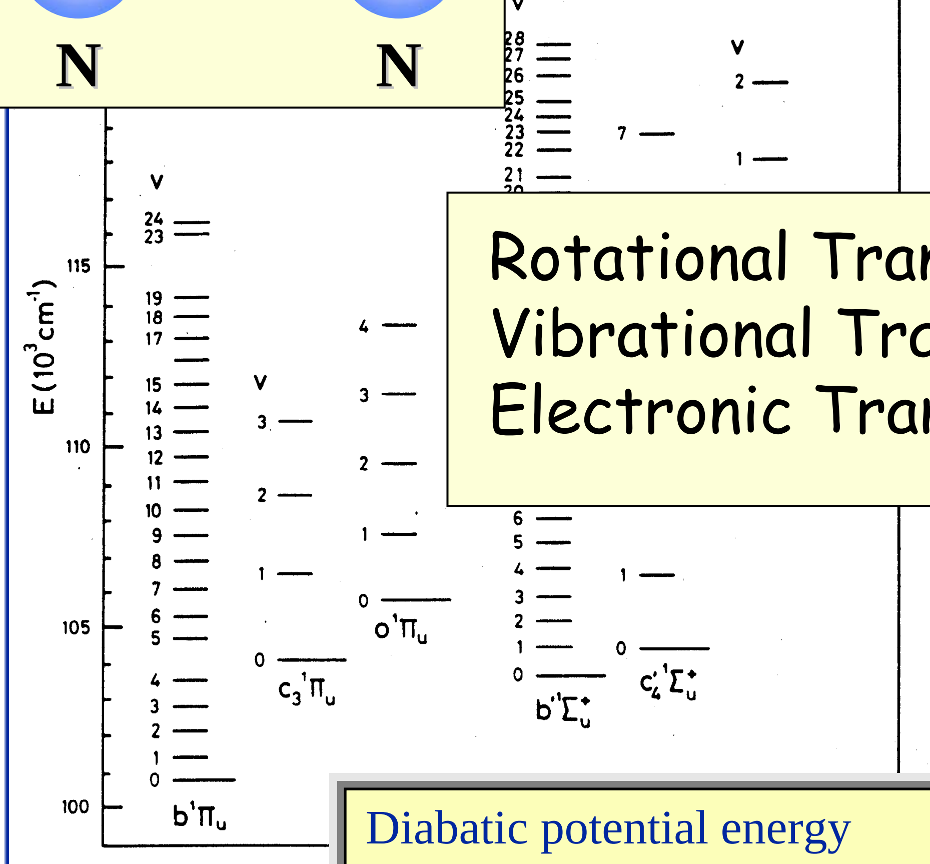
• Based on an “effective interaction” or a “density functional”
The (small number of) parameters of the effective interaction are fixed by general considerations (no local adjustments)
• Pairing correlations are included at the BCS or better HFB level

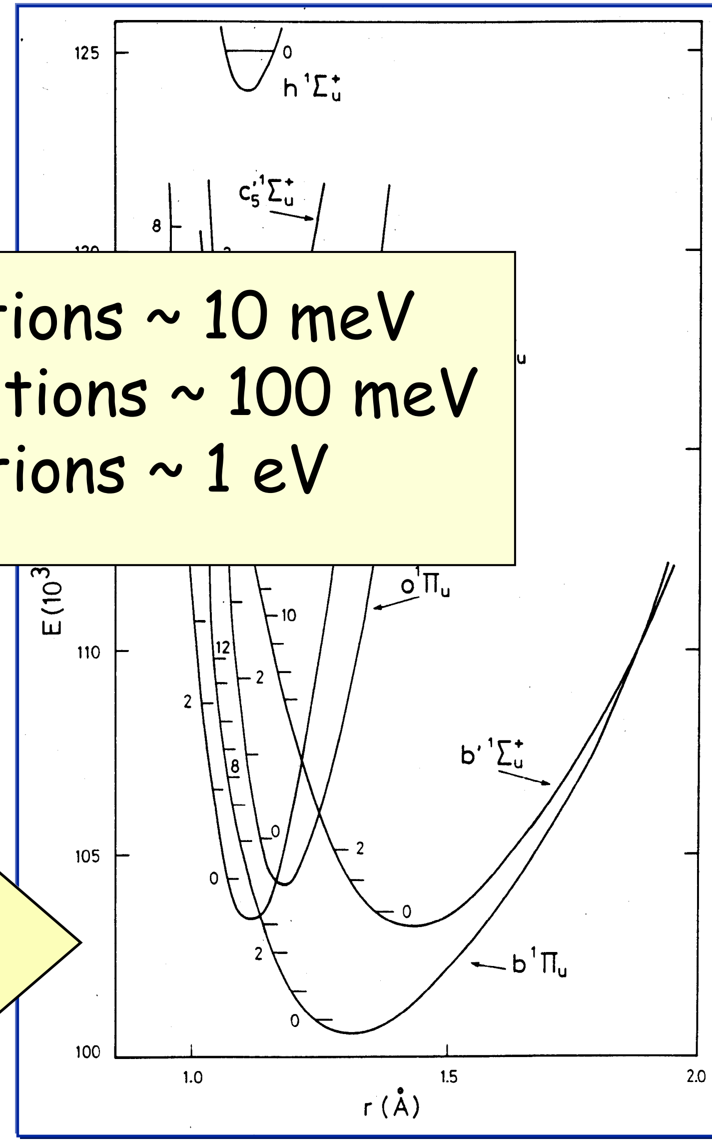
• Full self-consistency
• No restrictions to a few shells, mean-field equations are solved as precisely as one wishes.
• Spherical and deformed nuclei are treated on the same footing, no “parametric deformation”

Excitation spectrum of N_2 molecule



excited $^1\Sigma_u^+$ and $^1\Pi_u$ states





Rotational Transitions ~ 10 meV
Vibrational Transitions ~ 100 meV
Electronic Transitions ~ 1 eV

Diabatic potential energy surfaces for excited electronic configurations of N_2

Deformation of the nucleus introduced by a Lagrange multiplier:

$$H \Rightarrow H - \lambda q$$

by varying λ, one obtains solutions for different deformations

Missing ingredient: pairing correlations (superconductivity)

They can be introduced using the BCS theory:

- single particle states are occupied with a probability v^2 between 1 and 0
- nucleons are grouped in pairs of opposite spin projections

The nuclear density becomes: $$\rho(\vec{r}) = \sum_i v_i^2 \, |\varphi_i(\vec{r})|^2$$

HF equations with modified densities
+ BCS equations to determine the occupations
Total wave function with only the right mean particle number!

An example of an effective interaction: the Gogny force

It contains:

- A finite range central term:

10

$$V_C = \sum_{i=1,2} (V_W^i + V_M^i P^r + V_B^i P^\sigma + V_H^i P^\sigma P^r) exp(-r^2/b_i^2)$$

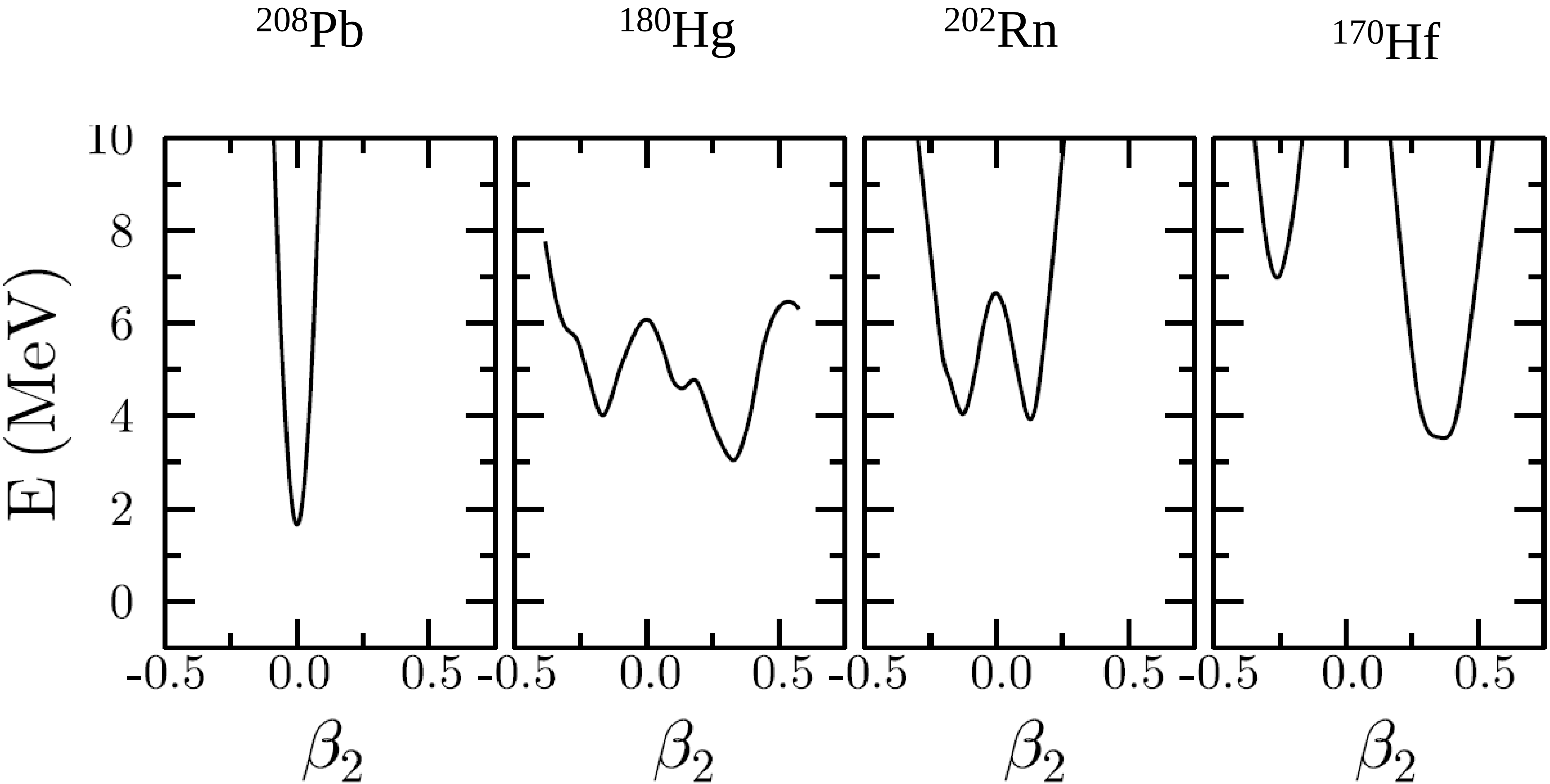
- A zero range density dependent term

1-3

$$t_3(1 + x_3 P^\tau)\rho(\overrightarrow{r})^\alpha$$

1 - Spin orbit and Coulomb

Parameters are adjusted on nuclear matter properties (saturation, …)
properties of a few magic nuclei



Mean-field energy curves (β_2 proportional to Q)

The main approaches

Three families:

- Gogny: finite range including a density dependence, same interaction for HF and pairing

(Bruyères le Chatel, Madrid, some Japanese groups)

- Skyrme: zero range, specific interaction for pairing, easy

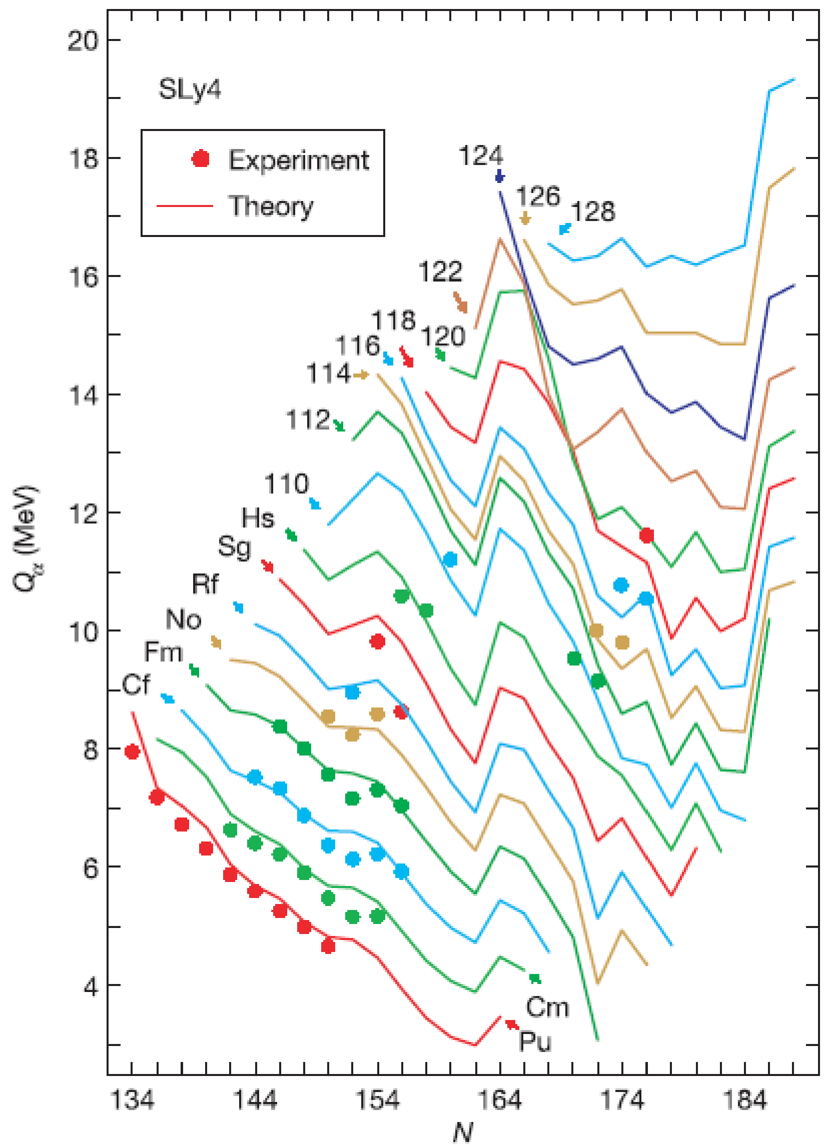
(France, Poland, Belgium, P. Rheinardt et al., Japanese groups,…)

- RMF: relativistic but no exchange, pairing non relativistic

(Munich-Zagreb, ….)

Skyrme HFB

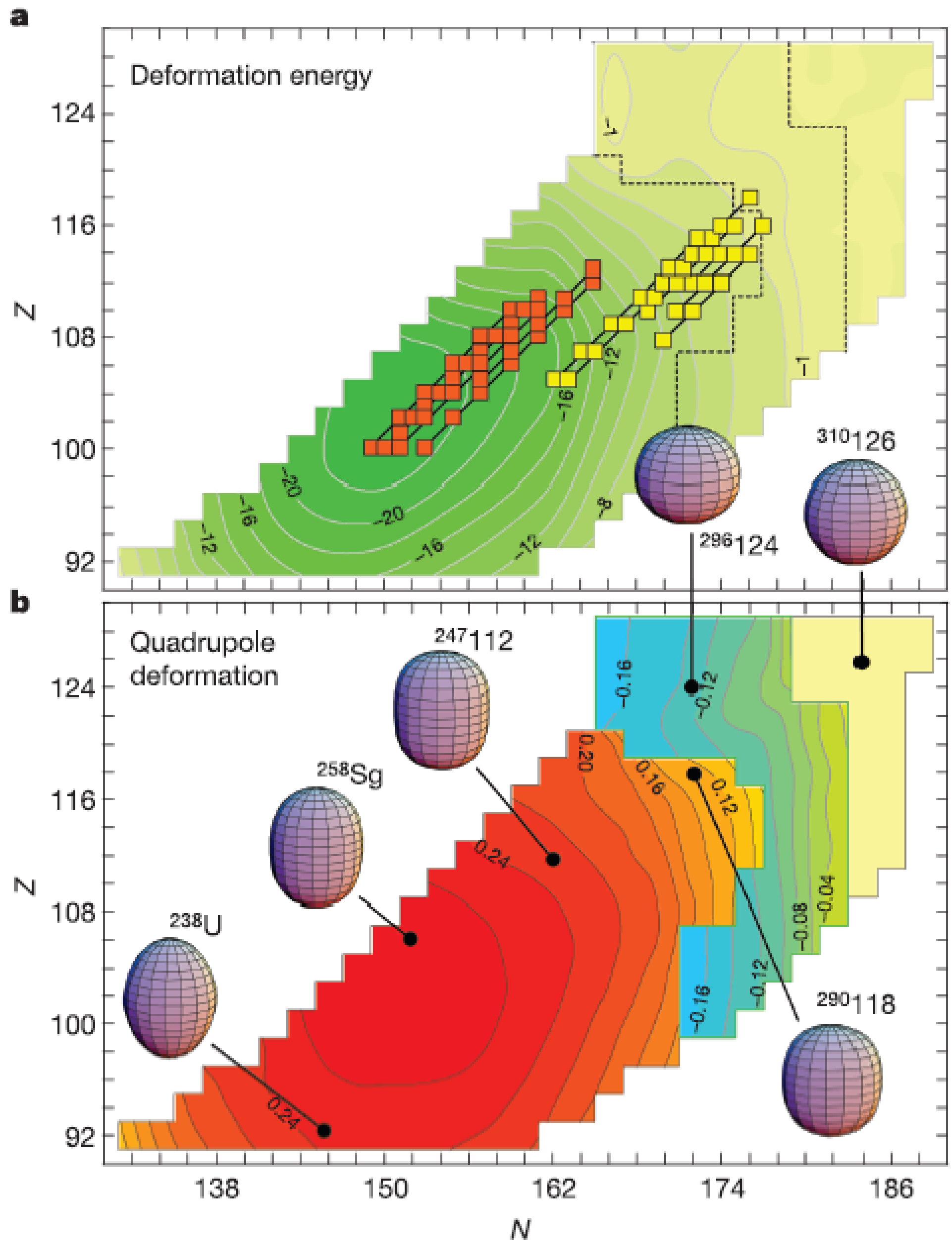
Q_α for isotopic chains for super heavy elements (only even-even)



Cwiok, Heenen, Nazarewicz
Nature 2005

Skyrme HFB

Deformation properties of super-heavies



Beyond ground state properties of even-even nuclei

Breaking of time reversal invariance

by a cranking constraint:

$H' = H - \omega J_x$ rotational bands for deformed nuclei

by quasi particle excitations:

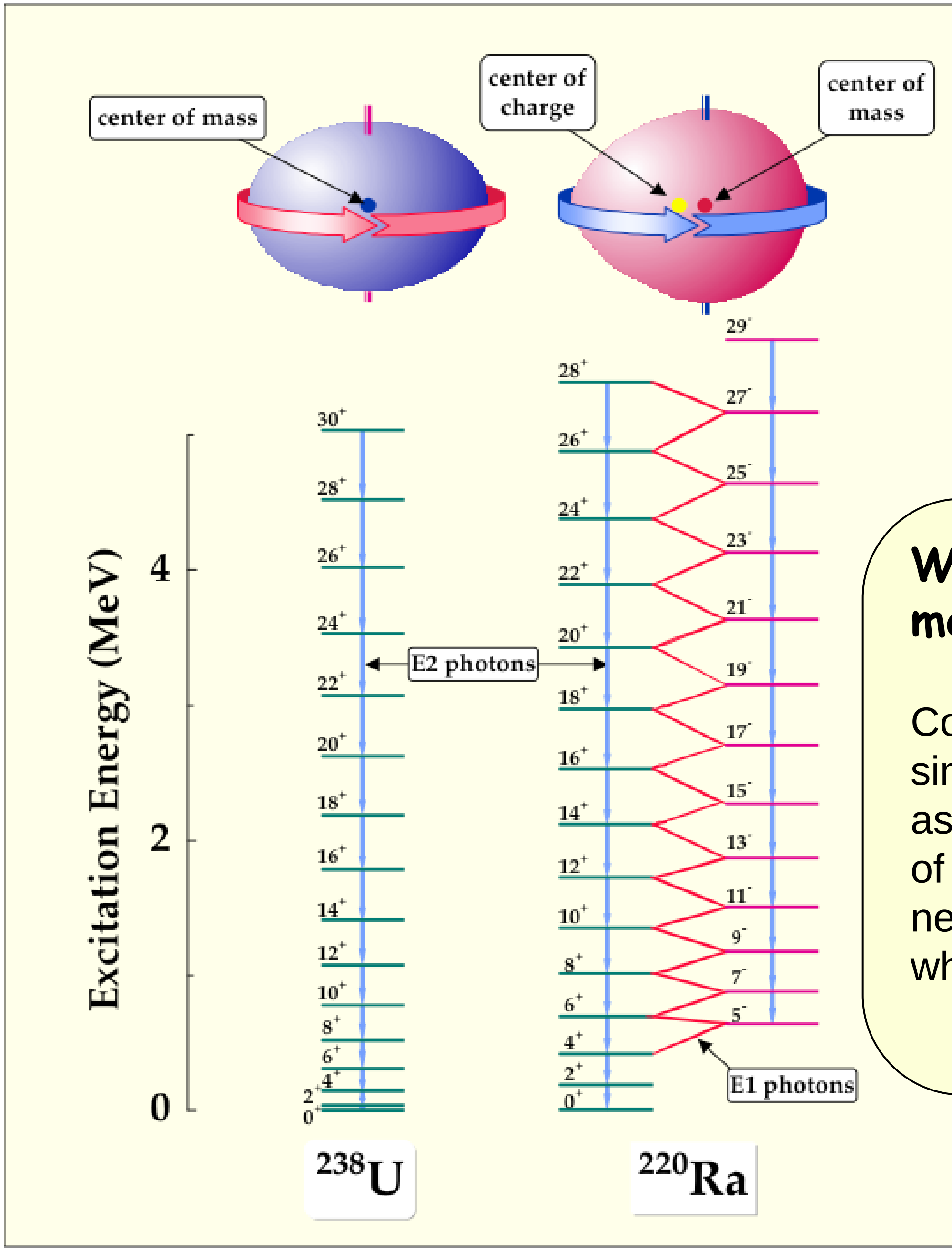
Odd nuclei : 1 qp states: $\beta_i^\dagger |0\rangle$

Even nuclei: 2qp states

Still a mean-field method

Full self-consistency for mean-field and pairing

Nuclear collective motion



Rotational Transitions ~ 0.2-2 MeV
Vibrational Transitions ~ 0.5-12 MeV
Nucleonic Transitions ~ 7 MeV

What is the origin of ordered motion of complex nuclei?

Complex systems often display astonishing simplicities. Nuclei are no exception. It is astonishing that a heavy nucleus, consisting of hundreds of rapidly moving protons and neutrons can exhibit collective motion, where all particles slowly dance in unison.

Moments of inertia

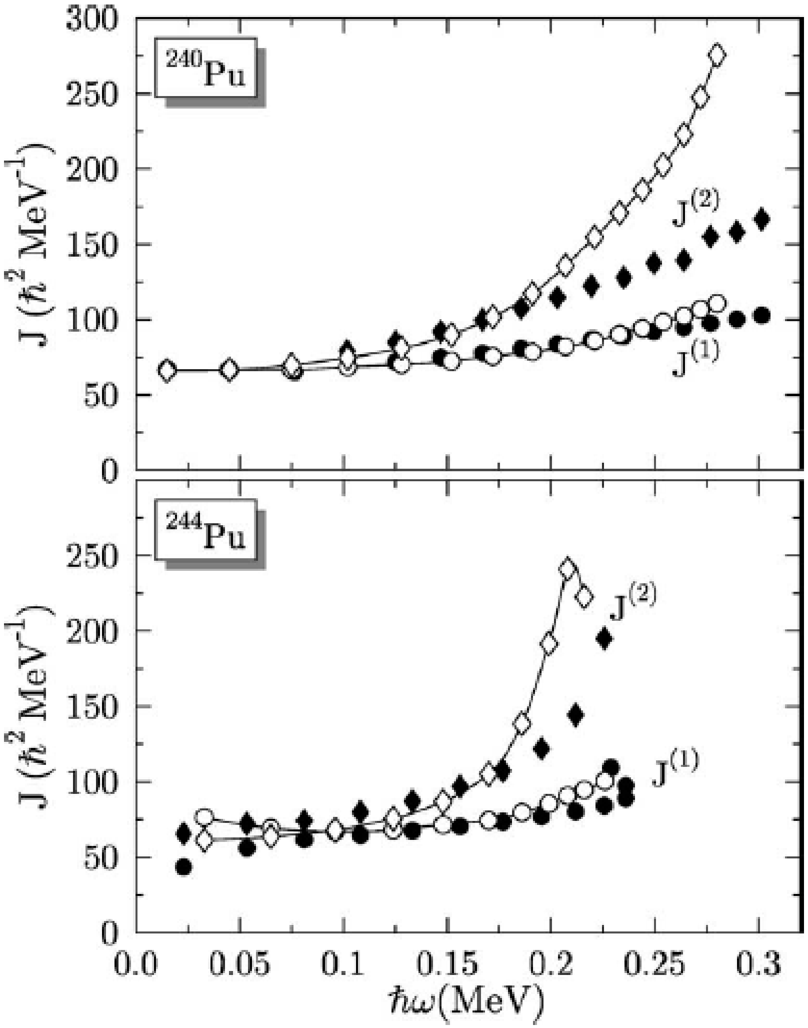
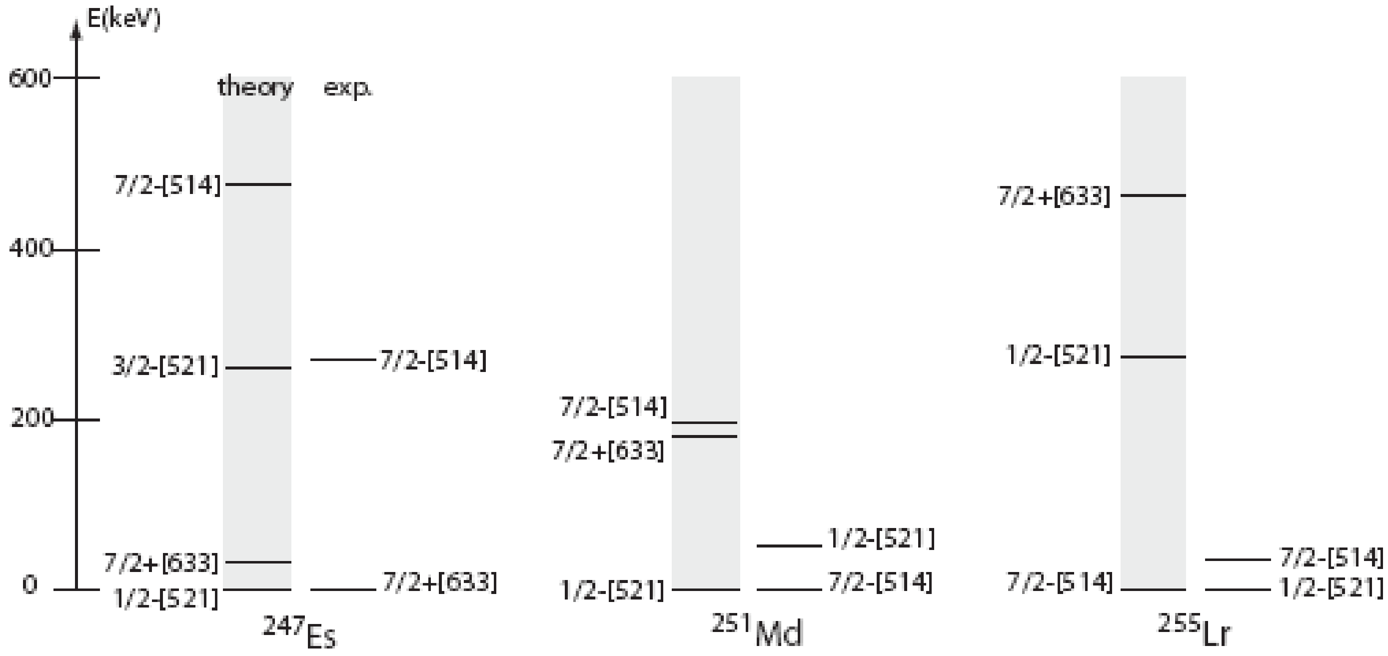


Fig. 3. Kinematical (circles) and dynamical (diamonds) moment of inertia for ^{240}Pu (top) and ^{244}Pu (bottom). Open (filled) markers denote calculated (experimental) values.

Spectra of odd Z nuclei

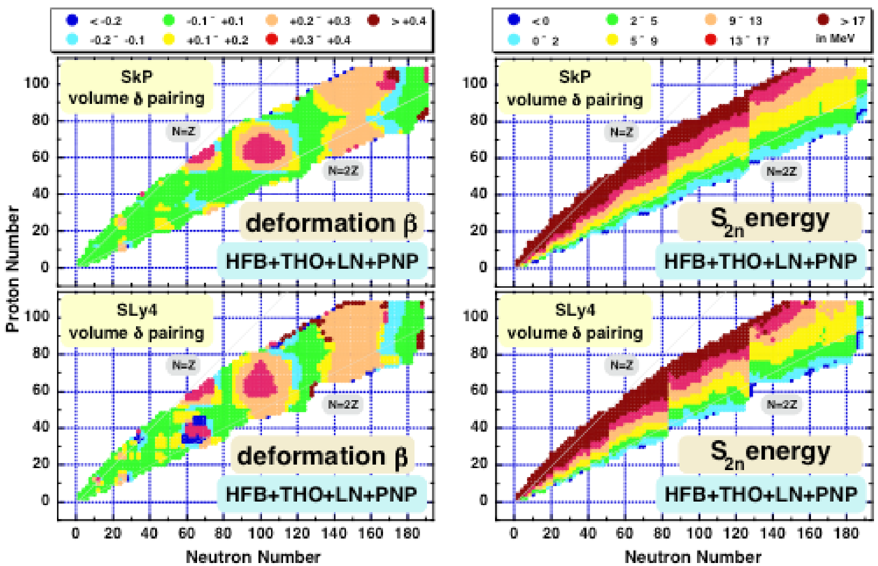


Nuclear DFT

From Qualitative to Quantitative!

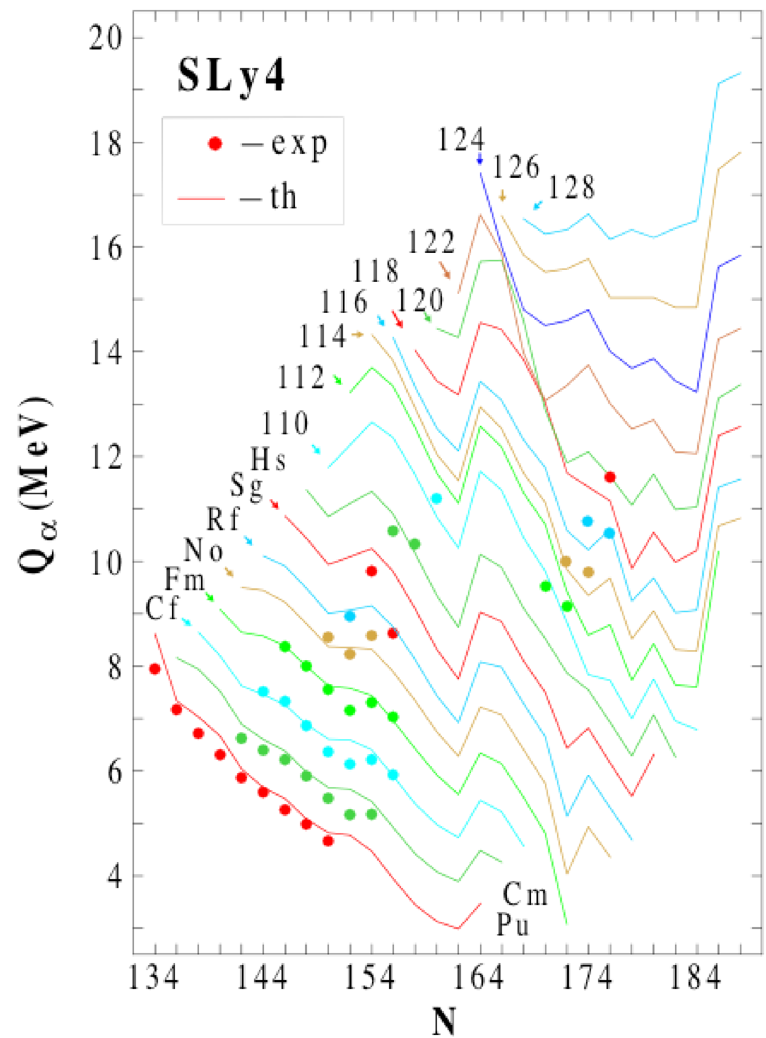
Microscopic Mass Table

M.V. Stoitsov et al., nucl-th/0406075
J. Dobaczewski et al., nucl-th/040407



Deformed Mass Table in one day!

S. Cwiok, P.H. Heenen, W. Nazarewicz



Towards Nuclear Energy Density Functional
(unified description of nuclei and nuclear matter)

- Self-consistent mean-field theory (HF, HFB, RMF)
- Nuclear density functional theory
- Symmetry breaking crucial
- Symmetry restoration essential (projection techniques, GCM, QRPA)
- Pairing channel extremely important but poorly know

Challenges:

- better understanding of isovector and density dependence of p-h and p-p interaction
  - how to extrapolate in isospin and mass?
  - time-odd fields
  - spin and isospin pieces
- improved treatment of many-body correlations
  - microscopic treatment
- nuclear matter equation of state at low and high temperatures
  - low density limit and clustering
  - isovector dependence of the symmetry energy

Energy Scales in Nuclear Physics